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Some Matrix Inequalities Related to χ_s-Orthogonal Matrices

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Abstract: In this paper we introduced the concept of s-partial ordering and derived some results related to χ_s – orthogonal matrices

Key words: χ_s **-orthogonal matrices, s-orthogonal matrices, s-partial ordering.**

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I. Introduction

The secondary type matrices and results related to secondary type matrices was introduced and discussed in [1-3]. The concept of χ_s-orthogonal matrices was introduced in [4]. Let O_{χ_s} be this set of all χ_s-orthogonal matrices and O_s be this set of all s-orthogonal matrices. In this paper we introduce this concept of s-partial ordering and derived some results related to χ_s-orthogonal matrices. Also we have to discussed this same related to minus partial ordering.

II. Main Results

Definition 2.1. The s-partial order denoted by $\underset{S}{\leq}$ is a relation on $\mathbb{R}$ defined by $A\underset{S}{\leq}B$ if there exists a A^S such that $A^S A = A^S B$ and $AA^S = BA^S$.

Definition 2.2. The Minus Partial order denoted by $\overset{-}{\leq}$ is a relation $\mathbb{R}$ defined by $A\overset{-}{\leq}B$ if there exists a A^- such that $A^- A = A^- B$ and $AA^- = BA^-$.

Definition 2.3 [6]. The lowener s-partial order denoted by $\underset{s}{\leq}$ is a relation $\mathbb{R}$ defined by $A\underset{s}{\leq}B$ if there exists a B such that $A^2 = AB.$

Theorem 2.4. Let $A \in O_{\chi_s}$ and $SA \underset{S}{\leq} AS$ then A is s-orthogonal.

Proof. Let $SA\underset{S}{\leq}AS$

$$\Rightarrow (SA)^S(SA) = (SA)^S(AS)$$
$$\Rightarrow A^S S\, SA = A^S\, S\, AS$$
$$\Rightarrow SA^{-1}\, S^{-1}\, SSA = SA^{-1}\, S^{-1}\, SAS$$
$$\Rightarrow S(SA)^{-1}A = S(SA)^{-1}SAS$$
$$\Rightarrow A^S A = S(SA)(SA)^{-1}S$$
$$\Rightarrow A^S A = S\,I\,S$$
$$\Rightarrow A^S A = S^2$$
$$\Rightarrow A^S A = I \qquad (1)$$

$SA\underset{S}{\leq}AS$

$$\Rightarrow (SA)(SA)^S = (AS)(SA)^S$$
$$\Rightarrow (SA)\,A^S S = (AS)(A^S S)$$
$$\Rightarrow (SA)\,SA^{-1}S^{-1}\,S = (AS)(SA^S)$$
$$\Rightarrow (SA)\,S\,(SA)^{-1}S = AS(SA^S)$$
$$\Rightarrow (SA)(SA)^{-1}SS = A\,I\,A^S$$
$$\Rightarrow I\,SS = AA^S$$
$$\Rightarrow SS = AA^S$$
$$\Rightarrow I = AA^S$$

From (1) and (2) we have $A^S A = AA^S = I$. Therefore A is χ_s –orthogonal

Theorem 2.5. Let $A, B \in O_{\chi_s}$ and $AS = SA$, $SB = BS$ then $A\underset{T}{\leq}B \Rightarrow AS\underset{S}{\leq}BS$

Proof. $A\underset{T}{\leq}B \Rightarrow A^T A = A^T B$ and $AA^T = BA^T$. Take,

$$A^T A = A^T B$$
$$A^{-1}A = A^{-1}B$$
$$S^{-1}A^S SA = S^{-1}A^S SB$$
$$SA^S SA = SA^S SB$$
$$A^S SA = A^S SB$$

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Pre-multiply by S^S

$$S^S A^S S A = S^S A^S S B$$
$$(AS)^S (SA) = (AS)^S (SB)$$
$$(AS)^S (AS) = (AS)^S (BS)$$

Take $AA^T = BA^T$

$$AA^{-1} = BA^{-1}$$
$$A S^{-1} A^S S = B S^{-1} A^S S$$
$$AS A^S S = BS A^S S$$
$$AS A^S = BS A^S$$

Post multiply by S^S

$$AS A^S S^S = BS A^S S^S$$
$$(AS)(SA)^S = (BS)(SA)^S$$
$$(AS)(AS)^S = (BS)(AS)^S$$

From (1) and (2), we get $AS \underset{S}{\leq} BS$. Therefore

$$A \underset{T}{\leq} B \Rightarrow AS \underset{S}{\leq} BS$$

Theorem 2.6. $A \overset{-}{\leq} B$ if and only if $A^- A = A^- B$ and $AA^- = BA^-$. Let $SA \overset{-}{\leq} AS$ and $A \in O_{\chi_s}$, then s-orthogonal.

Proof. Let $SA \overset{-}{\leq} AS$

$$(SA)^{-1}(SA) = (SA)^{-1}(AS)$$

Pre-multiply by S

$$S(SA)^{-1}(SA) = S(SA)^{-1}(AS)$$
$$A^S SA = S(SA)^{-1}(SA)$$
$$A^S SA = S$$

Post multiply by S

$$A^S S AS = S^2$$
$$A^S S SA = S^2$$
$$A^S A = I$$

By Definition $SA \leq AS$

$$(SA)(SA)^{-1} = (AS)(SA)^{-1}$$

Pre multiply by A^{-1}

$$A^{-1}(SA)(SA)^{-1} = A^{-1} AS (SA)^{-1}$$
$$A^{-1}(AS)(AS)^{-1} = S(SA)^{-1}$$
$$A^{-1} A S S^{-1} A^{-1} = A^S$$
$$A^{-1} = A^S$$
$$A^{-1} A = A^S A$$
$$I = A^S A$$

Theorem 2.7. Let A and B be the orthogonal matrices and $A, B \in O_{\chi_s}$ such that $AS = SA$, $SB = BS$ then

$$A \underset{S}{\leq} B \Rightarrow AS \underset{T}{\leq} BS.$$

Proof. By Definition

$$A^S A = A^S B \qquad (1)$$
$$SA^{-1} S^{-1} A = SA^{-1} S^{-1} B$$
$$S^T A^T SA = S^T A^T S B$$
$$(AS)^T (AS) = (AS)^T (SB) \qquad (1)$$
$$AA^S = BA^S$$
$$AS A^{-1} S^{-1} = BS A^{-1} S^{-1}$$
$$AS A^T S = BS A^T S$$
$$AS A^T S^T = BS A^T S^T$$
$$(AS)(SA)^T = (BS)(SA)^T \qquad (2)$$
$$(AS)(AS)^T = (BS)(AS)^T$$

From (1) and (2) we have $A \underset{S}{\leq} B \Rightarrow AS \underset{T}{\leq} BS$.

Theorem 2.8. Let $A, B \in \mathbf{R}_{n \times n}$, $A \overset{S}{\leq} B$, if and only if $A^2 = AB$, let $A \in O_{\chi_s}$ and $SA \overset{s}{\leq} AS$, then A is s-orthogonal.

Proof.

$$A^2 = AB$$
$$(SA)^2 = (SA)(AS)$$
$$S\,AS\,A = S A^2 S$$
$$AS\,A = A^2 S$$
$$AS\,A = AAS$$
$$SA = AS$$
$$A^{-S} S = AS$$
$$A^{-S} = A$$

Theorem-2.9. For $A, B \in C_{n \times n}$ and S is the orthogonal matrix with units in the secondary diagonal.

$$A \underset{S}{\leq} B \Leftrightarrow SA \underset{S}{\leq} SB \Leftrightarrow AS \underset{S}{\leq} BS$$

Proof.

$A \underset{S}{\leq} B \Leftrightarrow A^S A = A^S B$ and $AA^S = BA^S$ (By definition)

$\Leftrightarrow A^S SSA = A^S SSB$ and $SAA^S S = SBA^S S$

$\Leftrightarrow (SA)^S SA = (SA)^S SB$ and

$$SA(SA)^S = (SB)(SA)^S$$

Therefore,

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$A \underset{S}{\leq} B \Leftrightarrow SA \underset{S}{\leq} SB$

Similarly, we can prove $A \underset{S}{\leq} B \Leftrightarrow AS \underset{S}{\leq} BS$

Hence, $A \underset{S}{\leq} B \Leftrightarrow SA \underset{S}{\leq} SB \Leftrightarrow AS \underset{S}{\leq} BS$

Theorem-2.10. Let A and B be χ_S -orthogonal and non negative definite. Then $A^2 \overset{*}{\leq} B^2$ iff $A \overset{*}{\leq} B$.

Example-2.11. Let $A = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix}$,

$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Then $A^2 \overset{*}{\leq} B^2$, but not $A \overset{*}{\leq} B$.

Corollary-2.12. Let A and B be χ_S -orthogonal matrices.

If $A \overset{*}{\leq} B$ then $AB = BA$.

Example-2.13. $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Therefore, $AB = BA$.

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