International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-09, Dec 2018

Nano δG-Closed Sets In Nano Topological Spaces

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Abstract - **The aim of this paper is to define and study the new class of closed sets namely Nano δ closed sets and then Using this Nano δ closed sets ,we are introduce Nano Gδ closed sets and Nano δG closed sets in Nano topological spaces and study some of their properties. Also we investigate the relationships between the other existing Nano closed sets .Further, we define and study the concept of Nano δG open sets and Nano δG closed sets in Nano topological spaces and study some of their properties.**

Keywords* — *Nano δ closed sets, Nano δG closed sets and Nano Gδ closed sets, ,Nano-δG Interior , Nano δG -Closure

2010 AMS Subject Classification:54 B05, 54C05

I. Introduction

The concept of Nano topology was introduced by Lellis Thivagar [6] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Nano generalized closed sets introduced by K. Bhuvaneswari[1] et.al., The aim of this paper is to define and study the new class of closed sets namely Nano δ closed sets in Nano topological spaces and then Using this Nano δ closed sets ,we are introduce Nano δG closed sets and Nano Gδ closed sets in Nano topological spaces and study some of their properties. Also we investigate the relationships between the other existing Nano closed sets..Further, we define and study the concept of Nano δG open sets and Nano δG closed sets in Nano topological spaces and study some of their properties.

II. Preliminaries

Definition 2.1 [6]:

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$

That is $L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$.

Where R(x) denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$$U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [6]

If(U, R) is an approximation space and $X, Y \subseteq U$, then

i) $L_R(X) \subseteq X \subseteq U_R(X)$
ii) $L_R(\phi) = U_R(\phi) = \phi$
iii) $L_R(U) = U_R(U) = U$
iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
v) $U_R(X \cap Y) \subseteq U_R(X) \cup U_R(Y)$
vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
ix) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3 [6]

Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$.

Let $\tau_R(X) = N\tau = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$.

Then $\tau_R(X)$ is a topology on U , called as the Nano topology with respect to X.

Elements of the Nano topology are known as the Nano-open sets in U and $(U, N\tau)$ is called the Nano topological space.

 DOI : 10.18231/2454-9150.2018.1165

$[N\tau]^C$ is called as the dual Nano topology of $N\tau$. Elements of $[N\tau]^C$ are called as Nano closed sets. `

Definition 2.4

Let $(U, N\tau)$ be a Nano topological space and $A \subseteq U$,then A is said to be

(i) .Nano semi-open set[6] if $A \subseteq Ncl(Nint(A))$.
(ii). Nano-regular open set [6] if $A= Nint(Ncl(A))$.
(iii).Nano Pre-open set [6] if $A \subseteq Nint(Ncl(A))$
(iv).Nano α-open set [6]if $A \subseteq Nint(Ncl(Nint(A))$
(v).The finite union of Nano regular open sets is said to be [8]Nano π-open

Definition 2.5

Let $(U, N\tau)$ be a Nano topological space and $A \subseteq U$,then A is said to be

(i).Nano g-closed [1]if Nano $cl(A) \subseteq Q$ whenever $A \subseteq Q$ and Q is Nano open.
(ii).Nano gp-closed[2] if Nano $pcl(A) \subseteq Q$whenever $A \subseteq Q$ and Q is Nano open.
(iii).Nano πgp-closed[9] if Nano $P\text{-}c1(A) \subseteq Q$ whenever $A \subseteq Q$ and Q is Nano π -open
(iv).Nano πgs-closed [10]if Nano $S\text{-}cl(A) \subseteq Q$ whenever $A \subseteq Q$ and Q is Nano π -open
(v).Nano gα-closed set [11]if Nano $\alpha cl(A) \subseteq Q$ whenever $A \subseteq Q$ and Q is Nano α open .

Definition 2.6.[6]

Nano -closure of a Subset A of U is denoted by Nano cl(A) and is defined as the intersection of all Nano closed sets containing A. Therefore

Nano $cl(A) = \cap \{F \subseteq U: A \subseteq F$ and F is Nano closed set $\}$.

Definition 2.7.[6]

Nano interior of subset A of U is denoted by Nano -(int A) and is defined as the union of all Nano -open sets contained in A .Therefore

Nano -int(A)= $\cup \{G \subseteq U: G \subseteq A$ and G is Nano open set$\}$.

III. GENERALIZED Δ CLOSED SETS IN NANO TOPOLOGICAL SPACES

In this section first we introduce Nano δ closed sets and then Using this Nano δ closed sets ,we introduce Nano Gδ closed sets and Nano δG closed sets in Nano topological spaces

Definition 3.1

Let $(U, N\tau)$ be a Nano topological space and $A \subseteq U$,then A is said to be

(i).Nano δ-closed if $A=Ncl_\delta(A)$, where

$Ncl_\delta (A)=\{x \in U:Nint(Ncl(Q)) \cap A \neq \phi, Q \in N\tau$ and $x \in Q\}$.

(ii).Nano δG-closed set if $N\delta cl(A) \subseteq Q$ whenever $A \subseteq Q$,Q is Nano open in $(U, N\tau)$.
(iii).Nano Gδ-closed set if $Ncl(A) \subseteq Q$ whenever $A \subseteq Q$, Q is Nδ-open in $(U,N\tau)$.

Example 3.2

Let $U= \{a_1, a_2, a_3,a_4\}$ with $U/R=\{\{a_1, a_2\},\{ a_3,a_4\}\}$
Let $X=\{a_1, a_2\} \subseteq U$.
Then $N\tau =\{U, \phi ,\{ a_1, a_2\}\}$.
(i).Nano δ-closed set= $\{U, \phi\}$
(ii).Nano δG-closed set=$\{U, \phi,\{a_3\},\{a_4\},\{a_1,a_3\},\{a_1, a_4\},\{a_2, a_3\},\{a_2, a_4\},\{a_3, a_4\},\{a_1, a_2,a_3\},a_1, a_3,a_4\}, \{a_1, a_2,a_4\}, \{a_2, a_3,a_4\}\}$
(iii).Nano Gδ-closed set $=\{ U, \phi,\{a_1\},\{a_2\},\{a_3\},\{a_4\}, \{a_1, a_2\},\{a_1, a_3\},\{a_1, a_4\},\{a_2, a_3\}, \{a_2, a_4\},\{a_3, a_4\}, \{a_1, a_2,a_3\},\{a_1, a_3,a_4\},\{a_1, a_2,a_4\},\{a_2, a_3,a_4\}\}$

Definition 3.3.

Nano δG-closure of a Subset A of U is denoted by Nano δG -cl(A) and is defined as the intersection of all Nano δG-closed sets containing A. Therefore

Nano δG $cl(A) = \cap \{F \subseteq U: A \subseteq F$ and F is Nano δG -closed set $\}$.

Definition 3.4.

Nano δG -interior of subset A of U is denoted by Nano δG -(int A) and is defined as the union of all Nano δG -open sets contained in A . Therefore

Nano δG -int(A)= $\cup\{G \subseteq U:G \subseteq A$ and G is Nano δG- open set$\}$.

Theorem 3.5.

Every Nδ-Closed set is Nano δG-closed set.

Proof:

Let A be a Nδ-Closed set and Q be any Nano open set containing A. Since A is Nδ-Closed, $N\delta\text{-}Cl(A)=A \subseteq Q$. Hence A is Nano δG-closed set.

The converse of the above theorem need not be true by the following example.

Example 3.6.

Let $U= \{a_1, a_2, a_3,a_4\}$ with $U/R= \{\{a_1\},\{a_3\},\{ a_2,a_4\} \}$
Let $X=\{a_1, a_2\} \subseteq U$.
Then $N\tau =\{U, \phi ,\{a_1\},\{a_2, a_4\},\{a_1, a_2,a_4\}\}$.
The set $\{a_1, a_2,a_3\}$is a Nano δG-closed set but not a Nδ-Closed set in $(U, N\tau)$.

Theorem 3.7.

Every Nano δG-closed set is Nano g-closed set.

Proof:

Let A be a Nano δG-closed set and Q be an Nano open set containing A. Since A is Nano δG-closed, $N\delta\text{-}Cl(A) \subseteq Q$, and since $Ncl(A) \subseteq N\delta\text{-}Cl(A) \subseteq Q$, A is Nano g-closed set.

The converse of the above theorem need not be true by the following example.

Theorem 3.8.

Every Nano δG-closed set is Nano Gδ-closed set.

Proof:

Let A be a Nano δG-closed set and Q be an Nano δ-open set containing A. Since every Nano δ-open set is Nano open, $N\delta Gcl(A) \subseteq Q$, and since $Ncl(A) \subseteq N\delta\text{-}Cl(A) \subseteq Q$, A is Nano Gδ -closed set.

The converse of the above theorem need not be true by the following example.

Example 3.9.

Let $U= \{a_1, a_2, a_3,a_4\}$ with $U/R=\{\{a_1, a_2\},\{ a_3,a_4\}\}$
Let $X=\{a_1, a_2\} \subseteq U$.
Then $N\tau=\{U, \phi ,\{a_1, a_2\}\}$.
Then the set $\{a_2\}$is a Nano Gδ -closed set but not a Nano δG-closed set in $(U, N\tau)$.

 DOI : 10.18231/2454-9150.2018.1165

Theorem 3.10.

Every Nano δG-closed set is Nano πgp-closed set.

Proof:

Let A be a Nano δG-closed set and Q be an Nano δ-open set containing A. Since every Nano δ-open set is Nano open, Nδ-Cl(A)⊆Q, and since Npcl(A)⊆Nδ-Cl(A)⊆Q, A is Nano πgp-closed set.

The converse of the above theorem need not be true by the following example.

Example 3.11.

Let U= {1, 2, 3} with U/R= {{1},{2,3}}

Let X={2,3}⊆ U.

Then Nτ ={U, ϕ ,{2, 3}}.

Then the set {b} is a Nano πgp closed set but not a Nano δG-closed set in (U, Nτ)..

Theorem 3.12.

Every Nano δG-closed set is Nano πgs-closed set.

Proof:

Let A be a Nano δG-closed set and Q be an Nano δ-open set containing A. Since every Nano δ-open set is Nano open, Nδ-Cl(A)⊆Q, and since Nano scl(A)⊆Nδ-Cl(A)⊆Q, A is Nano πgs-closed set.

The converse of the above theorem need not be true by the following example.

Example 3.13.

Let U= { a_1, a_2, a_3} with U/R= {{a_1},{ a_2, a_3}}

Let X={ a_1}⊆U.

Then Nτ ={U, ϕ ,{a_1}}.

Then the set { a_1} is a Nano πgs-closed set but not a Nano δG-closed set in (U, Nτ).

Theorem 3.14.

Every Nano δG-closed set is Nano gp-closed set.

Proof:

Let A be a Nano δG-closed set and Q be an Nano open set containing A. Since A is Nano δG-closed, Nδ-Cl(A)⊆Q, and since Nano pcl(A)⊆Nδ-Cl(A)⊆Q, A is Nano gp-closed set.

The converse of the above theorem need not be true by the following example.

Example 3.15.

Let U= {a_1, a_2, a_3,a_4} with U/R= {{a_1, a_2},{ a_3,a_4} }

Let X={a_1, a_2}⊆ U.

Then Nτ={U, ϕ ,{a_1, a_2}}.

Then the set {a_1} is a gp-closed but not a Nano δG-closed in (U ,Nτ)

Theorem: 3.16.

Let (U, Nτ) be a Nano topological space and A⊆U . Then

(i) .Every Nano closed set is Nano δG closed set.

(ii) .Every Nano semi closed set is Nano δG closed set.

(iii) .Every Nano pre closed set is Nano δG closed set.

(iv) .Every Nano α closed set is Nano δG closed set.

(v). Every Nano regular closed set is Nano δG closed set.

(vi). Every Nano sg closed set is Nano δG closed set.

(vii) .Every Nano gα closed set is Nano δG closed set.

Proof:

Obvious

Example 3.17.

Let U= { a_1, a_2, a_3} with U/R= {{a_2},{ a_1, a_3}}

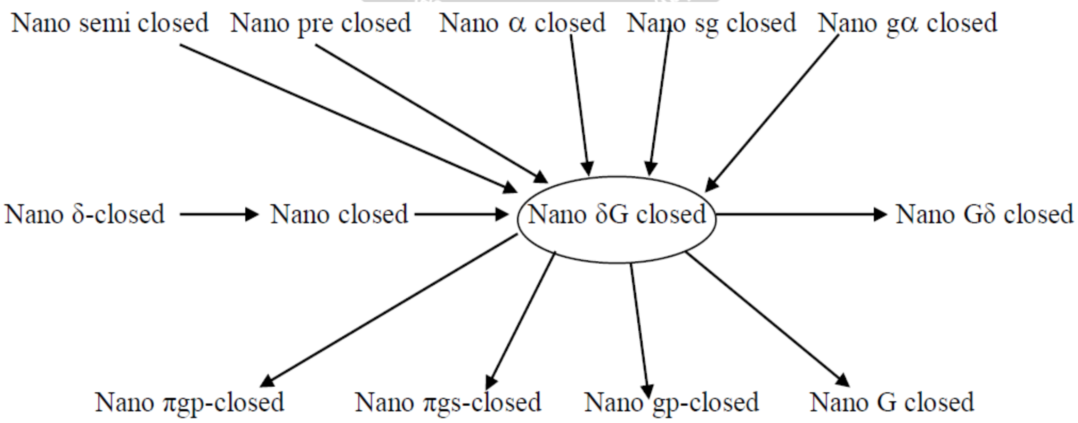
Let X={a_2}⊆ U.

Then Nτ ={U, ϕ ,{a_2}}.

Then the set ,{a_1, a_2}is a a Nano δG-closed set but not in Nano closed,Nano semi closed set, Nano pre closed set, Nano α closed set , Nano regular closed set , Nano sg closed set, Nano gα closed set (U, Nτ).

Diagram

Here the following diagram shows the relationships of Nano δG closed sets with other sets.



IV. PROPERTIES OF NANO ΔG CLOSED SETS

Theorem 4.1

The union of two Nano δG-closed subsets of U is a Nano δG-closed subset of U.

Proof:

Assume that A and B are Nano δG-closed sets in U. Let Q be Nano open in U such that A∪B⊆Q.Then A⊆Q and B⊆Q. Since A and B are Nano δG closed, Nδ-Cl(A)⊆Q and Nδ-Cl(B)⊆Q.HenceNδ-Cl(A∪B)=Nδ-Cl(A)∪Nδ-Cl(B)⊆Q. That is Nδ-Cl(A∪B)⊆Q.Therefore A∪B is an Nano δG-closed set in U.

 DOI : 10.18231/2454-9150.2018.1165

Remark 4.2

Intersection of any two Nano δG-closed sets in (U ,Nτ) need not be Nano δG- closed set.

Example 4.3

Let U= $\{a_1, a_2, a_3, a_4\}$ with U/R=$\{\{a_1, a_2\},\{a_3, a_4\}\}$

Let X=$\{a_1, a_2\}\subseteq$U.

Then Nτ={U, ϕ ,$\{a_1, a_2\}$}.

Then A=$\{a_1, a_3\}$and B= $\{a_1, a_4\}$are Nano δG-closed set but A∩B=$\{a_1\}$ is not a Nano δG-closed set in (U, Nτ).

Theorem 4.4

Let A be a Nano δG-closed set of (U ,Nτ) Then Nδ-Cl(A)\A does not contain any non-empty Nano closed set.

Proof:

Suppose that A is Nano δG-closed set. Let K be a Nano closed set contained in Nδ-Cl(A)\A.Then F^C is an Nano open set of (U ,Nτ) such that A⊆ K^C. Since A is Nano δG-closed, Nδ-Cl(A)⊆K^C.Thus K⊆(Nδ-Cl(A))^C.Also K⊆Nδ-Cl(A)\A. Therefore K⊆(Nδ-Cl(A))^C∩(Nδ-Cl(A))=ϕ. Hence K=ϕ.

Theorem 4.5

If A is an Nano open and Nano δG-closed subset of (U, Nτ), then A is a Nδ-Closed subset of
(U, Nτ).

Proof:

Since A is Nano open and Nano δG-closed, Nδ-Cl(A)⊆A. Hence A is Nδ-Closed set.

Theorem 4.6

The intersection of a Nano δG-closed set and Nδ-Closed set is a Nano δG-closed set.

Proof:

Let A be Nano δG-closed and let K be Nδ-Closed set. If Q is an Nano open set such that A∩K⊆Q, then A⊆Q∪F^C and so Nδ-Cl(A)⊆Q∪F^C.Now Nδ-Cl(A∩K)⊆Nδ-Cl(A)∩K⊆Q. Hence A∩K is Nano δG-closed set.

Theorem 4.7

If A is a Nano δG-closed set in a space (U ,Nτ) and A⊆B⊆Nδ-Cl(A), then B is also a Nano δG-closed set.

Proof:

Let Q be an Nano open set of (U ,Nτ) such that B⊆Q. Then A⊆Q. Since A is a Nano δG-closed set, Nδ-Cl(A)⊆Q. Since B⊆Nδ-Cl(A), Nδ-Cl(B)⊆Nδ-Cl(A).Therefore Nδ-Cl(B)⊆Q and hence B is Nano δG-closed set.

Theorem 4.8

Let A be a Nano δG-closed set of (U ,Nτ) Then A is Nδ-Closed if and only if Nδ-Cl(A)\A is closed set.

Proof:

Necessity: Let A be a Nδ-Closed subset of U. Then Nδ-Cl(A)=A and so Nδ-Cl(A)\A= y which is Nano closed set.

Sufficiency:

Since A is Nano δG-closed, by theorem 4.4. Nδ-Cl(A)\A does not contain any non empty Nano closed set. Therefore Nδ-Cl(A)\A= ϕ. That is Nδ-Cl(A)=A.Hence A is Nδ-Closed set.

Theorem 4.9

A subset A of (U ,Nτ) is Nano δG-open if and only if K⊆N δ-int(A) whenever K is Nano closed and K⊆A.

Proof:

Assume that A is Nano δG-open in U. Let K be Nano closed and K⊆A. This implies F^C is Nano open and A^C⊆ K^C since A^C is Nano δG-closed, Nδ-Cl(A^C)⊆K^C since Nδ-Cl(A^C)=(Nδ-int(A))^C, (Nδ-int(A))^C⊆K^C. Therefore K⊆Nδ-int (A).

Conversely assume that K⊆ Nδ-int(A) whenever K is Nano closed, and K⊆A. Let Q be a Nano open set in U containing A^C. Therefore Q^C is a Nano closed set contained in A. By hypothesis Q^C⊆Nδ-int(A),Q⊇Nδ-Cl(A^C). Therefore A^C is Nano δG-closed in U. Hence A is Nano δG-open in U.

Theorem 4.10

If A is a Nano δG-open subset of (U ,Nτ) and N δ-int(A)⊆B⊆A, then B is Nano δG-open.

Proof:

Let K⊆B and K be a Nano closed subset of U. Since A is Nano δG-open and K⊆A, K⊆N δ-int(A) and then K⊆N δ-int(B). Hence B is Nano δG-open.

Theorem 4.11

If a subset A of (U ,Nτ) is Nano δG-closed, then Nδ-Cl(A)-A is Nano δG-open.

Proof:

Let K⊆Nδ-Cl(A)\A, where K be Nano closed in U. Then by theorem 4.4, K= ϕ and so K⊆ N δ-int(Nδ-Cl(A)\A). This shows that Nδ-Cl(A)\A is Nano δG-open.

V. CONCLUSIONS

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, In this paper we defined and studied the new class of closed sets namely Nano δ closed sets in Nano topological spaces.and then Using this Nano δ closed sets ,we are introduced Nano δG closed sets and Nano Gδ closed sets in Nano topological spaces and study some of their properties. Also we investigate the relationships between the other existing Nano closed sets..Further, we define and study the concept of Nano δG open sets and Nano δG closed sets in Nano topological spaces and study some of their properties.This shall be extended in the future Research with some applications

VI. ACKNOWLEDGMENT

I wish to acknowledge friends of our institution and others those who extended their help to make this paper as successful one. I acknowledge the Editor in chief and other friends of this publication for providing the timing help to publish this paper.

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