International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-09, Dec 2018

On Some Double Integral Transformation Of Aleph-Function

Yashwant Singh, Department of Mathematics, Government College, Kaladera, Jaipur (Rajasthan), India. dryashu23@yahoo.co.in

Abstract - In this paper, the author will establish a double integral transform of Aleph $(\aleph)$ -function which leads to yet another interesting process of augmenting the parameters in the Aleph $(\aleph)$ -function. The result is given of general character and on specializing the parameters suitably, yields several interesting results as special cases.

Key words: Aleph $(\aleph)$ -function, Euler Transformation, Hypergeometric Function, Integral Transformation.

(2010 Mathematics subject classification: 33C99)

I. Introduction

Many researchers like Rainville [6, p.104], Abdul Halim and Al-Salam [1] have shown that the single and double Euler transformations of the hypergeometric function are effective tools for augmenting its parameters. Srivastava and Singhal [9] and Srivastava and Joshi [10] have discussed some similar interesting properties of ${}_pF_q$ in double $\aleph$ -function and double Whittaker transforms respectively.

We have used the symbols $(a_r, A_r), \Delta(r,a), \Delta(r,\pm a), \Delta((r,a_p))$ to denote the set of parameters $(a_1, A_1),...,(a_r, A_r); \frac{a}{r}, \frac{a+1}{r},..., \frac{a+r-1}{r}; \Delta(r,a), \Delta(r,-a)$ and $\Delta(r,a_1), \Delta(r,a_2),..., \Delta(r,a_p)$ respectively.

The $\aleph$ -function introduced by Suland et.al. [11] will be represented and defined as follows:

$$\aleph[x] = \aleph^{m,n}_{p_i,q_i;\tau_i;r}[x] = \aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[x\left|\begin{matrix}(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \theta(s)ds \tag{1.1}$$

where $i = \sqrt{-1}$

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - B_j s)\prod_{j=1}^{n}\Gamma(1-a_j + A_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji} + B_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji} - A_{ji}s)\right\}} \tag{1.2}$$

$p_i, q_i (i = 1,...,r), m, n$ are integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i, (i = 1,...,r), r$ is finite A_j, B_j, A_{ij}, B_{ji} are real and a_j, b_j, a_{ji}, b_{ji} are complex numbers such that

$A_j(b_h + v) \ne B_h(a_j - v - k)$ for $v, k = 01, 2,...$

We shall use the following notations:

$A^* = (a_j, A_j)_{1,n}, [\tau_i(a_{ji}, A_{ji})]_{n+1,p_i}; B^* = (b_j, B_j)_{1,m}, [\tau_i(b_{ji}, B_{ji})]_{m+1,q_i}$

$A^{**} = (c_j, C_j)_{1,r}, [\tau_i(c_{ji}, C_{ji})]_{r+1,u_i}; B^{**} = (d_j, D_j)_{1,s}, [\tau_i(d_{ji}, D_{ji})]_{s+1,v_i}$

If we take $\tau_i = 1, r = 1$ in (1.1), the $\aleph$ - function reduces to the Fox's H-function [4].

II. Main Result

In this section, we have established the following double integral transform of $\aleph$ -function:

If s, k and r are positive integers, then

 DOI : 10.18231/2454-9150.2018.1205

ISSN : 2454-9150 Vol-04, Issue-09, Dec 2018

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^{\mu}\aleph^{f,g}_{u_i,v_i;\tau_i;r}\left[\gamma(x+y)\Big|^{C*}_{D*}\right]$$

$$\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[tx^s y^k(x+y)^r\Big|^{A*}_{B*}\right]dxdy =$$

$$(2\pi)^{(1-D)\left(f+g-\frac{1}{2}u-\frac{1}{2}v\right)+\frac{1}{2}}D^{\sum_1^v d_j-\sum_1^u c_j+\left(A-\frac{1}{2}\right)(u-v)}$$

$$\frac{s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\mu}(s+k)^{\alpha+\beta-\frac{1}{2}}}\aleph^{m+Dg,n+\rho+Df}_{p_i+\rho+Dv_i,q_i+\rho+Du_i;\tau_i;r}$$

$$\left[\frac{t\delta D^{D(v-u)}}{\gamma^D}\Big|^{\Delta((D,1-A-D_f)),\Delta(s,1-\alpha),\Delta(k,1-\beta),A*,\Delta(D,1-A-d_{f+1}),...,\Delta(D,1-A-d_u)}_{\Delta((D,1-A-C_g)),B*,\Delta(k+s,1-\alpha-\beta),\Delta(D,1-A-c_{g+1}),...,\Delta(D,1-A-c_v)}\right] \quad (2.1)$$

Where $\lambda=\frac{s^s k^k}{(s+k)^{s+k)}}, \rho=s+k, D=s+k+r, A=\alpha+\beta+\mu,$

$0\le Dg\le Du\le Dv<Du+q-p, u+v-2g\le 2f\le 2v, 0\le n\le p,$

$p+q-2n<2m\le 2q,$

$$\mathrm{Re}\left(\min\frac{d_i}{\delta_i}+D\min\frac{b_{ji}}{\beta_{ji}}\right)>\mathrm{Re}(-A)>\mathrm{Re}\left[D\left(\frac{s-\alpha}{s},\frac{k-\beta}{k},a_l\right)+C_t-D-1\right]$$

$i=1,2,...,f; j=1,2,...,m; l=1,2,...n; t=1,2,...,g; u, \mathrm{Re}(\min C_i+A)-v,$

$$\mathrm{Re}\left(\max\frac{d_j}{\delta_j}+A\right)-uD+v+\frac{1}{2}D\left(Dv-Du+1\right)>D(Dv-Du),$$

$$\mathrm{Re}\max\left(\frac{s-\alpha}{s},\frac{k-\beta}{k},a_l\right),$$

$i=1,2,...,u; j=1,2,...,v; l=1,2,...,u; |\arg\gamma|\le\left(f+g-\frac{1}{2}u-\frac{1}{2}v\right)\pi,$

$|\arg t|<\left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi, \mathrm{Re}\left(\alpha+s\frac{b_{ji}}{\beta_{ji}}\right)>0, \mathrm{Re}\left(\beta+k\frac{b_{ji}}{\beta_{ji}}\right)>0, j=1,2,...,m$

And the double integral converges.

Proof: To prove (2.1), we start with the following known result [2, p. 177]

$$\int_0^\infty\int_0^\infty\theta(x+y)x^{\alpha-1}y^{\beta-1}dxdy=B(\alpha,\beta)\int_0^\infty\theta(z)z^{\alpha+\beta-1}dz \quad (2.2)$$

Which is valid for $\mathrm{Re}(\alpha)>0$ and $\mathrm{Re}(\beta)>0$.

It is easy to prove by following the technique of reversing the order of integrations, that

$$\int_0^\infty\int_0^\infty\theta(x+y)x^{\alpha-1}y^{\beta-1}\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[tx^s y^k(x+y)^r\Big|^{A*}_{B*}\right]dxdy=$$

$$\sqrt{2\pi}\frac{s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$\int_0^\infty\phi(z)z^{\alpha+\beta-1}\aleph^{m,n+\rho}_{p_i+\rho,q_i+\rho;\tau_i;r}\left[t\delta z^D\Big|^{\Delta(s,1-\alpha),\Delta(k,1-\beta),A*}_{B*,\Delta(k+s,1-\alpha-\beta)}\right]dz \quad (2.3)$$

Where s, k and r are positive integers,

 DOI : 10.18231/2454-9150.2018.1205

$$\delta=\frac{s^s k^k}{(s+k)^{s+k}}, \rho=s+k, D=s+k+r, p+q<2(m+n),$$

$$|\arg t|<\left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi,$$

$$\mathrm{Re}\left(\alpha+s\frac{b_j}{\beta_j}\right)>0, \mathrm{Re}\left(\beta+k\frac{b_j}{\beta_j}\right)>0,\ j=1,2,...,m.$$

In (2.3), taking

$$\theta(x)=z^{\mu}\aleph_{u_i,v_i:\tau_i:r}^{f,g}\left[\gamma x\middle|_{B^{**}}^{A^{**}}\right]$$

And evaluating the integral on the right hand side using [7, p.401] the result (2.1) follows.

III. PARTICULAR CASES

On choosing the parameters suitably in (2.1), several known and unknown results are obtained as particular cases. However, we mention some of the interesting results here.

(a) Taking

$$f=v=2, g=0, u=1, c_1=\frac{1}{2}, d_1=v, d_2=-v, \mu=\sigma+\frac{1}{2}, \alpha_j=\beta_j=\delta_j=\lambda_j=1, r=1, \tau_i=1$$

in (2.1) and using [3, p.216, (5)]

$$H_{1,2}^{2,0}\left[z\middle|_{(b,1),(-b,1)}^{\left(\frac{1}{2},1\right)}\right]=\pi^{-\frac{1}{2}}e^{-\frac{1}{2}x}K_b\left(-\frac{1}{2}z\right),$$

We obtain

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^{\sigma+\frac{1}{2}}a^{-\frac{1}{2}\gamma(x+y)}K_v\left\{-\frac{1}{2}\gamma(x+y)\right\}$$

$$H_{p_i,q_i:1}^{m,n}\left[tx^s y^k(x+y)^r\middle|_{B^*}^{A^*}\right]dxdy=$$

$$(2\pi)^{-\frac{1}{2}(2-D)\left(f+g-\frac{1}{2}u-\frac{1}{2}v\right)+\frac{1}{2}}D^{A-1}\sqrt{\pi}\frac{s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\mu}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D,q+\rho+D}^{m,n+\rho+2D}\left[\frac{t\delta D^D}{\gamma^D}\middle|_{B^*,\Delta((D,\frac{1}{2}-A)),\Delta(k+s,1-\alpha-\beta)}^{\Delta((D,1-A\mp v)),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*}\right], \quad (3.1)$$

Where δ, D and γ have the same value as (2.1) and

$$A=\sigma+\alpha+\beta+\frac{1}{2}; p+q<2(m+n), \mathrm{Re}(\alpha+s\frac{b_{ji}}{\beta_{ji}}\pm v)>0, \mathrm{Re}(\beta+s\frac{b_{ji}}{\beta_{ji}}\pm v)>0,$$

$$\mathrm{Re}\left(\alpha+\beta+\mu\pm v+D\frac{b_{ji}}{\beta_{ji}}+\frac{1}{2}\right)>0,\ j=1,2,...,m; \mathrm{Re}(\lambda)>0,$$

$$|\arg t|<\left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi$$

(b) Further, replacing q, t and (a_p,α_p) by $q+1, -t$ and $(1-a_p,\alpha_p)$ respectively and then putting $m=1, n=p, b_1=1, b_{j+1}=b_j (j=1,2,...q)$, using the result [3,p. 215, (1)] and [3,p. 4, (11)], we obtain an interesting result obtained by Srivastava and Singhal [9]:

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^{\mu+\frac{1}{2}}a^{-\frac{1}{2}\lambda(x+y)}K_v\left\{-\frac{1}{2}\gamma(x+y)\right\}$$

 DOI : 10.18231/2454-9150.2018.1205

$${}_pF_q\left[\begin{smallmatrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{smallmatrix}:tx^sy^k(x+y)^r\right]dxdy=$$

$$\frac{\sqrt{\pi}\Gamma\left(\alpha+\beta+\sigma\pm v+\frac{1}{2}\right)}{\gamma^{\alpha+\beta+\sigma+\frac{1}{2}}\Gamma\left(\alpha+\beta+\sigma+1\right)}B(\alpha,\beta)$$

$${}_{p+3s+3k+2r}F_{q+2s+2k+r}\left[t\delta\left(\frac{s+k+r}{\gamma}\right)^{s+k+r}\left|\begin{smallmatrix}\Delta(s+k+r,\alpha+\beta+\sigma\pm v+\frac{1}{2},),\Delta(s,\alpha),\Delta(k,\beta),(a_p,1)\\(b_q,1),\Delta(s+k,\alpha+\beta),\Delta(k+s,\alpha+\beta+\sigma+1)\end{smallmatrix}\right.\right], \quad (3.2)$$

provided $\mathrm{Re}(\sigma+\alpha+\beta\pm v+\frac{1}{2})>0,\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0.$

(c) Setting $v=f=2,g=0,u=0,c_1=\sigma,d_1=-\frac{1}{2}-v,d_2=-\frac{1}{2}+v,r=1,\tau_i=1$ in (2.1) and using the known formula [3,p.216,(6)]

$$H_{1,2}^{2,0}\left[z\left|\begin{smallmatrix}(1-k,1)\\(\frac{1}{2}+m,1),(\frac{1}{2}-m,1)\end{smallmatrix}\right.\right]=e^{-\frac{1}{2}z}W_{k,m}(z),$$

We have

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^{\mu}e^{-\frac{1}{2}\gamma(x+y)}W_{\lambda,v}[\gamma(x+y)]$$

$$H_{p,q}^{m,n}\left[tx^sy^k(x+y)^r\left|\begin{smallmatrix}A^*\\B^*\end{smallmatrix}\right.\right]dxdy=$$

$$(2\pi)^{\frac{1}{2}(2-D)}D^{\sigma+A-\frac{1}{2}}\frac{s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\gamma}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D,q+\rho+D}^{m,n+\rho+2D}\left[\frac{t\delta D^D}{\gamma^D}\left|\begin{smallmatrix}\Delta(D,\frac{1}{2}-A\pm v),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*\\B^*,\Delta(k+s,1-\alpha-\beta),\Delta(D,\sigma-A)\end{smallmatrix}\right.\right], \quad (3.3)$$

Where

D,ρ,δ and A are given in (2.1);

$$p+q<2(m+n),|\arg t|<\left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi,\mathrm{Re}(\gamma)>0,\mathrm{Re}(\alpha+sb_j>0,$$

$$\mathrm{Re}(k+sb_j)>0,\mathrm{Re}\left(m+n+\mu+Db_j\pm v+\frac{1}{2}\right)>0,\ j=1,2,...,m.$$

(d) Further, replacing q,t and (a_p,α_p) by $q+1,-t$ and $(1-a_p,\alpha_p)$ respectively and then putting $m=1,n=p,b_1=1,b_{j+1}=b_j(j=1,2,...,q)$ and using the result [p.215,(1)], (3.3) reduces to a result due to Srivastava and Joshi [9,p.19,(2.3)]

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^{\eta}e^{-\frac{1}{2}\gamma(x+y)}W_{\mu,v}\{\gamma(x+y)\}$$

$${}_pF_q\left[\begin{smallmatrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{smallmatrix}:tx^sy^k(x+y)^r\right]dxdy=$$

$$\frac{\Gamma\left(\alpha+\beta+\eta\pm v+\frac{1}{2}\right)}{\lambda^{\alpha+\beta+\eta}\Gamma\left(\alpha+\beta+\eta-\mu+1\right)}B(\alpha,\beta)$$

 DOI : 10.18231/2454-9150.2018.1205

$$ {}_{p+3s+3k+2r}F_{q+2s+2k+r}\left[t\delta\delta' \left| \begin{matrix} \Delta(s+k+r,\alpha+\beta+\eta\pm v+\frac{1}{2},),\Delta(s,\alpha),\Delta(k,\beta),(a_p,1) \\ (b_q,1),\Delta(s+k,\alpha+\beta),\Delta(k+s+r,\alpha+\beta+\eta-\mu+1) \end{matrix} \right. \right] \quad (3.4) $$

Where

$$ \delta = \frac{s^s k^k}{(s+k)^{s+k}}, \delta' = \left(\frac{s+k+r}{\gamma} \right)^{s+k+r} $$

$\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0, \mathrm{Re}(\gamma) > 0, \mathrm{Re}\left(\alpha+\beta+\eta \pm v + \frac{1}{2} \right) > 0$ and the resulting hypergeometric series converges.

With $\mu = 0, v = \pm\frac{1}{2}$ and $\eta = -\frac{1}{2}$,(3.4) reduces to the earlier results of Jain [5] and Singh [8].

(e) Choosing

$$ f = g = u = 1, v = 2, c_1 = 1-k, d_1 = \frac{1}{2}+M, d_2 = \frac{1}{2}-M, \alpha_j = \beta_j = \delta_j = \gamma_j = 1, r = 1, \tau_i = 1 $$

in (2.1) and using the known result

$$ H_{1,2}^{1,1}\left[z \left| \begin{matrix} (1-k,1) \\ (\frac{1}{2}+m,1),(\frac{1}{2}-m,1) \end{matrix} \right. \right] = \frac{\Gamma\left(\frac{1}{2}+k+m\right)}{\Gamma(2m+1)} e^{-\frac{1}{2}z} M_{k,m}(z), $$

We obtain

$$ \int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\eta e^{-\frac{1}{2}\gamma(x+y)} M_{k,m}[\gamma(x+y)] $$

$$ H_{p_i,q_i:1}^{m,n}\left[tx^s y^k (x+y)^r \left| \begin{matrix} A* \\ B* \end{matrix} \right. \right] dxdy = $$

$$ (2\pi)^{\frac{1}{2}(2-D)} D^{k+A-\frac{1}{2}} \frac{\Gamma(2m+1)}{\Gamma\left(k+m+\frac{1}{2}\right)} \frac{s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\eta}(s+k)^{\alpha+\beta-\frac{1}{2}}} $$

$$ H_{p+\rho+2D,q+\rho+D}^{m+D,n+\rho+D}\left[\frac{t\delta D^D}{\gamma^D} \left| \begin{matrix} \Delta(D,\frac{1}{2}-A-m),\Delta(s,1-\alpha),\Delta(k,1-\beta),\Delta(D,\frac{1}{2}-A+m) \\ \Delta(k+s,1-\alpha-\beta),\Delta(D,k-A) \end{matrix} \right. \right], \quad (3.5) $$

Where

D, ρ, δ and A are given in (2.1);

$$ p+q < 2(m+n), |\arg t| < \left(m+n-\frac{1}{2}p-\frac{1}{2}q \right)\pi, \mathrm{Re}(\gamma) > 0, \mathrm{Re}(\alpha+sb_j) > 0, $$

$$ \mathrm{Re}(\beta+kb_j) > 0, \mathrm{Re}\left(\alpha+\beta+\eta+Db_j+m+\frac{1}{2} \right) > 0,\ j = 1,2,\dots,m. $$

(f) Substituting $f = 1, g = u = 0, v = 2, d_1 = \frac{1}{2}v, d_2 = -\frac{1}{2}v, r = 1, \tau_i = 1$

and using the result [3,p.216,(3)]

$$ H_{0,2}^{1,0}\left[z \left| \begin{matrix} - \\ (\frac{1}{2}\mu,1),(-\frac{1}{2}\mu,1) \end{matrix} \right. \right] = J_\mu(2\sqrt{z}), $$

(2.1) reduces to

$$ \int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\sigma J_\mu(2\sqrt{\gamma(x+y)}) H_{p,q}^{m,n}\left[tx^s y^k \left| \begin{matrix} A* \\ B* \end{matrix} \right. \right] dxdy $$

 DOI : 10.18231/2454-9150.2018.1205

$$= \sqrt{2\pi}\, \frac{D^{2A-1} s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{\gamma^{\alpha+\beta+\eta} (s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H^{m,n+\rho+D}_{p+\rho+2D,q+p}\left[t\delta\left(\frac{D}{\gamma}\right)^{D} \middle| \begin{matrix} \Delta\left(D,1-A-\frac{1}{2}\mu\right),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*,\Delta\left(D,1-A+\frac{1}{2}\mu\right) \\ B^*,\Delta(k+s,1-\alpha-\beta) \end{matrix} \right] \quad (3.6)$$

Where δ, D, ρ and A have the same values given in (2.1);

$$p+q<2(m+n), |\arg t| < \left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi, \mathrm{Re}(\gamma)>0, \mathrm{Re}(\alpha+sb_j)>0,$$

$$\mathrm{Re}(\beta+kb_j)>0, \mathrm{Re}\left(\alpha+\beta+\eta+\frac{1}{2}\mu+Db_j\right)>0,\ j=1,2,...,m;$$

$$\mathrm{Re}\left(\alpha+\beta+\eta-D+Da_i\right)<\frac{1}{4}, i=1,2,...,n.$$

In view of the numerous properties of $\aleph$-function, on specializing the parameters suitably, a large number of interesting results may be obtained as particular case.

References

[1] Abdul-Halim,N. and Al-Salam, W.A.; Double Euler transformation of certain hypergeometric functions, Duke Math. J., 30(1963), 51-62.

[2] Edwards, J.; A Treatise on Integral Calculus, Chelsea, New York (1954).

[3] Erdelyi, A. et. al. ; Higher Transcendental Functions, vol. I, McGraw-Hill, New York (1953).

[4] Fox, C.; The G and H function as symmetric Fourier kernels Trans. Amer. Math. Soc. 98 , 1961 , 395-429 .

[5] Jain, R.N.; Some double integral transformations of certain hypergeometric functions, Math. Japon, 10(1965), 17-26.

[6] Rainville, E.D.; Special Functions, Macmillan, New York (1960).

[7] Saxena, R.K.; Some theorems on generalized Laplace transforms, Proc. Nat. Inst. Sc. India, 26 A(1960), 400-413.

[8] Singh, R.P.; A note on double transformation of certain hypergeometric functions, Proc. Edinburgh. Math. Soc. (2),1491965), 221-227.

[9] Srivastava, H.M. and Singhal, J.P.; Double Meijer's transformation of certain hypergeometric functions, Proc. Camb. Phil. Soc. 64(1968).

[10] Srivastava, H.M. and Joshi, C.M.; Certain Double Whittaker transforms of generalized hypergeometric functions, The Yokohama Mathematical Journal, vol. XV, (1) (1967), 17-32.

[11] Sudland, N., Baumann, B. and Nonnenmacher, T.F.; Open problem: who knows about the Aleph($\aleph$)-functions? Frac. Calc. Appl. Annl. 1(4),(1998), 401-402.

 DOI : 10.18231/2454-9150.2018.1205