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f – Derivations of e – commutative BF_1 – algebra

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Abstract: In this article authors introduced the notion of f − derivation, (left - right) – f – derivation, (right – left) – f − derivation and regular f −derivations of an e-commutative BF_1-algebra and some related properties have been investigated.

***Keywords —** BF_1 – algebra, e-commutative BF_1 – algebra, derivation, f-derivation, left – right and right-left derivations.*

I. INTRODUCTION

The classical result of Posner [12], which states that the existence of a non zero centralizing derivation on prime ring implies that the ring has to be commutative, made a great impact on the research work related to derivations of prime and semi prime rings. This result attracted many researchers and was subsequently extended in a number of ways. The concept of reverse derivations of prime rings was introduced by Bresar and Vukman [19]. Relations between derivations and reverse derivations with examples were given by Samman and Alyamani [21]. Recently great deal of work done by many authors on commutativity and centralizing mappings on prime rings and semi prime rings in connection with derivations, skew derivations, reverse derivations, skew reverse derivations. Vukman [16], Mohammad Ashraf [20], have studied the concepts of Symmetric biderivations in prime rings and semi prime rings. Ajda Fosner [1], Faiza Shujat and Abuzaid Ansari [13], Basudeb Dhara and Faiza Shujat [6] have extended and studied the concepts of symmetric skew 3-derivations, 4-derivations and *n*-derivations. Recently Jayasubba Reddy, Vijay Kumar and Hemavati [11], has studied the concepts of symmetric skew reverse derivations.

Jun and Xin [26] have applied the concept of derivation in BCI-algebras, similar to that of derivation in rings and near rings. After this work, many articles have come up with new ideas like (left–right), (right-left)-derivations, regular derivations, *f*-derivations on BCI-algebras [14]. In 2007, Andrez Walendziak [2] introduced the notion of BF, BF_1 and BF_2 – algebras, which is generalization of B-algebras [15]. Satyanarayana and Mastan [3]- [5] extended their study on BF-algebra and found relationship between family of BF-algebras with BG-[10], BH-[25], BM-[8], BN-[9], BP-[22]- [23], QS-[24], B-[15], and G-algebra [7]. Inspired by the above works the authors introduced the notion of *f*-derivations of an e-commutative BF_1-algebra and proved related theorems, which may be a contribution to the theory of *propositional calculi*, [17]-[18].

Throughout this article, authors used the notations D: $e*(e*x)=x$, E: $x*(e*y)=y*(e*x)$, F: $y*(y*x)=x$, G: $(e*x)*(e*y)=y*x=e*(x*y)$, $(BF_1)^e$: X is an e – commutative BF_1–algebra, Δ: derivation, $(l,r)-\Delta$: (l,r)– derivation, $(r,l)-\Delta$: (r,l)– derivation and f : endomorphism on X, for all $x,y,z\in X$ and for any fixed $e\in X$.

II. PRELIMINARIES

Definition 3.1. [2, definition 2.1] The algebraic structure $(X,*,e)$ is said to be *BF - algebra*, if it satisfies the identities $(I)\ x*x=e$, $(II)\ x*e=x$, $(BF)\ e*(x*y)=y*x$, for all $x,y\in X$ and for any fixed $e\in X$.

Definition 3.2. [2, definition 2.7] A BF - algebra is called a *BF_1 - algebra*, if it obeys (BG): $(x*y)*(e*y)=x$, for all $x,y\in X$.

Definition 3.3. [2, definition 2.7] A BF – algebra is called BF_2 -algebra, if it obeys (BH): $x*y=e=y*x$ implies that $x=y$, for all $x,y\in X$.

Definition 3.4. [3, definition 2.4] Let X be a non-empty set equipped with a binary operation "$*$" and fixed element "e". Then the algebraic structure $(X,*,e)$ is said to be *e – commutative*, if it satisfies the axiom $x*(e*y)=y*(e*x)$, for all $x,y\in X$

Definition 3.5. [3, definition 2.5] The BF_1 - algebra $(X,*,e)$ is said to be *e – commutative BF_1* – algebra if it satisfies the axiom $x*(e*y)=y*(e*x)$, for all $x,y\in X$.

Proposition 3.6. [3, proposition 3.5] If $(X,*,e)$ is an e – commutative BF_1 – algebra then $(e*x)*y=(e*y)*x$, $\forall\, x,y\in X$.

Proposition 3.7. [3, proposition 3.2] Let $(X,*,e)$, for any fixed $e\in X$ be a BF_1 – algebra. Then X is an $(BF_1)^e$ if and only if $(e*x)*(e*y)=y*x=e*(x*y)$, for all $x,y\in X$.

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Definition 3.8. [3, definition 3.6] Let $(X,*,e)$ is an $(BF_1)^e$. Then the partial order $"\leq"$ is defined as $x\leq y$ if and only if $x*y=e$, for all $x,y\in X$ and $x\wedge y$ is defined as, $x\wedge y=y*(y*x)$, for all $x,y\in X$.

Definition 3.9. [5, definition 3.7] Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta: X\to X$ is said to be $(l,r)-\Delta$ of X, if it satisfies the identity $\Delta(x*y)=(\Delta(x)*y)\wedge(x*\Delta(y))$, for all $x,y\in X$.

Definition 3.10. [5, definition 3.8]Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta: X\to X$ is said to be $(r,l)-\Delta$ of X if, it satisfies the identity $\Delta(x*y)=(x*\Delta(y))\wedge(\Delta(x)*y)$ for all $x,y\in X$.

Definition 3.11. [5, definition3.9]Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta: X\to X$ is said to be a derivation of X if, it is both $(l,r)-\Delta$ and $(r,l)-\Delta$ of X.

Proposition 3.12. [10, Lemma 2.4] Cancellation Laws holds well in BG – algebra.

Proposition 3.13. [3, Lemma 4.1] Cancellation Laws holds well in an $(BF_1)^e$.

Definition 3.14. Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta_f: X\to X$ is said to be a regular f - derivation of X, if $\Delta_f(e)=e$, where e is any fixed element of X.

Definition 3.15. Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta_f: X\to X$ is said to be a (l,r)- f - Δ of X, if $\Delta_f(x*y)=(\Delta_f(x)*f(y))\wedge(f(x)*\Delta_f(y))$, $\forall x,y\in X$.

Definition 3.16. Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta_f: X\to X$ is said to be a $(r,l)-f-\Delta$ of X, if $\Delta_f(x*y)=(f(x)*\Delta_f(y))\wedge(\Delta_f(x)*f(y))$, $\forall x,y\in X$.

Definition 3.17. Let $(X,*,e)$ is an $(BF_1)^e$. A self map $\Delta_f: X\to X$ is said to be a f –derivation of X if, Δ_f is both $(l,r)-f-\Delta$ and $(r,l)-f-\Delta$ of X.

Example 3.18. Let $X=\{e,a,b,c\}$ and $*$ be the binary operation defined on X as shown below.

*	0	1	2	3
0	0	1	3	2
1	1	0	2	3
2	2	3	0	1
3	3	2	1	0

Define a map $\Delta_f: X\to X$ such that

$$\Delta_f(x)=\begin{cases}1, & if\ x=0\\ 0, & if\ x=1\\ 3, & if\ x=2\\ 2, & if\ x=3\end{cases},\quad f(x)=\begin{cases}0, & if\ x=0\\ 1, & if\ x=1\\ 2, & if\ x=2\\ 3, & if\ x=3\end{cases}$$

Then one can easily verify that Δ_f is a f - derivation of X, where $f: X\to X$ is an endomorphism on X.

Remark 3.19. From example 3.18, it is evident that Δ_f is not a regular f - derivation of X, as $\Delta_f(0)\neq f(0)$.

IV. RESULTS ON $(l,r)-f$, $(r,l)-f$ AND f- DERIVATIONS OF AN e-COMMUTAIVE BF_1-ALGEBRA

Proposition 4.1. Let $(X,*,e)$ is an $(BF_1)^e$. If $\Delta_f: X\to X$ is a $(l,r)-f-\Delta$ of X then $\Delta_f(x*y)=\Delta_f(x)*f(y)$, $\forall x,y\in X$.

Proof: Since $\Delta_f: X\to X$ is a $(l,r)-f-\Delta$ of X then,

$\Delta_f(x*y)=(\Delta_f(x)*f(y))\wedge(f(x)*\Delta_f(y))$
$=(f(x)*\Delta_f(y))*((f(x)*\Delta_f(y))*(\Delta_f(x)*f(y)))$
$=e*(((f(x)*\Delta_f(y))*(\Delta_f(x)*f(y)))*(f(x)*\Delta_f(y)))$
$=(e*((f(x)*\Delta_f(y))*(\Delta_f(x)*f(y))))*(e*(f(x)*\Delta_f(y))$
$=((\Delta_f(x)*f(y))*(f(x)*\Delta_f(y))*(e*(f(x)*\Delta_f(y)))$
$=\Delta_f(x)*f(y)$

Hence, $\Delta_f(x*y)=\Delta_f(x)*f(y)$, $\forall x,y\in X$.

Corollary 4.2. Let $(X,*,e)$ is an $(BF_1)^e$. If $\Delta_f: X\to X$ is a $(l,r)-f-\Delta$ of X then $\Delta_f(e*x)=\Delta_f(e)*f(x)$, $\forall x\in X$.

Proof: Proof is straight forward by proposition 4.1.

Proposition 4.3 Let $(X,*,e)$ is an $(BF_1)^e$. If $\Delta_f: X\to X$ is a $(r,l)-f-\Delta$ of X.then $\Delta_f(x*y)=f(x)*\Delta_f(y)$, $\forall x,y\in X$.

Proof: Proof is similar to the proof of the proposition **4.1.**

Corollary 4.3. Let $(X,*,e)$ is an $(BF_1)^e$. If $\Delta_f: X\to X$ is a $(r,l)-f-\Delta$ of X. Then $\Delta_f(e*x)=e*\Delta_f(x)$, $\forall x\in X$.

Proof: Proof is straight forward by proposition 4.3.

Remark 4.3. Let $(X,*,e)$ is an $(BF_1)^e$. If $\Delta_f: X\to X$ is a f – derivation of X then $\Delta_f(x*y)=\Delta_f(x)*f(y)=f(x)*\Delta_f(y)$, $\forall x,y\in X$.

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Proposition 4.4. Let $(X,*, e)$ is an $(BF_1)^e$. If $\Delta_f : X \to X$ is a $(r,l)-f-\Delta$ of X then $(e*f(x))*\Delta_f(y) = (e*f(y))*\Delta_f(x), \forall x,y \in X$.

Proof: Since $(X,*,e)$ is an e-commutative BF_1 – algebra then $(e*x)*y = (e*y)*x$, $\forall x,y \in X$.

$(e*f(x))*\Delta_f(y) = (f(e)*f(x))*\Delta_f(y) = f(e*x)*\Delta_f(y)$
$= \Delta_f((e*x)*y) = \Delta_f((e*y)*x) = f(e*y)*\Delta_f(x)$
$= (f(e)*f(y))*\Delta_f(x) = (e*f(y))*\Delta_f(x)$.

Hence, $(e*f(x))*\Delta_f(y) = (e*f(y))*\Delta_f(x), \forall x,y \in X$.

Proposition 4.5. Let $(X,*,e)$ is an $(BF_1)^e$ and Δ_f be the $(r,l)-f-\Delta$ of X. Then $\Delta_f(x) = e*(e*\Delta_f(x)), \forall x \in X$.

Proof: Since $\Delta_f : X \to X$ be the $(r,l)-f-\Delta$ on an e – commutative BF_1 – algebra $(X,*,e)$, then $e*(e*x) = x, \forall x \in X$. Consider, $e*(e*\Delta_f(x)) = = e*(f(e)*\Delta_f(x))$
$= e*\Delta_f(e*x)) = f(e)*\Delta_f(e*x)) = \Delta_f(e*(e*x))$ $\Delta_f(x)$.

Hence, $e*(e*\Delta_f(x)) = \Delta_f(x), \quad \forall x \in X$.

Proposition 4.6. Let $\Delta_f : X \to X$ be the $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $f(x)*(e*\Delta_f(y)) = f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Proof: Since $(X,*,e)$ is an e – commutative BF_1 – algebra, then $x*(e*y) = y*(e*x), \forall x,y \in X$,

Consider, $f(x)*(e*\Delta_f(y)) = f(x)*(f(e)*\Delta_f(y))$
$= f(x)*\Delta_f(e*y)) = \Delta_f(x*(e*y)) = \Delta_f(y*(e*x))$
$= f(y)*\Delta_f(e*x)) = f(y)*(f(e)*\Delta_f(x)) = f(y)*(e*\Delta_f(x))$.

Hence, $f(x)*(e*\Delta_f(y)) = f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Proposition 4.7. Let $\Delta_f : X \to X$ be the $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $f(x)*(e*\Delta_f(y)) = f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Proof: Since $(X,*,e)$ is an e – commutative BF_1 – algebra then $\forall x,y \in X$, $x*(e*y) = y*(e*x)$.

Consider, $f(x)*(e*\Delta_f(y)) = \Delta_f(y)*(e*f(x))$
$= \Delta_f(y)*(f(e)*f(x)) = \Delta_f(y)*f(e*x) = \Delta_f(y*(e*x))$
$= \Delta_f(x*(e*y)) = \Delta_f(x)*f(e*y)) = \Delta_f(x)*(f(e)*f(y))$
$= \Delta_f(x)*(e*f(y)) = f(y)*(e*\Delta_f(x))$.

$\therefore f(x)*(e*\Delta_f(y)) = f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Theorem 4.8. Let $\Delta_f : X \to X$ be the f derivation of an e – commutative BF_1 – algebra $(X,*,e)$. Then $f(x)*(e*\Delta_f(y)) = f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Proof: Combining the proofs of proposition 4.6 and proposition 4.7, the theorem can be proved.

Proposition 4.9. Let $(X,*,e)$ is an $(BF_1)^e$ and Δ_f be the f derivation of X. Then $(e*\Delta_f(x))*f(y) = (e*\Delta_f(y))*f(x), \forall x,y \in X$.

Proof: Since $(X,*,e)$ is an e – commutative BF_1 – algebra then $(e*x)*y = (e*x)*y$, $\forall x,y \in X$.

Consider, $(e*\Delta_f(x))*f(y) = (f(e)*\Delta_f(x))*f(y)$
$= \Delta_f(e*x))*f(y) = \Delta_f((e*x))*y) = \Delta_f((e*y))*x)$
$= \Delta_f((e*y))*x) = \Delta_f(e*y)*f(x)$
$= (f(e)*\Delta_f(y))*f(x) = (e*\Delta_f(y))*f(x)$.

Hence, $(e*\Delta_f(x))*f(y) = (e*\Delta_f(y))*f(x), \forall x,y \in X$.

Proposition 4.10. Let $(X,*,e)$ is an $(BF_1)^e$ and Δ_f be the $(l,r)-f-\Delta$ of X. Then $(\Delta_f(e)*f(x))*f(y) = (\Delta_f(e)*f(y))*f(x), \forall x,y \in X$.

Proof: Consider, $(\Delta_f(e)*f(x))*f(y) = \Delta_f(e*x)*f(y)$
$= \Delta_f((e*x)*y) = \Delta_f((e*y)*x) = \Delta_f(e*y)*f(x)$
$= (\Delta_f(e)*f(y))*f(x)$.

Hence, $(\Delta_f(e)*f(x))*f(y) = (\Delta_f(e)*f(y))*f(x), \forall x,y \in X$.

Proposition 4.11. Let $(X,*,e)$ is an $(BF_1)^e$ and Δ_f be the f derivation of X. Then $(e*\Delta_f(x))*(e*\Delta_f(y)) = \Delta_f(y)*\Delta_f(x) = e*(\Delta_f(x)*\Delta_f(y))$ if and only if $\Delta_f(x)*(e*\Delta_f(y)) = \Delta_f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Proof: Suppose that $(e*\Delta_f(x))*(e*\Delta_f(y)) = \Delta_f(y)*\Delta_f(x) = e*(\Delta_f(x)*\Delta_f(y)), \forall x,y \in X$ holds good.

Consider, $\Delta_f(x)*(e*\Delta_f(y)) = e*((e*\Delta_f(y))*\Delta_f(x)))$
$= (e*(e*\Delta_f(y)))*(e*\Delta_f(x)) = \Delta_f(y)*(e*\Delta_f(x))$

$\therefore \Delta_f(x)*(e*\Delta_f(y)) = \Delta_f(y)*(e*\Delta_f(x)), \forall x,y \in X$.

Conversely, suppose that $\Delta_f(x)*(e*\Delta_f(y)) = \Delta_f(y)*(e*\Delta_f(x)), \forall x,y \in X$, holds good.

Consider, $(e*\Delta_f(x))*(e*\Delta_f(y)) = \Delta_f(y)*(e*(e*\Delta_f(x)))$
$= \Delta_f(y)*\Delta_f(x) = e*(\Delta_f(x)*\Delta_f(y))$

$$\therefore (e*\Delta_f(x))*(e*\Delta_f(y)) = \Delta_f(y)*\Delta_f(x) = e*(\Delta_f(x)*\Delta_f(y)), \forall x,y \in X.$$

Proposition 4.12. Let $(X,*,e)$, for any fixed $e \in X$ be an e – commutative BF_1 – algebra and $\Delta_f : X \to X$ is a $(l,r)-f-\Delta$ of X. Then (1) $\Delta_f(a) = \Delta_f(e)*(e*f(a))$, $\forall a \in X$, (2) $\Delta_f(a) = f(a)*(e*\Delta_f(e)), \forall a \in X$, if X is an e - commutative BF_1 – Algebra, (3) $\Delta_f(a) = f(a)$ if $\Delta_f(e) = e$, where f is an Endomorphism on X.

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Proof:

(1) $\Delta_f(a) = \Delta_f(e*(e*a))$

$= (\Delta_f(e)* f(e*a)) \wedge (f(e)* \Delta_f(e*a))$

$= (\Delta_f(e)* f(e*a)) \wedge (e* \Delta_f(e*a))$

$= (e* \Delta_f(e*a))*((e* \Delta_f(e*a))*(\Delta_f(e)* f(e*a)))$

$= e*(((e* \Delta_f(e*a))*(\Delta_f(e)* f(e*a)))*(e* \Delta_f(e\ a)))$

$= (((\Delta_f(e)* f(e*a)))*(e* \Delta_f(e*a)))*(e*(e* \Delta_f(e*a)))$

$= \Delta_f(e)* f(e*a) = \Delta_f(e)*(f(e)* f(a))$

Hence, $\Delta_f(a) = \Delta_f(e)*(e* f(a))$ (i)

(2) From (i), $\Delta_f(a) = \Delta_f(e)*(e* f(a)) = f(a)*(e* \Delta_f(e))$

(3)From (i), $\Delta_f(a) = \Delta_f(e)*(e* f(a)) = e*(e* f(a)) = f(a)$.

Definition 4.13. Let $(X,*, e)$ is an $(BF_1)^e$. If $\Delta_f{}^1, \Delta_f{}^2$ be the two f- derivations of X then we define $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x) = \Delta_f{}^1(x) \wedge \Delta_f{}^2(x), \forall x \in X$.

Proposition 4.14. If $\Delta_f{}^1, \Delta_f{}^2$ be the two $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta_f{}^1 \wedge \Delta_f{}^2$ is also a $(l,r)-f-\Delta$ of X.

Proof: To prove that $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x*y) = (\Delta_f{}^1 \wedge \Delta_f{}^2)(x)*f(y), \forall x,y \in X$.

Consider, $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x*y) = \Delta_f{}^1(x*y) \wedge \Delta_f{}^2(x*y)$

$= (\Delta_f{}^1(x)* f(y)) \wedge (\Delta_f{}^2(x)* f(y))$

$= (\Delta_f{}^2(x)* f(y))*((\Delta_f{}^2(x)* f(y))*(\Delta_f{}^1(x)* f(y)))$

$= (\Delta_f{}^2(x)* f(y))*(e*((\Delta_f{}^1(x)* f(y)) *(\Delta_f{}^2(x)* f(y))))$

$= ((\Delta_f{}^1(x)* f(y))*(\Delta_f{}^2(x)* f(y))) *(e*(\Delta_f{}^2(x)* f(y)))$

$= \Delta_f{}^1(x)* f(y) = (\Delta_f{}^2(x)*(\Delta_f{}^2(x)* \Delta_f{}^1(x)))* f(y)$

$= (\Delta_f{}^1(x) \wedge \Delta_f{}^2(x))* f(y) = (\Delta_f{}^1 \wedge \Delta_f{}^2)(x)* f(y)$

Hence, $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x*y) = (\Delta_f{}^1 \wedge \Delta_f{}^2)(x)* f(y)$, $\forall x,y \in X$.

$\therefore \Delta_f{}^1 \wedge \Delta_f{}^2$ is also a $(l,r)-f-\Delta$ of X.

Proposition 4.15. If $\Delta_f{}^1, \Delta_f{}^2$ be the two $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta_f{}^1 \wedge \Delta_f{}^2$ is also a $(r,l)-f-\Delta$ of X.

Proof: Proof is similar to the proof of the proposition 4.14.

Theorem 4.16. If $\Delta_f{}^1$, $\Delta_f{}^2$ be the two f derivations of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta_f{}^1 \wedge \Delta_f{}^2$ is also a derivation of X.

Proof: Combining the proofs of proposition 4.14 and proposition 4.15, the theorem can be proved.

Definition 4.17. Let $(X,*,e)$ is an $(BF_1)^e$ and $\Delta_f{}^1, \Delta_f{}^2$ be two derivations of X. Then the composition mapping of $\Delta_f{}^1$ and $\Delta_f{}^2$ is denoted by $\Delta_f{}^1 o \Delta_f{}^2$ and is defined as, $(\Delta_f{}^1 o \Delta_f{}^2)(x) = \Delta_f{}^1(\Delta_f{}^2(x))$, where f is an endomorphism on X.

Proposition 4.18. Let $(X,*,e)$ is an $(BF_1)^e$ and $\Delta_f{}^1, \Delta_f{}^2$ be the two $(l,r)-f-\Delta s$ of an e – commutative BF_1 – algebra X. Then $(\Delta_f{}^1 o \Delta_f{}^2)(x*y) = (\Delta_f{}^1 o \Delta_f{}^2)(x)* f^2(y), \forall x,y \in X$, where f is an endomorphism on X.

Proof: Since $\Delta_f{}^1, \Delta_f{}^2$ be the two $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$ then $\Delta_f{}^1(x*y) = \Delta_f{}^1(x)* f(y), \forall x,y \in X$ and $\Delta_f{}^2(x*y) = \Delta_f{}^2(x)* f(y), \forall x,y \in X$.

Consider, $(\Delta_f{}^1 o \Delta_f{}^2)(x*y) = \Delta_f{}^1(\Delta_f{}^2(x*y))$

$= \Delta_f{}^1(\Delta_f{}^2(x)*f(y)) = \Delta_f{}^1(\Delta_f{}^2(x))* f(f(y))$

$= (\Delta_f{}^1 o \Delta_f{}^2)(x*y)*(fof)(y)$. Hence,

$(\Delta_f{}^1 o \Delta_f{}^2)(x*y) = (\Delta_f{}^1 o \Delta_f{}^2)(x)* f^2(y), \forall x,y \in X$.

Remark 4.19. From proposition 4.18, it is clear that $\Delta_f{}^1 o \Delta_f{}^2$ is not a $(l,r)-f-\Delta$ of X. But if $f^2(x) = f(x), \forall x \in X$, then $\Delta_f{}^1 o \Delta_f{}^2$ is a $(l,r)-f-\Delta$ of X.

Proposition 4.20. Let $\Delta_f{}^1, \Delta_f{}^2$ be the two $(r,l)-f-\Delta$ of an e–commutative BF_1–algebra $(X,*,e)$. Then $(\Delta_f{}^1 o \Delta_f{}^2)(x*y) = f^2(x)*(\Delta_f{}^1 o \Delta_f{}^2)(y), \forall x,y \in X$, where f is an endomorphism on X.

Proof: Similar to the proof of the proposition 4.18.

Remark 4.21. From proposition 4.20, it is clear that $\Delta_f{}^1 o \Delta_f{}^2$ is not a $(r,l)-f-\Delta$ of X. But if $f^2(x) = f(x), \forall x \in X$, then $\Delta_f{}^1 o \Delta_f{}^2$ is a $(r,l)-f-\Delta$ of X.

Theorem 4.22. Let $\Delta_f{}^1, \Delta_f{}^2$ be the two derivations of a BF_1 – algebra $(X,*,e)$. Then the composition mapping $\Delta_f{}^1 o \Delta_f{}^2$ is also a derivation of X, if $f^2(x) = f(x)$, $\forall x \in X$, where f is an endomorphism on X.

Proof: The proof can be easily obtained by combining the proofs of proposition 4.18 and proposition 4.20.

Definition 4.23. Let $(X,*,e)$ be an e – commutative BF_1 – algebra and Δ_f be the f– derivation of X. Define

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$\Delta^2{}_f : X \to X$ such that $\Delta^2{}_f(x) = (\Delta_f o \Delta_f)(x) = \Delta_f(\Delta_f(x)), \forall x \in X$, where f is an endomorphism on X.

Theorem4.24. Let Δ_f be the $(l,r)-f-\Delta$ of an e–commutative BF_1 – algebra $(X,*,e)$. Then $\Delta^2{}_f(x*y) = \Delta^2{}_f(x) * f^2(y), \forall x,y \in X$.

Proof: Consider, $\Delta^2{}_f(x*y) = (\Delta_f o \Delta_f)(x*y) = \Delta_f(\Delta_f(x*y)) = \Delta_f(\Delta_f(x)*f(y)) = \Delta_f(\Delta_f(x)) * f(f(y)) = (\Delta_f o \Delta_f)(x) * (fof)(y) = \Delta^2{}_f(x) * f^2(y)$.

$\therefore \Delta^2{}_f(x*y) = \Delta^2{}_f(x) * f^2(y), \forall x,y \in X$.

Remark 4.25. From proposition 4.24, it is clear that $\Delta^2{}_f$ is not a $(l,r)-f-\Delta$ of X. But if $f^2(x) = f(x), \forall x \in X$, then $\Delta^2{}_f$ is a $(l,r)-f-\Delta$ of X.

Theorem 4.26. Let Δ_f be the $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta^2{}_f(x*y) = f^2(x)*\Delta^2{}_f(y), \forall x,y \in X$, where f is an endomorphism on X.

Proof: Consider, $\Delta^2{}_f(x*y) = (\Delta_f o \Delta_f)(x*y) = \Delta_f(\Delta_f(x*y)) = \Delta_f(f(x)*\Delta_f(y)) = f(f(x))*\Delta_f(\Delta_f(y)) = f^2(x)*(\Delta_f o \Delta_f)(y) = f^2(x)*\Delta^2{}_f(y), \forall x,y \in X$.

$\therefore \Delta^2{}_f(x*y) = f^2(x)*\Delta^2{}_f(y), \forall x,y \in X$.

Remark 4.27. From proposition 4.26, it is clear that $\Delta^2{}_f$ is not a $(r,l)-f-\Delta$ of X. But if $f^2(x) = f(x), \forall x \in X$, then $\Delta^2{}_f$ is a $(r,l)-f-\Delta$ of X.

Remark 4.28. From proposition 4.24 and proposition 4.26, it is clear that $\Delta^2{}_f$ is not a derivation of X. But if $f^2(x) = f(x), \forall x \in X$, then $\Delta^2{}_f$ is a derivation of X.

Theorem 4.29. Let Δ_f be the f derivation of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta^2{}_f(x*y) = (\Delta_f of)(x) * (fo\Delta_f)(y), \forall x,y \in X$, where f is an endomorphism on X.

Proof: Consider, $\Delta^2{}_f(x*y) = (\Delta_f o \Delta_f)(x*y) = \Delta_f(\Delta_f(x*y)) = \Delta_f(f(x)*\Delta_f(y)) = \Delta_f(f(x))*f(\Delta_f(y))$

$\therefore \Delta^2{}_f(x*y) = (\Delta_f of)(x) * (fo\Delta_f)(y), \forall x,y \in X$.

V. REGULAR DERIVATIONS OF AN e – COMMUTATIVE BF$_1$-ALGEBRA

Theorem 5.1. If $\Delta_f{}^1, \Delta_f{}^2$ be the two regular $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x) = f(x), \forall x \in X$, where f is an endomorphism on X.

Proof: Since $\Delta_f{}^1, \Delta_f{}^2$ are two regular $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$ then (i) $\Delta_f{}^1(e) = e, \Delta_f{}^2(e) = e$ and (ii) $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x*y) = f(x) * (\Delta_f{}^1 \wedge \Delta_f{}^2)(y), \forall x,y \in X$.

Now, substituting y by e in (ii), $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x*e) = f(x)*(\Delta_f{}^1 \wedge \Delta_f{}^2)(e)$,

$\Rightarrow (\Delta_f{}^1 \wedge \Delta_f{}^2)(x) = f(x)*(\Delta_f{}^1(e) \wedge \Delta_f{}^2(e))$

$= f(x)*(e*(e*e)) = f(x)*e = f(x)$.

Hence, $(\Delta_f{}^1 \wedge \Delta_f{}^2)(x) = f(x), \forall x \in X$.

Proposition 5.2. Let $\Delta_f : X \to X$ be a self map of an e – commutative BF_1 – algebra $(X,*,e)$. If Δ_f is regular $(l,r)-f-\Delta$ then $\Delta_f(x) \le f(x)\ \forall x \in X$.

Proof: Since $\Delta_f : X \to X$ is regular $(l,r)-f-\Delta$ of X then from proposition 4.1, $\Delta_f(x*y) = \Delta_f(x)*f(y), \forall x,y \in X$. Consider, $\Delta_f(e) = e \Rightarrow \Delta_f(x*x) = \Delta_f(e) \Rightarrow \Delta_f(x)*f(x) = e \Rightarrow \Delta_f(x) \le f(x)$, using the definition 3.8, $\forall x \in X$.

Proposition 5.3. Let $\Delta_f : X \to X$ be a self map of an e – commutative BF_1 – algebra $(X,*,e)$. If Δ_f is regular $(r,l)-f-\Delta$ of X then $f(x) \le \Delta_f(x), \forall x \in X$.

Proof: Proof is similar to the proof of the proposition 5.2.

Theorem 5.4. Let $\Delta_f : X \to X$ be a self map of an e – commutative BF_1 – algebra $(X,*,e)$. If Δ_f is regular f – derivation of X then $\Delta_f(x) = f(x), \forall x \in X$.

Proof: Combining the proofs of the proposition 5.2 & proposition 5.3, the theorem can be proved.

Alternatively if, Δ_f is regular $(r,l)-f-\Delta$ of X then $\Delta_f(x) = \Delta_f(x*e) = f(x)*\Delta_f(e) = f(x)*e = f(x)$.

Hence, $\Delta_f(x) = f(x), \forall x \in X$.

Proposition 5.5. Let $(X,*,e)$, for any fixed $e \in X$ is an e – commutative BF_1 – algebra and $\Delta_f : X \to X$ be a $(l,r)-f-\Delta$ of X. Then $\Delta_f(x) = \Delta_f(x) \wedge f(x), \forall x \in X$ if and only if Δ_f is regular on X.

Proof: Given that $\Delta_f : X \to X$ be a $(l,r)-f-\Delta$ of X and $\Delta_f(x) = \Delta_f(x) \wedge f(x), \forall x \in X$.

Let $x = e$, then $\Delta_f(e) = \Delta_f(e) \wedge f(e) = e \wedge e = e$. Therefore, $\Delta_f(e) = e$.

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Conversely, suppose that $\Delta_f(e)=e,\ e\in X$. Consider, $\Delta_f(x)=\Delta_f(x*e)=(\Delta_f(x)*f(e))\wedge(f(x)*\Delta_f(e))$ $=(\Delta_f(x)*e)\wedge(f(x)*e)=\Delta_f(x)\wedge f(x)$.
Hence, if $\Delta_f(e)=e$, then $\Delta_f(x)=\Delta_f(x)\wedge f(x),\ \forall x\in X$.

Proposition 5.6. Let $(X,*,e)$, for any fixed $e\in X$. be an e – commutative BF_1 – algebra and $\Delta_f : X\to X$ b be a $(r,l)-f-\Delta$ of X. Then $\Delta_f(x)=f(x)\wedge\Delta_f(x)$, $\forall x\in X$ if and only if Δ_f is regular on X.
Proof: Proof is similar to the proof of proposition 5.5

Theorem 5.7. Let Δ_f be the regular f derivation of an e – commutative BF_1 – algebra$(X,*,e)$. Then $\Delta^2_f(x)=(\Delta_f of)(x),\ \forall x,y\in X$.
Proof: Since Δ_f is the f derivation of an e – commutative BF_1 – algebra X then $\Delta_f(e)=e$ and $\Delta^2_f(x*y)=(\Delta_f of)(x)*(fo\Delta_f)(y),\ \forall x,y\in X$. Now replacing y by e, $\Delta^2_f(x*e) = (\Delta_f of)(x) * (fo\Delta_f)(e) =(\Delta_f of)(x) * f(\Delta_f(e))=(\Delta_f of)(x)*f(e)=(\Delta_f of)(x)*e=\Delta_f(f(x))$
Hence, $\Delta^2_f(x)=(\Delta_f of)(x)\ \forall x\in X$.

Corollary 5.8. Let Δ_f be the regular f derivation of an e – commutative BF_1 – algebra$(X,*,e)$. Then $\Delta^n_f(x)=\Delta^{n-1}_f(f(x)),\quad n\in Z^+,\quad \forall x\in X$, where f is an endomorphism on X.

Theorem 5.9. Let Δ_f be the regular f derivation of an e – commutative BF_1 – algebra$(X,*,e)$. Then $\Delta^2_f(e)=e$, $\forall x\in X$, where f is an endomorphism on X.
Proof: Since Δ_f is the regular f derivation of X, then $\Delta_f(e)=e$ and from theorem 5.7, $\Delta^2_f(x)=(\Delta_f of)(x)$, $\forall x\in X$. Now replacing x by e, $\Delta^2_f(e) = (\Delta_f of)(e)=\Delta_f(f(e))=\Delta_f(e)=e \Rightarrow \Delta^2_f(e)=e$.

Theorem 5.10. Let $(X,*,e)$, for any fixed $e\in X$ is an e – commutative BF_1 – algebra and $\Delta_f : X\to X$ be the regular$(r,l)-f-\Delta$ of X. Then $\Delta^2_f(x)=f^2(x),\ \forall x\in X$.
Proof: Since $\Delta_f : X\to X$ is regular $(r,l)-f-\Delta$ of X, then $\Delta^2_f(x)=(\Delta_f o\Delta_f)(x)=\Delta_f(\Delta_f(x))$
$=\Delta_f(\Delta_f(x*e))=\Delta_f(f(x)*\Delta_f(e))=\Delta_f(f(x)*e)=$
$=(fof)(x)*e=f^2(x)$
Hence, $\Delta^2_f(x)=f^2(x),\ \forall x\in X$.

Corollary 5.11 Let $(X,*,e)$, for any fixed $e\in X$ is an e – commutative BF_1 – algebra and $\Delta_f : X\to X$ be the regular$(r,l)-f-\Delta$ of X. Then $\Delta^n_f(x)=f^n(x),\ \forall x\in X$.

Corollary 5.12. Let Δ_f be the regular f derivation of an e – commutative BF_1 – algebra$(X,*,e)$. Then $\Delta^n_f(e)=e$, n $\in$ Z^+ i.e. $\Delta^n_f(x)$ is also a regular f derivation of X.

Proposition 5.13. Let $\Delta_f : X\to X$ be the regular $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $e*(e*\Delta_f(e))=e$.
Proof: Since $\Delta_f : X\to X$ be the $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$, then from proposition 4.5, $e*(e*\Delta_f(x))=\Delta_f(x),\forall x\in X$. Now taking $x=e$, $e*(e*\Delta_f(e))=\Delta_f(e)=e$. Hence, $e*(e*\Delta_f(e))=e$.

Corollary 5.14. Let $(X,*,\ e)$ is an $(BF_1)^e$. If $\Delta_f : X\to X$ is a regular $(l,r)-f-\Delta$ of X then $\Delta_f(e*x)=e\ *f(x),\ \forall\ x\in X$.

Proposition 5.15. Let $\Delta_f : X\to X$ be the regular $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta_f(x)=f(x),\forall x\in X$.
Proof: Since $\Delta_f : X\to X$ is the regular $(r,l)-f-\Delta$ of an e – commutative BF_1 – algebra$(X,*,e)$, then from the proposition 4.11, $f(x)*(e*\Delta_f(y))=f(y)*(e*\Delta_f(x))$, $\forall x,y\in X$. Now taking y by e $f(x)*(e*\Delta_f(e))=f(e)*(e*\Delta_f(x))\Rightarrow f(x)*(e*e)=e*(e*\Delta_f(x))$
$\Rightarrow\Delta_f(x)=f(x)*e$ Hence, $\Delta_f(x)=f(x),\forall x\in X$.

Proposition 5.16. Let $\Delta_f : X\to X$ is the regular $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $\Delta_f(x)=f(x),\forall x\in X$, where f is an endomorphism on X.
Proof: Since $\Delta_f : X\to X$ is a $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$, then from the proposition 4.11, $\Delta_f(x)*(e*f(y))=\Delta_f(y)*(e*f(x))$, $\forall x,y\in X$. Now taking y by e, $\Delta_f(x)*(e*f(e))=\Delta_f(e)*(e*f(x))\Rightarrow\Delta_f(x)*(e*e)=e*(e*f(x))$
$\Rightarrow\Delta_f(x)*e=f(x)$. Hence, $\Delta_f(x)=f(x),\forall x\in X$.

Theorem 5.17. Let $\Delta_f : X\to X$ be the regular f - derivation of an e – commutative BF_1 – algebra$(X,*,e)$. Then $\Delta_f(x)=f(x),\forall x\in X$.

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Proof: Combining the proofs of proposition 5.15 and proposition 5.16, the theorem can be proved.

Proposition 5.18. Let $\Delta_f : X \to X$ be the regular $(l,r)-f-\Delta$ of an e – commutative BF_1 – algebra $(X,*,e)$. Then $(e*f(x))*f(y)=(e*f(y))*f(x), \forall x,y \in X$.

Proof: Consider, $(e*f(x))*f(y)=(\Delta_f(e)*f(x))*f(y)$

$=\Delta_f(e*x)*f(y)= \Delta_f((e*x)*y)= \Delta_f((e*y)*x)$

$=\Delta_f(e*y)*f(x)=(\Delta_f(e)*f(y))*f(x)=(e*f(y))*f(x)$.

Hence, $(e*f(x))*f(y)=(e*f(y))*f(x), \forall x,y \in X$.

Proposition 5.19. Let $(X,*,e)$ be an $(BF_1)^e$ and Δ_f is a regular $(r,l)-f-\Delta$ of X. Then $\forall x,y \in X$, the following are true, where f is an endomorphism on X.

(1) $e*\Delta_f(x)=e*\Delta_f(y) \Leftrightarrow \Delta_f(x)=\Delta_f(y)$.

(2) $f(x)*\Delta_f(y)=e=f(y)*\Delta_f(x) \Leftrightarrow \Delta_f(x)=\Delta_f(y)$.

(3) $e*\Delta_f(x)=\Delta_f(y) \Leftrightarrow x=e*\Delta_f(y)$.

(4) $f(y)*(f(y)*\Delta_f(x))=\Delta_f(x)$.

Proof:

(1) Let $e*\Delta_f(x)=e*\Delta_f(y)$

$\Leftrightarrow e*(e*\Delta_f(x))=e*(e*\Delta_f(y))$

$\Leftrightarrow e*(f(e)*\Delta_f(x))=e*(f(e)*\Delta_f(y))$

$\Leftrightarrow e*\Delta_f(e*x)=e*\Delta_f(e*y)$

$\Leftrightarrow f(e)*\Delta_f(e*x)=f(e)*\Delta_f(e*y)$

$\Leftrightarrow \Delta_f(e*(e*x))=\Delta_f(e*(e*y))$

$\Leftrightarrow \Delta_f(x)=\Delta_f(y) \Leftrightarrow \Delta_f(x*e)=\Delta_f(y*e)$

$\Leftrightarrow f(x)*\Delta_f(e)=f(y)*\Delta_f(e)$

$\Leftrightarrow f(x)*e=f(y)*e \Leftrightarrow f(x)=f(y)$

(2) Let $f(x)*\Delta_f(y)=e \Rightarrow f(x)*\Delta_f(y)=f(y)*\Delta_f(y)$

$\Rightarrow f(x)=f(y)$, using RCL

Again let $f(y)*\Delta_f(x)=e \Rightarrow f(y)*\Delta_f(x)=\Delta_f(x*x)$

$\Rightarrow f(y)*\Delta_f(x)=f(x)*\Delta_f(x) \Rightarrow f(x)=f(y)$, by RCL.

Also if $x=y$ then $f(x)=f(y) \Rightarrow f(x)*\Delta_f(y)$

$=f(x)*\Delta_f(x)=\Delta_f(x*x)=e$

and $f(y)*\Delta_f(x)=f(y)*\Delta_f(y)=\Delta_f(y*y)=e$.

$\therefore f(x)*\Delta_f(y)=\Delta_f(e)=f(y)*\Delta_f(x) \Leftrightarrow x=y$

(3) Let $e*\Delta_f(x)=y \Leftrightarrow e*(e*\Delta_f(x))=e*\Delta_f(y)$

$\Leftrightarrow \Delta_f(x)=e*\Delta_f(y)$.

(4) Since $(X,*,e), e\in X$, is an e – commutative BF_1 – algebra then $y*(y*x)=x, \forall x,y\in X$.

$\Rightarrow f(y)*(f(y)*\Delta_f(x))=f(y)*\Delta_f(y*x)$

$=\Delta_f(y*(y*x))=\Delta_f(x)$.

$\therefore f(y)*(f(y)*\Delta_f(x))=\Delta_f(x), \forall x,y\in X$.

VI. CONCLUSION

Using the concepts of *(l, r)* and *(r, l)*-derivations, authors developed the new concepts such as *(l, r)-f-* and *(r, l)-f-*derivations of *e*-commutative BF_1-algebra and further applied the concept of regularity to *f*-derivations, which is useful in future work to establish the concepts of generalized derivations, fuzzy derivations, fuzzy intuitionistic derivations and cubic derivations.

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