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Prime Cordial Labeling of the graph P (3n+1,4m)

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Abstract - A bijection □ from the vertex set V of a graph G to {1,2,…..|V|} is called prime cordial labeling of G if each edge uv is assigned the label 1 if gcd(□ (u),□□ (v))=1 and 0 if gcd(□ (u),□□ (v)) > 1 where the number of edges labeled with 0 and the number of edges labeled with 1 differ atmost by 1. In this paper we exhibit prime cordial labeling of a special type of graph P (a, b).

Keywords: Graph labeling, prime labeling, cordial labeling, prime cordial labeling.

Subject Classification code: 05C78

I. Introduction

Graph labeling is a strong relation between numbers and structure of graphs. Various labeling schemes have been introduced so far and explored as well by many researchers.

A dynamic survey on different graph labeling problems with an extensive bibliography can be found in J.A.Gallian [4]. The concept of cordial labeling was introduced by Cahit [3]. Sundaram et al. [7] introduced the concept of prime cordial labeling. J.Basker Babujee [1,2] worked on prime cordial labeling on graphs . G.V.Ghodasara, J.P.Jena [5] established prime cordial labeling for certain graphs. M.A.Seoud and M.A.Salim[6] determined upper bounds of prime cordial graphs.

II. Some Definitions

Definition 2.1

The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (edge labeling).

Definition 2.2

A mapping □ : V(G) →{0,1} is called binary vertex labeling of G and □ (v) is called the label of the vertex v of G under □. If for an edge e = uv, the induced edge labeling □ *: E(G) →{0,1} is given by □ *(e) = |□□ (u)-□□ (v)|then we introduce following notations.

$v_{□}$ (i) = number of vertices of G having label i under □ where i =0 or 1

$e_{□}$ (i)= number of edges of G having label i under □ *

A binary vertex labeling □ of a graph G is called a cordial labeling if } |$v_{□}$ (0)-$v_{□}$ (1) |≤1 and |$e_{□}$ (0) - $e_{□}$ (1)| ≤ 1.

Definition 2.3

A prime labeling of a graph G of order n is an injective function □:V→{1,2,…n} such that for every pair of adjacent vertices u and v, gcd(□ (u),□□ (v))=1. The graph which admits prime labeling is called a prime graph.

Definition 2.4

A bijection □ from vertex set V(G) to { 1,2,3,… □V (G)□□□ of a graph G is called a prime cordial labeling of G if for each edge e=uv ∈ E (G)

□*(e=uv)=1; if gcd(□ (u),□□ (v))=1
=0;if gcd (□ (u),□□ (v))>1

and|$e_{□}$ (0)- $e_{□}$ (1)|≤ 1, where $e_{□}$ (0) is the number of edges labeled with 0 and $e_{□}$ (1) is the number of edges labeled with 1.

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III. MAIN RESULT

Definition 3.1

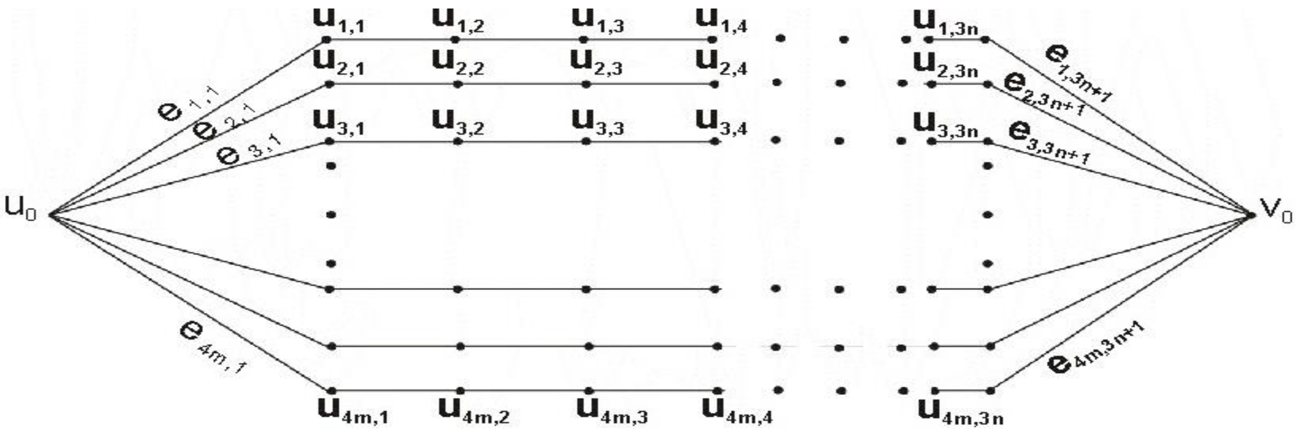
Let u and v be two fixed vertices. We connect u and v by means of “b” internally disjoint paths of length “a”. The resulting graph is denoted by P(a,b)

Theorem 3.1

The graph P (3n+1,4m) has prime cordial labeling for some specific values of n and m.

Proof

We name the vertices of the graph P (3n+1,4m) as follows:



This graph has 12mn+2 vertices and 12mn+4 edges.
Let $e_{1,1}, e_{2,1}, \ldots e_{4m,1}$ be the edges joining the vertex u_0 with $u_{1,1}, u_{2,1}, \ldots u_{4m,1}$ respectively

Similarly let $e_{1,3n+1}, e_{2,3n+1}, \ldots e_{4m,3n+1}$ be the edges joining the vertex v_0 with $u_{1,3n}, u_{2,3n}, \ldots u_{4m,3n}$ respectively.
Let $e_{i,j}$ be the edge joining $u_{i,j-1}$ and $u_{i,j}$ for i=1,2,….4m and j =1,2,…3n+1.

Case 1

Graph P (3n+1,4m) has prime cordial labeling for
n= 4,6,9,11,… and for $m \neq 2,4,7,9,\ldots$

Define □ : V(G) → {1,2,… 12mn+2} as follows:

□ (u_0) = 6n (2m-1) + 2
□ (v_0) = 6n (2m-1) + 3
□ $(u_{i,j})$ = 6n(i -1)+2 j -1
i = 1,2,… 2m-1
j = 1,2,… 3n
□ $(u_{2m,j})$ = 6n (2m-1)+2j-1 for j = 1,3,4,5…3n
□ $(u_{2m,2})$ =12 nm + 1
□ $(u_{i,j})$=6n(i -2m-1)+2 j
i = 2m+1, 2m+2 ,… 4m-1
j = 1 ,2,… 3n
□ $(u_{4m,1})$ = 12nm+2
□ $(u_{4m,j})$= 6n(2m-1)+2 j
j =2,3,… 3n

Clearly, f is one- one. It is clear that,

□*$(e_{i,j})$ =1 i = 1,2,… 2m
j =1,2,3,…3n+1
□*$(e_{i,j})$ =0 i = 2m+1,2m+2,… 4m
j =1,2,3,……,3n+1
□|e□ (0) – e□ (1)|= 0 □□1

This shows that the graph P (3n+1,4m) has a prime cordial labeling for n = 4,6,9,11,14,…
$m \neq 2,4,7,9,12,\ldots$

Example 3.2

Prime Cordial Labeling of P (13,4)

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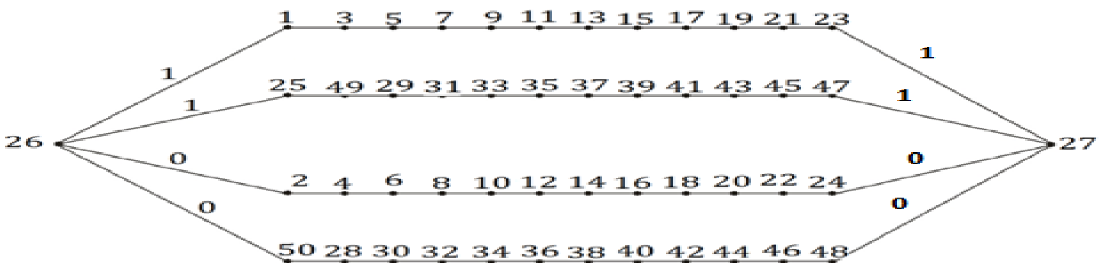


e □ (0)= 26 e□ (1) =26

Case 2

Graph P (3n+1,4m) has prime cordial labeling for
n= 2,3,7,8,12,13,… and for m≠ 5,6,10,11,15,16,…

Define □ : V(G) □{1,2,… 12mn+2} as follows:

□ (u_0) = 6n (2m-1) + 2

□ (v_0) = 6n (2m-1) + 3

□ $(u_{i,j})$ = 6n(i -1)+2j-1

i = 1,2 …2m-1

j = 1,2…3n

□ (u_{2m},j) = 6n (2m-1)+2j-1 for j= 1,3,4,5…3n

□ $(u_{2m,2})$ =12 nm + 1

□ $(u_{i,j})$=6n(i -2m-1)+2j

i = 2m+1, 2m+2,... 4m-1

j = 1 ,2,…3n

□ $(u_{4m,1})$ = 12nm+2

□ $(u_{4m,j})$= 6n(2m-1)+2 j

j =2,3,…3n

Clearly, □ is one- one. It is clear that,

□*(e $_{i,j}$)=1 i = 1,2,…2m

j =1,2,3,…3n+1

□*(e $_{i,j}$)=0 i = 2m+1,2m+2,…4m

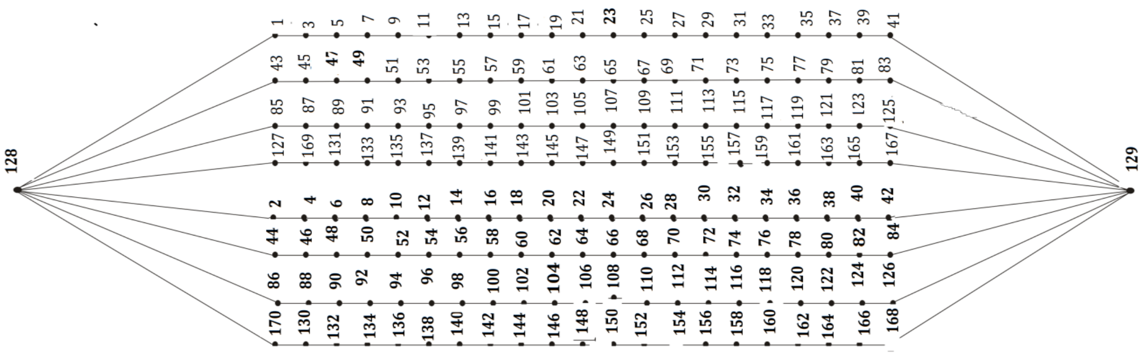
j =1,2,3,…3n+1

□|e□ (0) – e□ (1)|= 0 □□1

This shows that the graph P(3n+1,4m)has a prime cordial labeling for n = 2,3,7,8,12,13 … and for m ≠ 5,6,10,11,15,16, …

Example 3.3

Prime cordial labeling of P(22,8)



e □ (0) =88 , e □ (1)=88

Case 3

Graph P (3n+1,4m) has prime cordial labeling for n=5, 10, 15… and for all m=1, 2, 3…

Define f: V (G) □ {1, 2... 12mn+2} as follows:

□ (u_0) = 6n (2m-1) + 2

□ (v_0) = 6n (2m-1) + 3

□ $(u_{i,j})$ = 6n(i -1)+2j-1

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i = 1, 2 … 2m-1

j = 1, 2 … 3n

□ $(u_{2m,\,j}) = 6n\,(2m-1)+2\,j-1$ for j = 1,3,4,5 … 3n

□ $(u_{2m,2}) = 12\,nm + 1$

□ $(u_{i,\,j}) = 6n(i\ -2m-1)+2\,j$

i = 2m+1, 2m+2 … 4m-1

j = 1 ,2 … 3n

□ $(u_{4m,1}) = 12nm+2$

□ $(u_{4m,\,j}) = 6n(2m-1)+2\,j$

j =2,3 … 3n

Clearly, □ is one- one. It is clear that,

□ $*(e_{i,\,j}) = 1$ i = 1,2,… 2m

j =1,2,3,… 3n+1

□ $*(e_{i,\,j}) = 0$ i = 2m+1,2m+2,… 4m

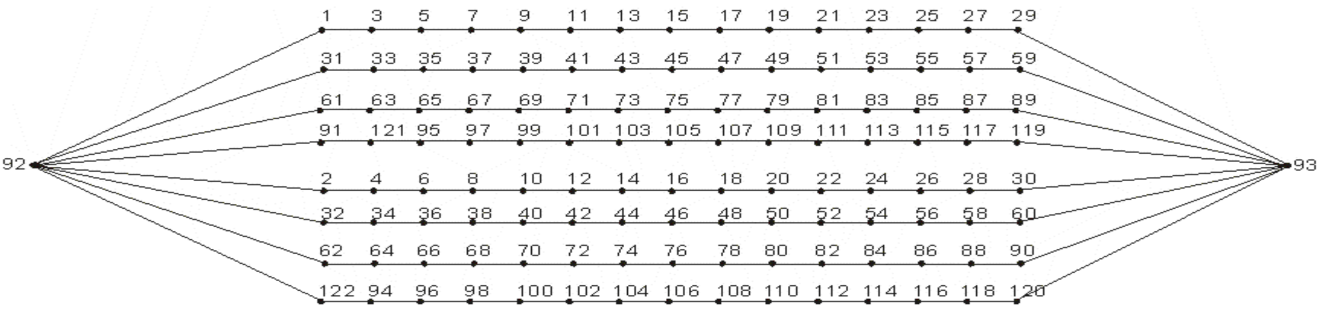
j =1,2,3,… 3n+1

□|e□ (0) – e□ (1)|= 0 □□1

This shows that the graph P(3n+1,4m)has a prime cordial labeling for n = 5,10,15,… and m = 1,2,3, …

Example 3.4

Prime cordial labeling of P (16, 8)



REFERENCE

[1] J.Baskar Babujee and S.Babitha, Prime Cordial Labeling on Graphs, International Journal of Mathematical Sciences, 7(1)(2013).

[2] J. Baskar Babujee and L. Shobana, Prime Cordial Labeling, International Review of Pure and Applied Mathematics,5(2)(2009), 277-282.

[3] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, ARS Combinatoria, 23(1987), 201-207.

[4] J.A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 19(2012),# DS6,1-260.

[5] G.V. Ghodasara , J.P. Jena, Prime Cordial Labeling of some special graph families, International Journal of Mathematics and soft computing, vol.4,No.2(2014),41-48.

[6] M.A. Seoud and M.A Salim, Two upper bounds of prime cordial graphs, Journal of Combinatorial Mathematics and Combinatorial Computing,75(2010),95-103.

[7] M. Sundaram, R. Ponraj and S. Somasundaram, Prime Cordial Labeling of graphs, Journal of Indian Academy of Mathematics, 27(2005), 373-390.

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