

# Reliability Analysis of Transport Network and Its Optimization using Meta-Heuristic Approach

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**ABSTRACT** - Most of the studies on system reliability are based on failure rate models and assumes that components of the system are independent of one another. During the design phase of a product or System, reliability and cost become two important factors in reliability engineering. In this present paper an attempt has been made to optimize the reliability of a transport system where time-to-failure of each component follows the weibull distribution. So comparing the transport system with a series-parallel system, a multi-objective optimization model of maximizing the reliability and minimizing the total initial and maintenance cost of a public transport system has been formulated here. The number of components in the system and an initial budget are the constraints of the model. The formulated problem has been solved here using multi objective genetic algorithm (MOGA) and multi objective particle swarm optimization (MOPSO). Also  $L_p$  norm strategy has been used to choose the best non-dominated solution from pareto optimal front. Finally a comparison has been given between the optimal designs of the transport system for different cases of shape parameters of weibull distribution.

**Keywords:** Reliability, Series-Parallel System, Weibull Distribution, Multi-Objective Genetic Algorithm (MOGA), Multi-Objective Particle Swarm Optimization (MOPSO).

## I. INTRODUCTION

From the very inception of history, human sensitivity has revealed an urge for mobility leading to a measure of society's progress. For a country to develop with right momentum, modern and efficient transport system is a basic need.

Transportation system is a network system that is composed of roadways, railways, waterways and airways. Among these, roadways play a vital role. Basically road and optimization of transport system reliability. Chunguang et al. [20] analyzed the reliability of urban transport system (road, street, bridge). The performance reliability of road networks under non-recurrent congestion had been assessed by Yin [13]. In [12], Chae studied the system reliability using binomial failure rate. Chow [10] presented a linear mathematical framework for modeling and optimization of road transport facility operations and reliability maintenance.

In general reliability optimization has received significant attention over the past few decades. The main aim of reliability engineering is to increase the system reliability. There are two ways to increase system reliability: 1) increase the reliability of components 2) uses of redundant components within subsystems. A majority of the work is devoted to solve redundancy allocation problems (RAP). Reliability-redundancy allocation problem (RRAP) is an

transport is essential for the economic development, trade and social integration. It facilitates smooth conveyance of both people and goods. Due to easy accessibility and flexibility of operations, road transport has earned an increasingly higher share of passenger and freight traffic than other modes of transportation. So for continuing the economic progress, road transport reliability maintenance is necessary.

Recently more and more reliability professionals have focused on the analysis

optimization technique that seeks to maximize system reliability through redundancy allocation. To optimize RRAP, component reliabilities are denoted as continuous values that lie between zero and one, whereas redundancy levels are integer values. Thus RRAP is a mixed non-linear integer programming problem with the goal of maximizing system reliability under different constraints.

Series-parallel arrangement is one of the essential arrangements which have key role in many real world applications such as telecommunication systems, power systems, transport systems, satellite systems etc. In a series parallel arrangement, there are few subsystems operating in series and each subsystem consists of several components in parallel. Such an arrangement is called a series-parallel arrangement. Mishra in [1, 2] optimized the reliability of series-parallel system using Lagrange multiplier approach and maximum principle approach. Sun

et al. [3] proposed an efficient algorithm for nonlinear integer programming problems arising in series-parallel reliability systems. In this paper, transport system is compared with a series-parallel system.

During the period of network design, a cost-benefit analysis should be performed to make some trade-off between the reliability improvement and the cost investment. So, researchers have not only focused on reliability maximization but also focused on cost minimization. Amara [11] suggested a cost optimization problem for series-parallel petroleum transportation pipelines under reliability constraints. In [4], Dogahe et al. proposed a new bi-objective model to optimize integrated redundancy allocation and reliability-centered maintenance problems in repairable system. A selective maintenance policy and redundancy allocation for series system is formulated by Gupta [5]. Safari [6] developed NSGA-II algorithm for multi-Objective reliability optimization of series parallel system with a choice of redundancy strategies. Also, in [8] Ardakan et al. proposed a GA approach for reliability optimization of series-parallel systems with mixed redundancy strategy in subsystems. Huang [7] optimized reliability redundancy allocation problems with the help of particle-based simplified swarm optimization. Garg [9] also optimized a bi-objective optimization model of the reliability-redundancy allocation problem for series parallel system using genetic algorithm and particle swarm optimization.

The hazard function (also known as failure rate) is the frequency rate at which a system or component fails per unit time. In real life, there are different types of components whose failure rate are increasing, decreasing or constant in nature with respect to time. The weibull distribution has an ability to model the hazard functions that are increasing, decreasing or constant. So in this paper, based on a real life transport network, a multi-objective optimization model is formulated where time to failure of each component follows weibull distribution. Two objective functions (reliability and cost) are considered here which are to be optimized. Cost function comprises of initial purchase cost and system maintenance cost. Here an initial budget is considered as a constraint. The problem has been solved using two different meta-heuristic approaches (MOGA and MOPSO). As this is a multi-objective optimization problem, so it gives pareto front as a solution. Although pareto front is an interesting result, decision makers are fascinated to find a unique solution. So here a  $L_p$  norm technique has been used to obtain a unique solution from all non-dominated solutions of pareto optimal front. No such work has been done so far using these approaches considering the transport system as a series-parallel network.

This paper is organized in 7 sections. Section I starts with the introduction. Some preliminaries are defined in section

II. Section III describes the formulation of a multi-objective optimization problem. Genetic Algorithm (GA) and particle swarm optimization (PSO) are discussed in section IV. In section V these concepts are implemented for a real life transport system. Finally discussion and conclusions are provided in sections VI and VII respectively.

**Notations:**

In this paper, the following notations have been used.

- $m$ = number of subsystems.
- $S_i$ =  $i^{th}$  subsystem.
- $\lambda_i$ = failure rate of  $i^{th}$  component.
- $R_i$ = reliability of  $i^{th}$  subsystem.
- $n_i$ = number of component in  $i^{th}$  subsystem.
- $C_i$ = cost of  $i^{th}$  component.
- $n_{i,min}$ = minimum number of component in  $i^{th}$  subsystem.
- $n_{i,max}$ = maximum number of component in  $i^{th}$  subsystem.
- $R_{s,min}$ = minimum reliability of a system

**II. PRELIMINARIES**

**A. Basic Definitions:**

**Definition 1:** Reliability: Let the random variable  $X$  be the lifetime or the time to failure of a component. The probability that the component survives until a specified period of time  $t$  is called the reliability  $R(t)$  of the component:

$$R(t) = P(X > t) = 1 - F(t)$$

Where,  $F$  is the distribution function of the component lifetime  $X$ .

For series system, the total system reliability:

$$R_s(t) = R_1(t)R_2(t) \dots \dots \dots R_n(t)$$

For parallel system, the system reliability is:

$$R(t) = 1 - [1 - R_1(t)]. [1 - R_2(t)] \dots \dots \dots [1 - R_n(t)]$$

Where  $R_i(t)$  = reliability of the  $i^{th}$  component.

**Definition 2:** Weibull distribution: The Weibull distribution describes the failure times of components when their failure rate either increases or decreases with time.

The failure time density function of Weibull distribution is

$$f(t) = \frac{\beta t^{\beta-1}}{\theta^\beta} e^{-\left(\frac{t}{\theta}\right)^\beta}, t \geq 0, \theta > 0, \beta > 0$$

Where  $\theta$  is the scale parameter and  $\beta$  is the shape parameter of the distribution.

Then the reliability function is

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}, t \geq 0$$

**Definition 3:** Non-dominated Pareto Optimal [18]: A vector  $x^* \in X$  is a pareto optimal if there does not exist another point  $x \in X$  such that  $f_t(x) \leq f_t(x^*)$  for all  $t = 1, 2, \dots, b$  (number of objectives) and  $f_s(x) < f_s(x^*)$  for at least one  $s$ . Such solution is called non-dominated pareto optimal solution.

**Defintion 4:** Pareto Set [18]: A set of non-dominated pareto optimal solutions  $\{x^* | \neg \exists x: x \succ x^*\}$  is said to be a pareto set.

**Definition 5:** Pareto Front [18]: The set of vectors in the objective space that are images of elements of a pareto set i.e,  $\{f(x^*) | \neg \exists x: x \succ x^*\}$ .

**B. Reliability of a Series-Parallel network:**

A system is a set of components working together as parts of a mechanism or an interconnecting network. The components may be connected in series or parallel or both series & parallel.

Series-Parallel System: The System consists of different components connected in series and each component contains different number of subcomponents which are connected in parallel. The general principle used is to reduce sequentially the complicated configuration by combining appropriate series and parallel branches of the reliability model until a single equivalent element is formed. The reliability of this equivalent element will represent the reliability of the original configuration.

Let us consider a series-parallel system where two subsystems are connected in series and each subsystem is configured by different number of components which are connected in parallel. Let subsystem 1 and 2 be configured by  $n_1$  and  $n_2$  number of components respectively.

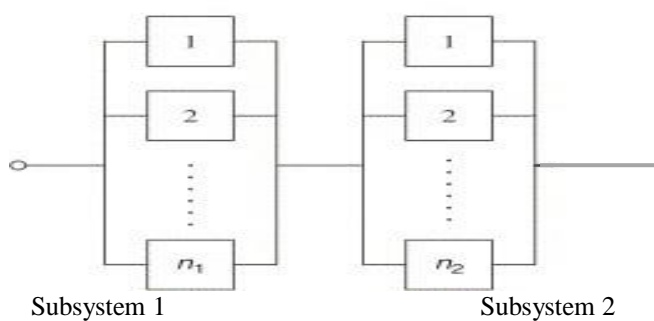


Figure 1: Series Parallel System

Let us consider,

Reliability of a component of subsystem 1 =  $R_1(t)$

So, reliability of the  $n_1$  parallel components =  $[1 - \{1 - R_1(t)\}^{n_1}]$

Reliability of a component of subsystem 2 =  $R_2(t)$

Similarly, reliability of the  $n_2$  parallel components =  $[1 - \{1 - R_2(t)\}^{n_2}]$

Reliability of the total system:

$$R_{sy}(t) = [1 - \{1 - R_1(t)\}^{n_1}] [1 - \{1 - R_2(t)\}^{n_2}]$$

$$= 1 - \{1 - R_1(t)\}^{n_1} - \{1 - R_2(t)\}^{n_2} + \{1 - R_1(t)\}^{n_1} \{1 - R_2(t)\}^{n_2}$$

If all the component of subsystems follow Weibull distribution,

$$R_{sy}(t) = 1 - \{1 - e^{-(\frac{t}{\theta_1})^\beta}\}^{n_1} - \{1 - e^{-(\frac{t}{\theta_2})^\beta}\}^{n_2} + \{1 - e^{-(\frac{t}{\theta_1})^\beta}\}^{n_1} \{1 - e^{-(\frac{t}{\theta_2})^\beta}\}^{n_2}$$

Where,  $\theta_1$  and  $\theta_2$  are the scale parameters of subsystem 1 and subsystem 2 respectively.

When  $\beta = 1$ , Weibull distribution reduces to an exponential distribution and total system reliability becomes,

$$R_{sy}(t) = 1 - \left\{1 - e^{-\left(\frac{t}{\theta_1}\right)}\right\}^{n_1} - \left\{1 - e^{-\left(\frac{t}{\theta_2}\right)}\right\}^{n_2} + \left\{1 - e^{-\left(\frac{t}{\theta_1}\right)}\right\}^{n_1} \left\{1 - e^{-\left(\frac{t}{\theta_2}\right)}\right\}^{n_2}$$

**C. L<sub>p</sub>-norm Strategy for best non-dominated solution [19]:**

Generally multi-objective optimization problems create non-dominated solutions and pareto optimal front. But it is tough for decision maker to choose a unique solution. So for finding the best non-dominated solution, the most appreciated method is L<sub>p</sub>-norm strategy. Here the normalized distance of pareto set and an ideal solution (Utopia point) is minimized using the formula:

$$\text{Minimize} \left( \sum_{i=1}^m \left( \frac{f_i(x) - f_i^{\min}}{f_i^{\max} - f_i^{\min}} \right)^p \right)^{1/p},$$

$$p = 1, 2, \dots, \dots, \dots, \infty$$

Where  $f_i^{\min}$  = minimum value of  $i^{\text{th}}$  objective in the pareto optimal set.

$f_i^{\max}$  = maximum value of  $i^{\text{th}}$  objective in the pareto optimal set.

Here all objective functions must be considered in minimization form.

**III. FORMULATION OF THE MULTI-OBJECTIVE OPTIMIZATION PROBLEM**

To obtain the optimal design policy of a system, reliability is one of the important attributes of performance because it directly influences the system's performance. Consider a series-parallel system containing m subsystems  $S_i$  ( $i = 1, 2, \dots, m$ ) in series arrangement as presented in figure 1. In real world problems however, many systems have components with increasing or decreasing failure rates. This indicates that as time passes by, the failure rates of the system components increase or decrease in comparison

to their initial failure rates. So in this paper every subsystem  $S_i$  contains a different number of components connected in parallel and time to failure of components are weibull distributed. Components are characterized according to their types and cost.

Total cost of the system  $C_T$  could be divided into two parts: initial costs and secondary costs. Initial costs ( $C_I$ ) include purchasing costs of the components which is a fixed cost while secondary costs refer to inspection and maintenance costs ( $C_M$ ) of the components.

So, the total cost is:  $C_T = C_I + C_M$

**Initial Cost:** Initial cost is calculated based on the purchase cost and number of the components implemented in subsystem at the beginning of running the system.

If we consider the purchase cost of different types of components as  $c_i$ ,

Then total initial cost of the system is  $C_I = \sum_{i=1}^m c_i n_i$

**Secondary Cost:** As mentioned before, secondary cost is the maintenance and inspection costs of the components.

The maintenance cost for the system is defined as

$$C_M = \sum_{i=1}^m C'_i [n_i + \exp(\gamma n_i)]$$

Where,  $\exp(\gamma n_i)$  is the additional cost spent due to the interconnection between parallel components.

And  $C'_i$  is considered as [2]:

$$C'_i = a \exp \left[ \frac{b}{1-R_i} \right]$$

a, b and  $\gamma$  are the parameters of the cost function.

**Constraints:** Total number of components of subsystems and an initial budget ( $B_0$ ) for purchasing components are considered as constraints of this proposed model.

In reliability optimization problems, the main goal is to minimize or maximize several objectives subject to several constraints. A designer is required to minimize the system cost with maximizing the system reliability. Therefore, multi objective optimization takes an important role in the reliability design of the system. In this paper, we consider two subsystems and these two subsystems contain  $n_1$  and  $n_2$  number of components respectively. Hence the appropriate optimization model considering the reliability and cost (Initial Cost + Maintenance Cost) as objectives is:

$$\begin{aligned} \text{Maximize } R_{sy}(t) &= 1 - \{1 - R_1(t)\}^{n_1} - \{1 - R_2(t)\}^{n_2} + \{1 - R_1(t)\}^{n_1} \{1 - R_2(t)\}^{n_2} \\ &= 1 - \{1 - e^{-\left(\frac{t}{\theta_1}\right)^\beta}\}^{n_1} - \{1 - e^{-\left(\frac{t}{\theta_2}\right)^\beta}\}^{n_2} + \{1 - e^{-\left(\frac{t}{\theta_1}\right)^\beta}\}^{n_1} \\ &\quad \{1 - e^{-\left(\frac{t}{\theta_2}\right)^\beta}\}^{n_2} \end{aligned}$$

$$\text{Minimize } C_T = C_I + C_M = \sum_{i=1}^m c_i n_i + \sum_{i=1}^m C'_i [n_i + \exp(\gamma n_i)]$$

Subject to:  $n_{i,min} \leq n_i \leq n_{i,max}$

$$\sum_{i=1}^m c_i n_i \leq B_0$$

$$n_1 + n_2 \leq N;$$

$$0 \leq R_i < 1 \text{ for } i=1, 2, \dots, m.$$

#### IV. META-HEURISTIC APPROACH:

The reliability optimization problem is a nonlinear integer programming problem. Due to complexity of such problems, meta- heuristic algorithms are used for solving these problems. Meta-heuristics are stochastic search methods for solving optimization problems. The approaches do not guarantee the determination of the exact solution, but shows the Pareto optimal front which is very close to the optimal solution. There are several solution methods for multi-objective optimization like Genetic Algorithm (GA), multi objective particle swarm optimization (MOPSO), Imperialist Competition Algorithm (ICA), Firefly Algorithm etc.

##### Genetic Algorithm Approach:

Genetic Algorithm (GA) is a stochastic global optimization technique that attempts to evolve a population of candidate solutions by giving preference of survival to quality solutions whilst allowing some low quality solutions to survive in order to maintain diversity in the population. Each candidate solution is coded into a string of digits called chromosomes. New offspring are obtained from probabilistic genetic operators such as selection, crossover, mutation and inversion. A comparison of new and old (parent) candidates is done based on a given fitness function retaining the best performing candidates into the next population. Thus characteristics of candidate solutions are passed from generation to generation through probabilistic selection, crossover, and mutation. The general flow of the GA approach is presented in flow chart.

##### Overall Multi-Objective GA procedure

The overall structure of the Multi objective GA for the optimization problems are consisting of initialization, selection, evaluation, crossover, mutation, replacement, and termination. The pseudo-code of the algorithm is given below:

##### Algorithm 1: Pseudo code for Multi-Objective GA [17]:

- 1: randomly generate initial population
- Repeat
  - 2: evaluation of fitness, objective:  $f(x)$ ,  $x = (x_1, x_2, \dots, x_h)$
  - 3: selection strategy
  - 4: crossover and mutation
  - 5: replacement

6: advance population; old pop = new pop

Until (termination criteria is satisfied)

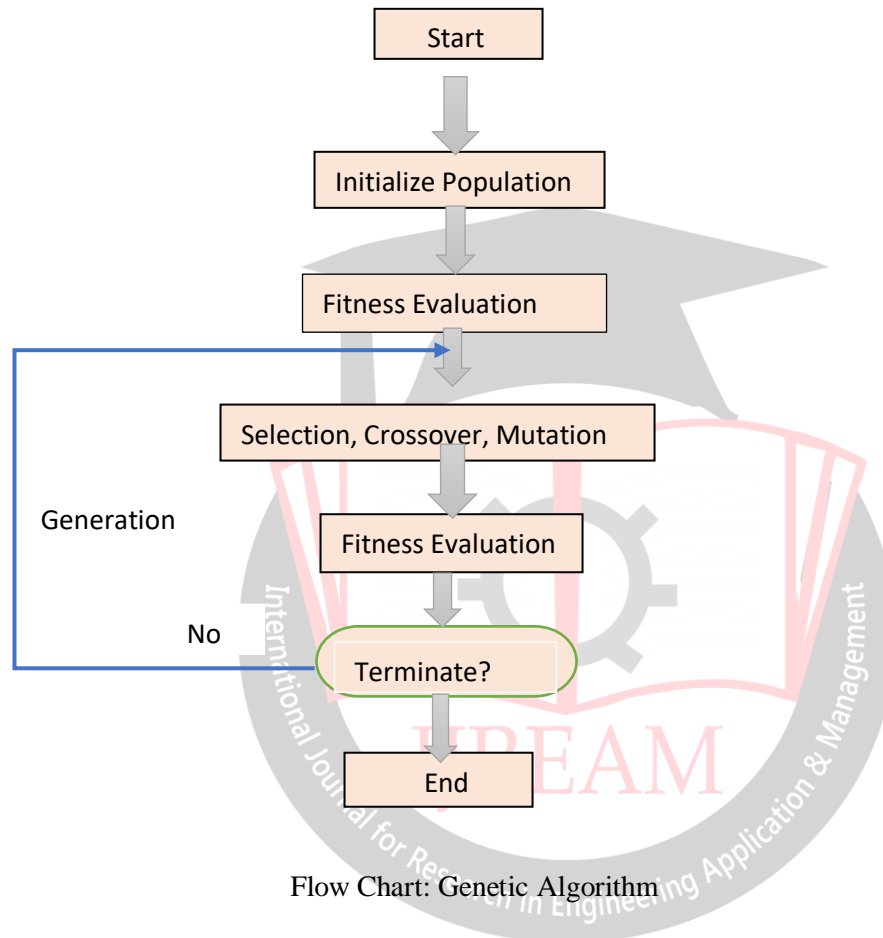
### Particle Swarm Optimization [14]:

The basic idea behind the algorithm is to use a collection of particles to explore the fitness landscape of a particular problem. Each particle is a vector that describes a candidate solution and can be evaluated (in the multi objective case) along several quality dimensions (or, equivalently, with several fitness functions). The algorithm is iterative, and at each iteration each particle moves

through the fitness landscape according to its current fitness values as well as those of nearby particles.

**Algorithm 2:** The basic steps of *MOPSO algorithm* [16]:

1. Initialize the swarm and archive.
2. For each particle in the swarm:
  - A. Select leader from the archive.
  - B. Update Velocity
  - C. Update Position
3. Update the archive of non-dominated solution.
4. Repeat



Flow Chart: Genetic Algorithm

### Pseudo Code of MOPSO algorithm [15]:

1. Randomly initialize population  $\overline{POP}$  of size  $\mu$ .
2. Initialize the speed  $\overline{VEL}$  of each particle.  
For all  $i \in \{1, 2, \dots, \mu\}$   $\overline{VEL}[i]=0$ .
3. Evaluate each particle in  $\overline{POP}$ .
4. Store non-dominated points in the repository  $\overline{REP}$ .
5. Generate hyper cubes (cubes in  $m$  dimensions, where  $m$  is the number of objectives) of the objective space explored so far. Do this by dividing the objective space explored by divisions in each dimension of it.
6. For each particle, determine in which hypercube it is positioned.
7. Initialize the memory  $\overline{PBESTS}$  of each particle.  
For all  $i \in \{1, \dots, \mu\}$   $\overline{PBESTS}[i]=\overline{POP}[i]$ .

8. While the maximum number of loops is not exceeded, do: compute the speed of each particle  $I$  (in each direction) while the formula  

$$\overline{VEL}[i]= w \times \overline{VEL}[i] + C_1 R_1 \times (\overline{PBESTS}[i] - \overline{POP}[i]) + C_2 R_2 \times (\overline{REP}[h] - \overline{POP}[i])$$
 where  $w$  is an inertia weight,  $R_1, R_2$  are random values between 0 and 1.

Compute the new position of each particle with

$$\overline{POP}[i]=\overline{POP}[i]+\overline{VEL}[i]$$

make sure each particle stays within the search space boundaries.

Update  $\overline{REP}$ ,  $\overline{PBESTS}$  the hyper cubes and the position of each particle within these hyper cubes.

### V. IMPLEMENTATION FOR A PUBLIC TRANSPORT ROUTE

Comparison of series-parallel system with a transport system:

In practice generally two types of buses (Volvo and non-ac) are used to transport from one city to another city. These are basically two individual transport systems. In our consideration, these two individual transport systems are connected in series and constitute the subsystems of the overall transport system. In turn, each subsystem contains different number of buses in parallel. If any one of the subsystem fails, then the overall transport system fails.

Implementation:

West Bengal is one of the developing states of India. Therefore, West Bengal government is trying to connect

all cities of West Bengal by bus transport network. This transportation is undertaken by WBSTC (West Bengal State Transport Corporation). In transport route, government wants to transport, mainly two types of buses: 1) Non-air condition state bus 2) Volvo bus, depending upon the public demand. Government also sanctioned a budget for initial purchasing of these buses. These two types of busses have different failure rate & different purchasing and maintenance cost. In this problem,

$\lambda_1$ = failure rate of Volvo buses,

$\lambda_2$ = failure rate of non-air conditioned buses.

$n_1$ = number of Volvo bus,

$n_2$ = number of non-air conditioned bus. Here, we endeavor to evaluate the total reliability of the transport system and then maximize the total system reliability.

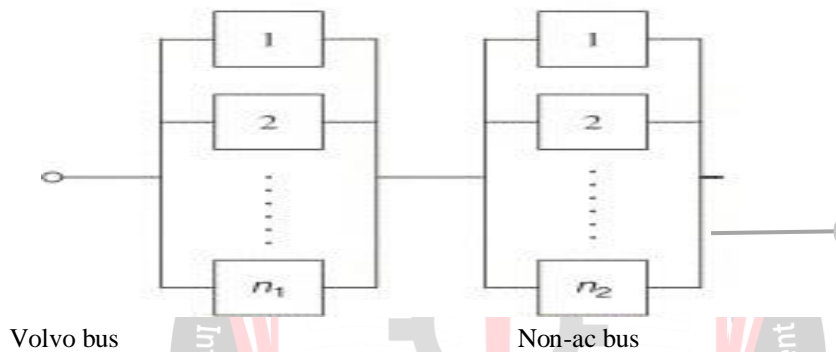


Figure 2: Public Road Transport System

Suppose, West Bengal Government wants to prepare a strong and effective transport system. This system is composed of two types of buses. The type, number, scale parameter and cost of components are given in the table below. If government wants to transport maximum 10

buses for each type and total 15 buses in the transport system then the problem is to determine the number of buses for maximizing the system reliability and minimizing the system cost.

Table 1:

Component	Type-1	Type-2
Type of component	Volvo Bus	Non-ac Bus
Number of component	$n_1$	$n_2$
Value of Scale Parameter	$\theta_1 = 30$	$\theta_2 = 20$
Purchase Cost of each component	60 lakhs	30 lakhs

Value of other parameters				
a	b	$\gamma$	$B_0$	N
1	0.20	0.25	1000	15

In this paper, time to failure of different components is considered as weibull distribution. So, for different values of shape parameter, model will be different.

**Model:** for the different values of shape parameter  $\beta$ , proposed model is as follows:

$$\text{Maximize } R_{sy}(t) = 1 - \{1 - e^{-\left(\frac{t}{\theta_1}\right)^\beta}\}^{n_1} - \{1 - e^{-\left(\frac{t}{\theta_2}\right)^\beta}\}^{n_2} + \{1 - e^{-\left(\frac{t}{\theta_1}\right)^\beta}\}^{n_1} \{1 - e^{-\left(\frac{t}{\theta_2}\right)^\beta}\}^{n_2}$$

$$\text{Minimize Cost } C_T = \sum_{i=1}^2 c_i n_i + \sum_{i=1}^2 a \exp\left[\frac{b}{1 - e^{-\left(\frac{t}{\theta_i}\right)^\beta}}\right] [n_i + \exp(\gamma n_i)]$$

$$\text{Sub to: } n_1 + n_2 \leq 15;$$

$$\sum_{i=1}^m c_i n_i \leq 1000,$$

$$1 \leq n_i \leq 10,$$

$n_1, n_2 \geq 1$  & are integer.

## VI. RESULTS AND DISCUSSION

After establishing the optimization model with two objective functions, the model was solved in MATLAB by MOGA and MOPSO approach as mentioned in previous part. For solving the model, the parameters of MOGA and MOPSO have been defined.

During the evolution, the integer variables  $n_i$  are considered as real variables, and in calculating the value of objective functions, the real values are converted to the nearest integer values. The inertia weight, cognitive component ( $C_1$ ) and social component ( $C_2$ ) of MOPSO algorithm are taken as  $w=0.5$ ,  $C_1=1$ , and  $C_2=2$  respectively. Maximum number of iterations and particles, both are considered as 100 which are used in computation. The parameter, crossover and migration of MOGA are 0.8 and 0.2 respectively. Intermediate crossover and Tournament selection process are used for reproduction. The termination criteria has been set either limited to a maximum number of 200 generations or to be the order of relative error equal to  $e^{-4}$  which is achieved first. The program has been run 200 times and the best values are chosen.

The application of MOGA and MOPSO for solving the models, results Pareto optimal solutions. The Pareto optimal solutions contain the solutions that were not dominated by the other solutions.

. Pareto Optimality:

A candidate is Pareto optimal if:

- It is at least as good as all other candidates for all objectives, and
- It is better than all other candidates for at least one objective.

We would say that this candidate *dominates* all other candidates.

Set of all Pareto optimal solutions (points in variable space) is called Pareto Set. Set of all Pareto objective vectors is called Pareto Front.

For obtaining the Pareto optimal solution of the optimization models, MOGA and MOPSO have been used with the parameter setting given in section VI. The corresponding results for different values of  $t=10, 20$  and  $\beta = 0.5, 1, 2$  are summarized in table 2, 3,4,5,6 and 7 corresponding to the proposed model. The Pareto optimal front for these models, are also shown in different figures below. The best non-dominated solutions are shown in table 8 using  $L_p$ -norm strategy ( $p=2$ ).

Table 2:

Pareto Optimal solution at $t=10$ and $\beta=0.5$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Cost	System Structure		reliability	Cost
$n_1$	$n_2$	$R_{sy}$	$C_T$	$n_1$	$n_2$	$R_{sy}$	$C_T$
7	8	0.9925	697.6859	3	5	0.8997	367.2545
1	4	0.6500	214.1571	5	8	0.9821	586.3007
3	3	0.8009	278.6891	4	7	0.9618	488.3885
3	5	0.8909	353.6525	5	6	0.9738	537.6624
2	3	0.7398	241.8568	3	3	0.7992	277.3697

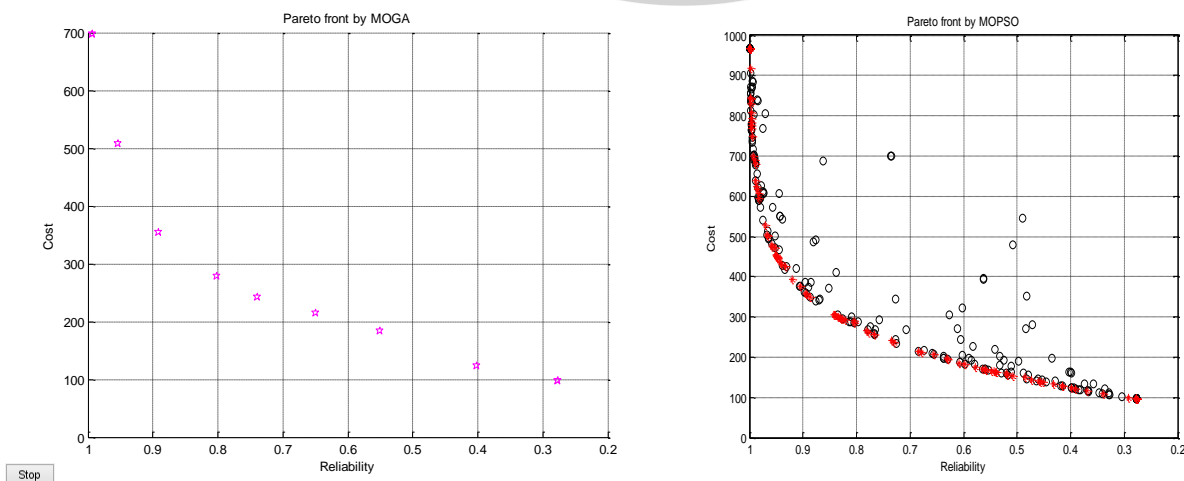


Figure 3: Pareto front by MOGA and MOPSO at  $t=10$  and  $\beta=0.5$

**Table 3:**

Pareto Optimal solution at $t= 10$ and $\beta= 1$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Total Cost	System Structure		reliability	Total Cost
$n_1$	$n_2$	$R_{sy}$	$C_T$	$n_1$	$n_2$	$R_{sy}$	$C_T$
7	8	0.9994	699.4014	6	9	0.9994	687.6156
3	7	0.9868	458.8965	4	6	0.9880	433.6742
3	3	0.9205	282.3288	4	5	0.9833	405.6311
2	5	0.9513	336.1573	2	2	0.7838	183.8786
3	4	0.9671	349.0318	3	5	0.9720	360.3358

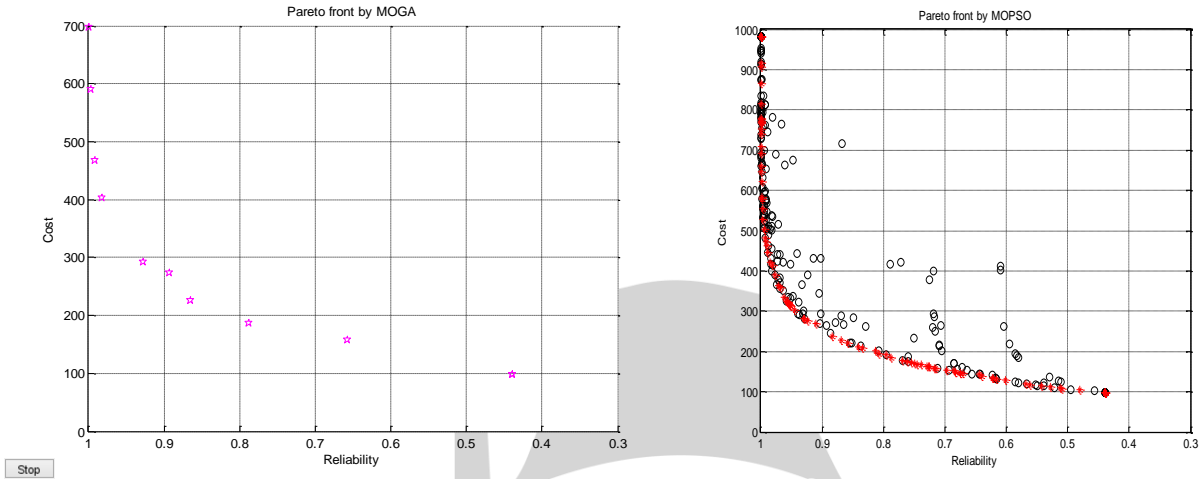


Figure 4: Pareto front by MOGA and MOPSO at  $t=10$  and  $\beta=1$

**Table 4:**

Pareto Optimal solution at $t= 10$ and $\beta= 2$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Total Cost	System Structure		reliability	Total Cost
$n_1$	$n_2$	$R_{sy}$	$C_T$	$n_1$	$n_2$	$R_{sy}$	$C_T$
4	7	0.9998	550.4819	4	6	0.9998	495.3475
3	7	0.9996	479.9039	3	5	0.9989	405.2508
6	9	0.9999	754.3615	6	9	0.9999	760.0092
2	2	0.9634	223.9294	3	3	0.9918	303.6788
1	2	0.8786	153.8422	2	3	0.9686	227.4587

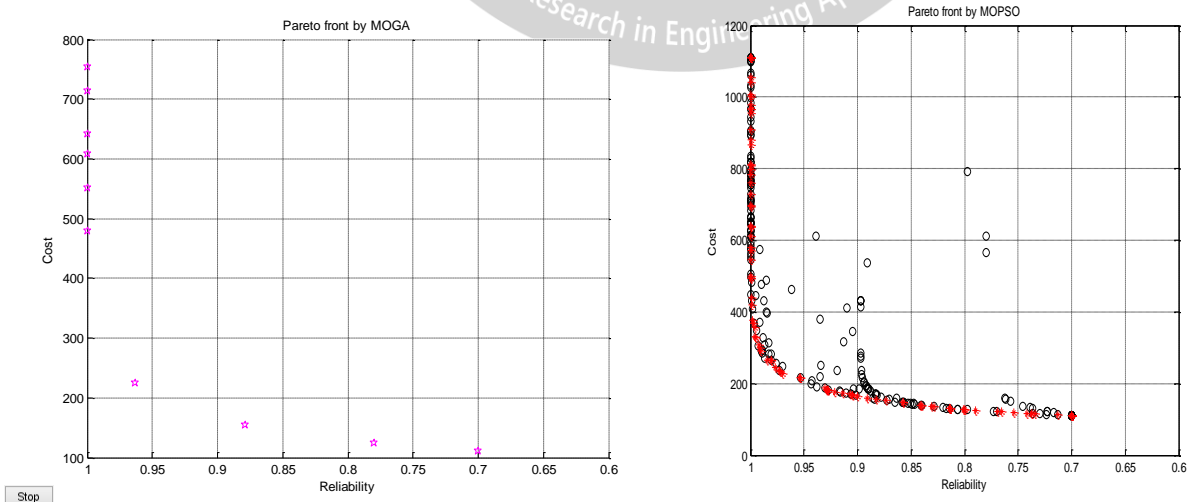


Figure 5: Pareto front by MOGA and MOPSO at  $t=10$  and  $\beta=2$



**Table 5:**

Pareto Optimal solution at $t= 20$ and $\beta= 0.5$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Cost	System Structure		reliability	Cost
$n_1$	$n_2$	$R_{sy}$	$C_T$	$n_1$	$n_2$	$R_{sy}$	$C_T$
7	8	0.9580	695.6437	5	7	0.9021	531.3518
6	7	0.9399	637.9085	6	8	0.9408	622.4851
5	6	0.8762	486.8433	5	6	0.8764	486.4162
3	6	0.8032	403.2138	3	5	0.7902	387.1680
3	4	0.7362	346.8626	2	3	0.4579	198.7441

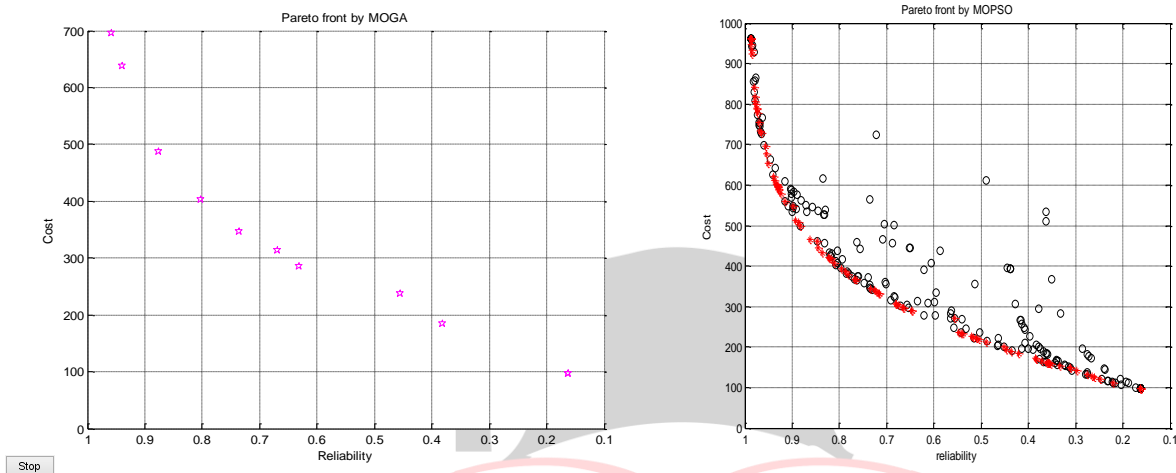


Figure 6: Pareto front by MOGA and MOPSO at  $t=20$  and  $\beta=0.5$

**Table 6:**

Pareto Optimal solution at $t= 20$ and $\beta= 1$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Total Cost	System Structure		reliability	Total Cost
$n_1$	$n_2$	$R_{sy}$	$C_T$	$n_1$	$n_2$	$R_{sy}$	$C_T$
6	9	0.9708	678.3747	5	8	0.9394	550.4153
4	8	0.9380	551.1833	4	7	0.8920	456.8989
4	6	0.8863	447.2550	4	6	0.8870	449.3349
3	6	0.8419	399.4245	3	6	0.8540	407.9645
2	5	0.7052	295.4695	2	5	0.7447	315.2493

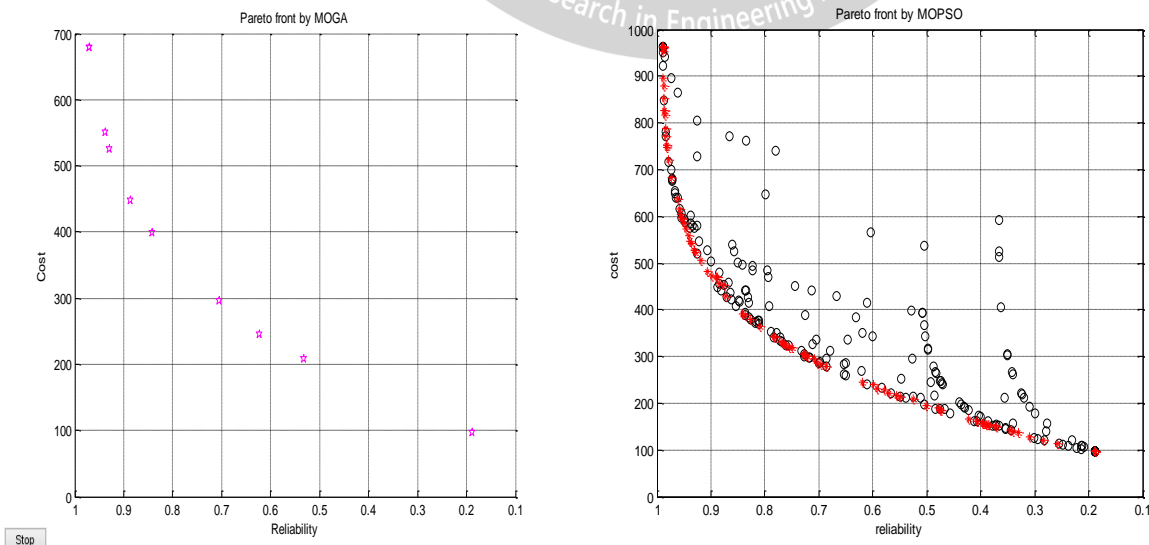


Figure 7: Pareto front by MOGA and MOPSO at  $t=20$  and  $\beta=1$

**Table 7:**

Pareto Optimal solution at $t=20$ and $\beta=2$							
Genetic Algorithm				Particle Swarm Optimization			
System Structure		Reliability	Total Cost	System Structure		reliability	Total Cost
$n_1$	$n_2$	$R_{Sy}$	$C_T$	$n_1$	$n_2$	$R_{Sy}$	$C_T$
5	10	0.9838	651.4917	5	10	0.9831	638.5542
5	6	0.9344	498.1565	4	9	0.9648	535.8841
5	8	0.9662	553.6624	3	7	0.9272	436.3120
4	5	0.8724	412.6554	3	5	0.8520	339.3569
2	6	0.8481	336.3883	2	4	0.7764	284.0008

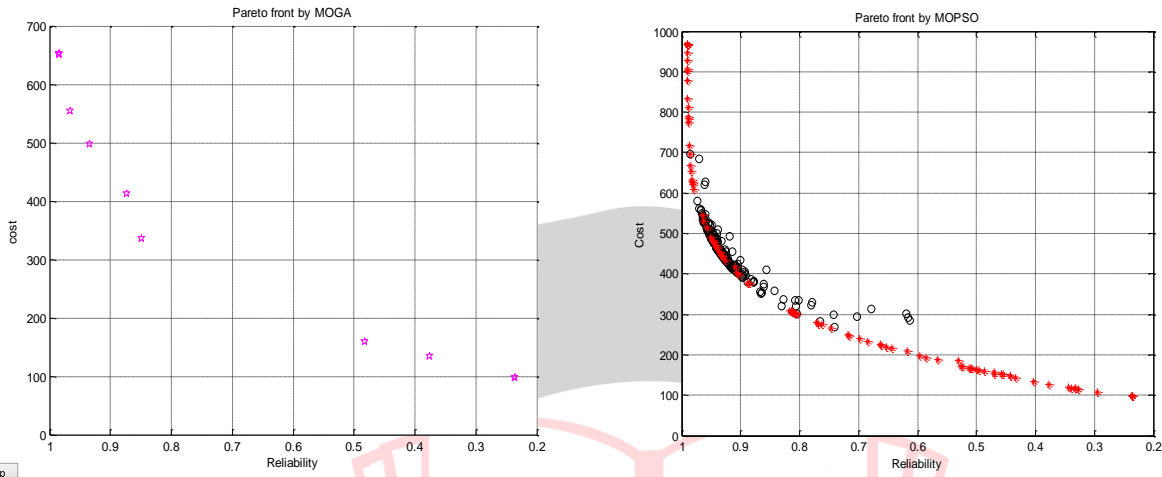


Figure 8: Pareto front by MOGA and MOPSO at  $t=20$  and  $\beta=2$

**Table 8:**

Best Non-Dominated Solution using $L_p$ - norm Strategy( $p=2$ )					
Time	Parameter Values	MOGA		MOPSO	
		Reliability	Cost	Reliability	Cost
t=10	$\beta=0.5$	0.8008586	278.6890	0.8044689	285.3582
	$\beta=1$	0.9205277	282.3287	0.9242966	275.6779
	$\beta=2$	0.9633510	223.9294	0.9699188	228.9282
t=20	$\beta=0.5$	0.7362234	346.8626	0.7272434	338.8045
	$\beta=1$	0.7052329	295.4694	0.7065473	288.9816
	$\beta=2$	0.8481311	336.3883	0.8516213	338.5793

Thus, from all these tables and figures, it has been shown that the results computed by MOGA and MOPSO with respect to individual shape parameter are almost similar. From table 8, it is also clear that for individual shape parameter ( $\beta$ ) of weibull distribution, the optimal reliability decreases and total cost increases when time increases. This observation justified that the formulated model is realistic and practical. Based on these results, the system analysts or decision makers may plan the proper design for optimal system's structure.

### VII. CONCLUSION

In this present paper, initially a multi objective optimization problem is formulated on series-parallel system where objectives are minimizing the system cost and maximizing the system reliability. Finally based on this concept a multi-objective optimization model on

transport system is proposed. Here, for the first time the transport reliability optimization problem with weibull distributed time-to-failure of each component have been solved with some budget constraints. In general, this type of problem is not easy to handle in real cases. Hence meta-heuristic approach has been used to solve such a hard and complex problem. Finding non-dominated solutions or Pareto optimal front is an attractive alternative for such type of problems. So, two well-known meta-heuristic techniques have been used for finding the pareto optimal front. After obtaining the non-dominated solutions,  $L_p$ -norm strategy has been used to choose the best non dominated solution. Based on these solutions, we are able to prove that this formulation is practical and realistic. It also provides flexibility to the decision-makers to obtain the best-compromise solution.

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