

Identity Based Encryption Schemes : A Survey

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ABSTRACT- Identity Based Encryption is an alternate to public key encryption, as it removes the necessity of digital certificates. Identity based encryption (IBE) is one of public key encryption scheme in which identity of a receiver is used as public key. Therefore, it eliminates the authentication of public key by digital certificates. In this paper we explain various IBE schemes such as IBE based on the quadratic residues, IBE based on weil pairing and Hierarchical IBE. IBE includes the applications such as give assistance to the deployment of public key encryption, public key revocation and decryption key delegation.

Keywords : Identity Based Encryption, IBE, Public Key Encryption and Hierarchical Identity Based Encryption

I. INTRODUCTION

In public key encryption if Alice wants to convey a message to Bob in an encrypted form he needs the public key of Bob. Digital certificates are used for the authenticity of that public key. Digital certificates are generated by Certificate Authorities (CAs), who are trusted and implement the process as follows: When Bob creates a key pair he presents PK_B and some valid identity to the CA. Then, CA generates a signature, under its verification key VK_{CA} which Bob appends to its public key. Here, signature is a sign of authentication to Bob's public key provided by CA [9,11]. Later, Alice can verify the signature of CA by using VK_{CA} . If it matches, he can gain the confidence that the key really belongs to Bob. However, in order to verify the signature Alice requires to know the true public key of CA [12,13]. To make it practically possible there should be less number of CAs with wide publicity of their public keys. Though, the entire process is difficult, as all new public keys should be registered with CA [5].

In Identity Based Encryption, Alice can send encrypted data to Bob, by using any one of his identity, such as email id, name, finger print etc., After receiving the encrypted data Bob generates the private key of that identity with the help of private key generator [2,3,4]. Here, no need for Alice to get the public key or certificate. Here, notice that the private key generator has the knowledge of the private keys and it should be a trusted party [5].

In 1984 Shamir has raised a problem for a public key encryption scheme where an arbitrary string can be used as a public key. Since the problem was raised many IBE schemes were proposed. Here, we want to discuss various schemes [1].

In section 2, we give basic definition of IBE scheme. We then give an overview of various proposed schemes.

Identity Based Encryption

The model for Identity Based Encryption composed of a private key generator that generates a pms_i (system parameters) and a master secret key msk_i [5].

This scheme is specified by four polynomial time probabilistic algorithms [15,16].

1. Setup(1^k)

Input : k is the security parameter

Output : System parameters pms_i , master secret key msk_i .

2. Extract(pms_i, msk_i, ID)

Input : System parameters pms_i , master secret key msk_i ,

$ID \in \{0,1\}^*$ is an Identity of the receiver

Output: sk_{ID} secret key of the identity ID

3. Encryption (pms_i, ID, x)

Input : System parameters pms_i , identity of the receiver ID , message to be encrypted $x \in M$

Output : Cipher text $y \in C$

4. Decryption (sk_{ID}, y)

Input : Secret key corresponding to that identity ID ,

cipher text $y \in C$

Output : Message $x \in M$

II. IBE BASED ON THE QUADRATIC RESIDUES

Suppose q is a prime which is greater than 2 and b is an integer then b is a quadratic residue modulo q if $b \not\equiv 0 \pmod{q}$ and the congruence $1^2 \equiv b \pmod{q}$ has a solution $1 \in \mathbb{Z}_q$. Here, the first practical IBE scheme was implemented by using quadratic residues. The security of this scheme is a dependant on the hardness of the quadratic residue problem. It consists of the following algorithms [6,2].

1. Setup()

The PKG generates a private key by using the following inputs.

- i. An RSA module $n_1 = rs$, where n_1 is a Blum integer which means r, s are two private distinct prime numbers which satisfy $r \equiv s \equiv 3 \pmod{4}$
- ii. A message space is indicated by $M = \{-1, 1\}$ and a cipher text space $C = \mathbb{Z}_n$
- iii. A secure common hash mapping $h_1: \{0, 1\}^* \rightarrow \mathbb{Z}_n$

2. Extract (pms, msk, id)

Input: parameters that are generated by Setup() and an arbitrary string ID

Output: the secret key sk_{id}

- i. Generate b such that $\left(\frac{b}{n_1}\right) = 1$ by a deterministic procedure ID.
- ii. Let $sk_{id} = b^{\frac{n_1+5-r-s}{8}} \pmod{n_1}$ such that $sk_{id}^2 = \pm b \pmod{n_1}$.

3. Encryption (pms, id, x)

Input: parameters that are generated by Setup (), ID of the sender and a message x

Output: Corresponding cipher text C

- i. Choose a random q such that $x = \left(\frac{q}{n}\right)$, where x is an arbitrary bit of x .
- ii. Let $y_1 = q + bq^{-1} \pmod{n_1}$ and $y_2 = q - bq^{-1} \pmod{n_1}$.
- iii. Send $y = \langle y_1, y_2 \rangle$ to the recipient.

4. Decryption

(sk_{id}, y)

Input : Secret key corresponding to that identity id , cipher text $y \in C$

Output : Message $x \in M$

- i. Let $\beta = y_1 + 2sk_{id}$ if $sk_{id}^2 = b$, otherwise $\beta = y_2 + 2sk_{id}$.
- ii. Return $m = \left(\frac{\beta}{n_1}\right)$

Correctness:

$$\begin{aligned} \text{Since, } \beta &= y_1 + 2sk_{id} \\ &= (q+bq^{-1})+2sk_{id} \\ &= q(1+bq^{-2}+2sk_{id}q^{-1}) \\ &= q(1+sk_{id}^2q^{-2}+2sk_{id}q^{-1}) \\ &= q(1+sk_{id}q^{-1})^2 \end{aligned}$$

$$\begin{aligned} \text{Take, } \left(\frac{\beta}{n_1}\right) &= \left(\frac{q(1+sk_{id}q^{-1})^2}{n_1}\right) \\ &= \left(\frac{q}{n_1}\right) \\ &= x \end{aligned}$$

III. IBE BASED ON THE WEIL PAIRING

This IBE scheme can be constructed from any bilinear map $e:G_1 \times G_1 \rightarrow G_2$ where G_1, G_2 are two groups of order q , as the Bilinear Diffie-Hellman problem (Though $\langle P, iP, jP, kP \rangle$ is given, it is difficult to compute $e(P, P)^{ijk}$, where P be the generator in G_1 and $I, j, k \in \mathbb{Z}_q^*$) in G_1 is

hard. One of the examples for such map is Weil pairing on the elliptic curve. In this scheme admissible bilinear map is used. The following properties are satisfied by an admissible bilinear map [5,8,9,].

- a) Bilinear: The bilinear map is a map, if $e(cR, dS) = e(R, S)^{cd}$ for all $R, S \in G_1$ and all $c, d \in \mathbb{Z}$.
- b) Non-Degenerate: The map does not send all the pairs in $G_1 \times G_1$ to the identity in G_2 .
- c) Computable: There is an efficient algorithm to generate the mapping.

This scheme composed of the following four algorithms.

1. Setup (1^l)

Input: l is the security parameter

Output: System parameters pms_i , master secret key msk_i .

- i. Run $G(1^l)$. G will generate a prime q and two groups G_1, G_2 of order q and a bilinear map which is admissible $e:G_1 \times G_2 \rightarrow G_2$
- ii. Select a random generator $g_r \in G_1$
- iii. Choose a random $r_m \in \mathbb{Z}_q^*$
- iv. $J_p = r_m g_r$
- v. Select a hash function $f_1: \{0,1\}^* \rightarrow G_1^*$, which is cryptographic
- vi. Select a hash function $f_2: G_2 \rightarrow \{0,1\}^n$ which is cryptographic for some n
- vii. Output $pms_i = \langle q, G_1, G_2, e, n, g_r, J_p, f_1, f_2 \rangle$
- viii. $msk_i = r_m$

2. Extract (pms_i, msk_i, ID)

Input : System parameters pms_i , master secret

Key $msk_i, ID \in \{0,1\}^*$ is an identity of the receiver

Output: sk_{id} secret key of the identity ID .

- i. Compute $R_{ID} = f_1(ID) \in G_1^*$
- ii. Set $e_{ID} = r_m R_{ID}$

3. Encryption (pms_i, ID, x)

Input: System parameters pms_i , Identity of the receiver ID , message to be encrypted $x \in M$

Output: Cipher text $y \in C$

- i. Compute $R_{ID} = f_1(ID) \in G_1^*$
- ii. Choose a random $t \in \mathbb{Z}_q^*$
- iii. $v_{ID} = e(R_{ID}, J_p) \in G_2$
- iv. $y = \langle tg_r, x \oplus f_2(v_{ID}^t) \rangle$

4. Decryption (sk_{ID}, y)

Input : Secret key corresponding to that identity ID , cipher text $y \in C$

Output : Message $x \in M$

- i. Let $y = \langle W, U \rangle \in C$
- ii. Compute $x = U \oplus f_2(e(e_{ID}, W))$

Correctness:

Take,

$$\begin{aligned} e(e_{ID}, W) &= e(r_m R_{ID}, tg_r) \\ &= e(R_{ID}, t)^{r_m g_r} \end{aligned}$$

$$= e(r_m R_{ID}, J_p)^t$$

$$= v_{ID}^t$$

Take, $U \oplus f_2(e(e_{ID}, W))$

$$= x \oplus v_{ID}^t \oplus f_2(e(e_{ID}, W))$$

$$= x \oplus v_{ID}^t \oplus v_{ID}^t$$

$$= x$$

IV. HIERARCHICAL IDENTITY BASED ENCRYPTION (H-IBE)

The Hierarchical IBE scheme is an expansion of IBE scheme based upon weil pairing. In H-IBE, every

user consists of an n-tuple ID in the hierarchy tree[14]. The n-tuple ID contains IDs of the user itself and its ancestors. The root is at level 0[7 8,10]. H-IBE contains following five algorithms: –

1. Root Setup()

- i. Based on l which is a security parameter l , generate a prime q using IG (BDH Parameter Generator).
- ii. Use q to generate two fields G_1 and G_2 , such that the map $e : G_1 \times G_1 \rightarrow G_2$ which is bilinear holds.
- iii. Pick an arbitrary element R_0 in G_1 , and then select a random number $T_0 \in \mathbb{Z}/q\mathbb{Z}$ as the master-key. Compute the system parameter $U_0 = T_0 R_0$.
- iv. Generate two hash functions $f_1 : \{0, 1\}^* \rightarrow G_1$, $f_2 : G_2 \rightarrow \{0, 1\}^n$.
- v. Lower-level Setup For each user $U_i \in L_i$, specify a random number $r_m \in \mathbb{Z}/q\mathbb{Z}$.

2. Extract()

- i. For each user U_i with $ID = \langle ID_1, ID_2, \dots, ID_i \rangle$, its father calculates $F_t = f_1 \langle ID_1, ID_2, \dots, ID_i \rangle \in G_1$, where r_0 is the identity of G_1 .
- ii. Return the private key $R_m = R_{m-1} + r_{m-1} F_t = \sum_{i=1}^m s_{i-1} P_i$ of U_i and parameter $U_i = r_i R_0$.

3. Encryption ()

- i. For a message x and $ID = \langle ID_1, ID_2, \dots, ID_i \rangle$, calculate: $F_i = f_1 \langle ID_1, ID_2, \dots, ID_i \rangle \in G_1$
- ii. For any $s \in \mathbb{Z}/q\mathbb{Z}$, return the cipher text: $y = \langle sR_0, sR_2, \dots, sR_i, x \oplus f_2(h^s) \rangle$, $h = e(U_0, F_i) \in G_2$

4. Decryption()

- i. For cipher text $y = \langle Y_0, Y_2, \dots, Y_i, W \rangle$ and $ID = \langle ID_1, ID_2, \dots, ID_i \rangle$,
- ii. return the message: $x = W \oplus f_2 \left(\frac{e(Y_0, R_m)}{\prod_{i=2}^m e(U_{i-1}, Y_i)} \right)$

Correctness:

Take, $\left(\frac{e(Y_0, R_m)}{\prod_{i=2}^m e(U_{i-1}, Y_i)} \right)$

$$= \left(\frac{e(sF_0, \sum_{i=1}^m r_{m-1} F_i)}{\prod_{i=2}^m e(r_{i-1} F_0, sR_i)} \right)$$

$$= e(U_0, F_1)^s$$

$$= h^s$$

Take, $W \oplus f_2 \left(\frac{e(Y_0, R_m)}{\prod_{i=2}^m e(U_{i-1}, Y_i)} \right)$

$$= x \oplus h^s \oplus f_2 \left(\frac{e(Y_0, R_m)}{\prod_{i=2}^m e(U_{i-1}, Y_i)} \right)$$

$$= x \oplus h^s \oplus h^s$$

$$= x$$

V. CONCLUSION

Here we have discussed three basic identity based encryption schemes. In addition to algorithms, this paper clearly depicts the step by step correctness of each scheme.

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