Stress Analysis of Laminated Composites under Thermal Gradient by Higher Order Shear and Normal Deformation Theory

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Abstract: Analytical solution for the simply supported composite laminated plates subjected to two different temperature variations through thickness based on higher order theory is presented. Analytical model presented herein includes the effect of transverse shear deformation which eliminates the need of shear correction factor to rectify the unrealistic variation of shear stress through thickness. Primary displacement field is expanded in the thickness direction using eleven degrees of freedom. Equilibrium equations in the present higher order shear and normal deformation theory (HOSNT11) are obtained using principle of virtual work. The Navier solutions for simply supported laminated composite plate subjected to varying thermal load through the thickness have been developed. Numerical results obtained are compared with semi-analytical model available in literature

Keywords — Analytical solution; composite laminates; higher order theory; thermal analysis

I. INTRODUCTION

Composite materials are widely used in many industries because of their special properties like high thermal resistance, high strength to weight ratio, long fatigue life etc. In the industries like aerospace, nuclear reactors and chemical plants composite laminates are often subjected to high temperature environment. The temperature stresses become governing factor for design of such structures therefore the accurate thermal stress analysis of laminated composite plates has constantly been an important area of research.

The classical plate theory (CPT) by Timoshenko and Woinowsky-Krieger [1] is inadequate for analysis of laminated composite plates as it neglects the effect of transverse normal and shear deformation. First order shear deformation theory (FSDT) by Reissner and Mindlin accounts for transverse shear stress and shear correction factors are needed to rectify the unrealistic variation of shear stress through thickness. To overcome the limitations of FSDT, higher order shear deformation theories (HSDT) were developed. Nelson and Lorch [2], Librescu [3] represented higher order displacement based shear deformation theories for the analysis of laminated plates. Reddy et al. [4] presented finite element formulation of laminates subjected to thermal loading based on FOST. Third-order shear deformation theory is developed by Reddy [5] for the mechanical and thermal analysis of laminated composite plates. Khdeir and Reddy [6] proposed refined plate theories using stress-space approach to study the thermal stresses and deformations of cross-ply rectangular laminates. Murakami [7] studied thermomechanical response of layered plates using various plate theories. Argyris and Tenek [8] used linear thermal variation across the thickness of the laminates to formulate FE model based on the first order shear deformation theory. A simple Co iso-parametric finite element model for the analysis of symmetric and unsymmetric laminates subjected to thermal gradient is presented by Kant and Khare [9]. 3D elasticity solution for temperature distribution and thermal stresses in simply supported rectangular laminates is derived by Tungikar and Rao [10]. Savoia and Reddy [11] presented transient heat conduction equation for exact temperature distribution across the thickness of laminates for 3D stress analysis of symmetric four-layered square laminate subjected to uniform temperature change. Bhaskar et al. [12] developed 3D elasticity solution for laminates under cylindrical and bi-directional bending by assuming linear variation of thermal profile through the thickness of the symmetric laminate. A displacement based higher order shear deformation theory for the thermal flexure analysis of symmetric laminated composite plates is developed by Ali, Bhaskar et al. [13]. Rohwer et al. [14] used higher order plate theories to predict the thermal stresses in layered plates. Third order zig-zag theory for composite laminates subjected various thermal profiles across the thickness is investigated by Kapuria and Achary [15]. A global higher order theory based on power series for prediction of inter-laminar stresses subjected to thermal loading is presented by Matsunaga [16]. Zhen and Wanji [17] developed finite element model based on global-local higher order theory to study the bending response of laminated composite plates subjected to thermal loading. Kant et al. [18] derived semi-analytical solution for constant and linear temperature variation through the thickness of a laminated composite and sandwich plates. Zhen et al. [19] used a refined global-

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local higher order theory to analyze angle-ply laminated composite plates subjected to thermomechanical loading. Exact solution is developed for thermo-elastic deformations in symmetric and antisymmetric cross-ply laminated arches by Khdeir [20]. Kant and Shiyekar [21] developed a complete analytical model, which incorporates shear deformation as well as transverse normal thermal strains is assessed for the thermal stress analysis of cross-ply laminates subjected to linear or gradient thermal profile across thickness of the laminate. A higher order computational model for the thermo-elastic analysis of laminated composite plates is used by Swaminathan and Fernandes [22].

The objective of this investigation is to present a higher order shear and normal deformation theory (HOSNT11) for thermal analysis of composite laminated plate. The constant and linear thermal load variation through thickness of laminate is considered for analysis. Numerical results of displacement and stresses are obtained using the HOSNT11 for simply supported composite laminated plate and are compared with the results available in the literature.

II. THEORETICAL FORMULATION

In order to approximate the three-dimensional elasticity problem to a two-dimensional plate problem, the displacement components $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ at any point in the plate space are expanded in a Taylor's series in terms of the thickness coordinate. The elasticity solution indicates that the transverse shear stresses vary parabolically through the plate thickness. This requires the use of a displacement field in which the in-plane displacements are expanded as cubic functions of the thickness coordinate. In addition, the transverse normal strain may vary nonlinearly through the plate thickness. The displacement field which satisfies the above criteria may be assumed in the form

$$u(x,y,z)=u_0(x,y)+z\theta_x(x,y)+z^2u_0^*(x,y)+z^3\theta_x^*(x,y)$$
$$v(x,y,z)=v_0(x,y)+z\theta_y(x,y)+z^2v_0^*(x,y)+z^3\theta_y^*(x,y)$$
$$w(x,y,z)=w_0(x,y)+z\theta_z(x,y)+z^2w_0^*(x,y)$$

The expansions of the in-plane displacements u and v imply nonlinear cubic variations of these through the laminate thickness. Thus, the warping of the transverse cross section is automatically incorporated. The expansion of transverse displacement w implies a nonvanishing transverse normal strain. Thus, the limitations of the usual Kirchhoff's hypothesis as well as the Mindlin -type FOSTs are completely eliminated. The parameters u_0, v_0 are the in-plane displacements and w_0 is the transverse displacement of a point (x, y) on the middle plane. The functions θ_x, θ_y are rotations of the normal to the middle plane about y and x axes, respectively. The parameters u_0^*, v_0^*, w_0^*, θ_x^*, θ_y^* are the higher-order terms in Taylor's series expansion and they represent higher-order transverse cross-sectional deformation modes. This model is named HOSNT11 because it has 11 middle surface parameters. The geometry of a composite laminated plate with positive set of coordinate axes are shown in Fig.1.0

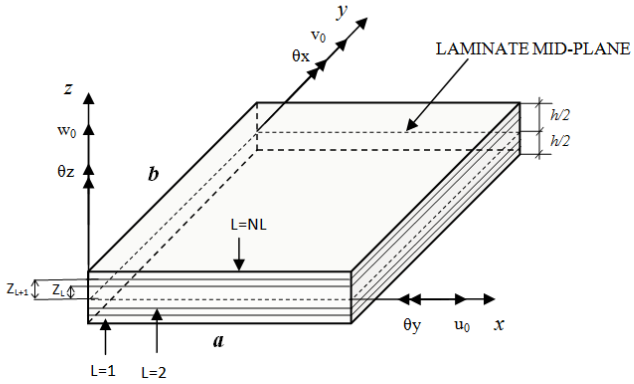

Fig. 1.0. Geometry composite laminated plate with positive set of reference axes and displacement components

III. STRAIN DISPLACEMENT RELATIONSHIP

The general linear strain-displacement relationships at any point within a plate are given as

$$\varepsilon_x=\frac{\partial u}{\partial x},\ \varepsilon_y=\frac{\partial v}{\partial y},\ \varepsilon_z=\frac{\partial w}{\partial z},$$
$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\ \gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y},\ \gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}$$

IV. CONSTITUTIVE EQUATIONS

From linear elasticity theory, the 3D stress strain constitutive relationship with stiffness matrix $[C_{ij}]$ for L^{th} lamina with reference to principal material coordinate system 1-2-3 can be written in matrix form as:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{12}\\ \tau_{23}\\ \tau_{13}\end{Bmatrix}^L=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}^L\begin{Bmatrix}\varepsilon_1-\alpha_1\Delta T\\ \varepsilon_2-\alpha_2\Delta T\\ \varepsilon_3-\alpha_3\Delta T\\ \gamma_{12}\\ \gamma_{23}\\ \gamma_{13}\end{Bmatrix}^L$$

Where, (σ_1, σ_2, σ_3, τ_{12}, τ_{23}, τ_{13}) are the stresses, (ε_1, ε_2, ε_3, γ_{12}, γ_{23}, γ_{13}) are the strains with respect to the lamina coordinate (1-2-3) and $[C_{ij}]$ are the elastic constants or stiffness matrix of L^{th} lamina. α_1, α_2, α_3 are the thermal expansion coefficients with respect to lamina reference axes and ΔT is rise in temperature with respect to reference temperature.

In the laminate coordinate (x,y,z) the stress strain relation for L^{th} lamina can be written as,

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$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0\\ Q_{12} & Q_{22} & Q_{23} & Q_{24} & 0 & 0\\ Q_{13} & Q_{23} & Q_{33} & Q_{34} & 0 & 0\\ Q_{14} & Q_{24} & Q_{34} & Q_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & Q_{55} & Q_{56}\\ 0 & 0 & 0 & 0 & Q_{56} & Q_{66}\end{bmatrix}^L \begin{Bmatrix}\varepsilon_x-\alpha_x\Delta T\\ \varepsilon_y-\alpha_y\Delta T\\ \varepsilon_z-\alpha_z\Delta T\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^L$$

Where, (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{xz}) are the stresses, (ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{xz}) are the strains with respect to laminate coordinate system (x-y-z) and $[Q_{ij}]$ are transformed elastic constants or stiffness matrix of L^{th} lamina with respect to laminate axes x,y,z. α_x, α_y, α_z are the thermal expansion coefficients with respect to laminate reference axes and are defined as,

$\alpha_x = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta$, $\alpha_y = \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta$, $\alpha_z = \alpha_3$

Where θ is the angle made by fiber direction to x-axis.

V. GOVERNING EQUATIONS OF EQUILIBRIUM

The equations of equilibrium for the stress analysis are obtained using the principle of minimum potential energy.

$$\delta u_0 : \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0,\ \ \delta v_0 : \frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}=0,$$

$$\delta w_0 : \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=0,\ \ \delta\theta_x : \frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0,$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-Q_y=0,\ \ \delta\theta_z : \frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}-N_z=0,$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x}+\frac{\partial N_{xy}^*}{\partial y}-2S_x=0,\ \ \delta v_0^* : \frac{\partial N_y^*}{\partial y}+\frac{\partial N_{xy}^*}{\partial x}-2S_y=0,$$

$$\delta w_0^* : \frac{\partial Q_x^*}{\partial x}+\frac{\partial Q_y^*}{\partial y}-2M_z^*=0,$$

$$\delta\theta_x^* : \frac{\partial M_x^*}{\partial x}+\frac{\partial M_{xy}^*}{\partial y}-3Q_x^*=0,\ \ \delta\theta_y^* : \frac{\partial M_y^*}{\partial y}+\frac{\partial M_{xy}^*}{\partial x}-3Q_y^*=0,$$

To obtain a 2D expression for the potential energy functional, stresses are integrated through the plate thickness, which yield the definition of the following set of stress resultants:

$$\begin{bmatrix} M_x & M_x^*\\ M_y & M_y^*\\ M_z & 0\\ M_{xy} & M_{xy}^*\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\end{Bmatrix}\begin{bmatrix} z & z^3\end{bmatrix}dz,$$

$$\begin{bmatrix} Q_x & Q_x^*\\ Q_y & Q_y^*\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}}\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}\begin{bmatrix} 1 & z^2\end{bmatrix}dz,$$

$$\begin{bmatrix} N_x & N_x^*\\ N_y & N_y^*\\ N_z & N_z^*\\ N_{xy} & N_{xy}^*\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\end{Bmatrix}\begin{bmatrix} 1 & z^2\end{bmatrix}dz,$$

$$\begin{bmatrix} S_x & S_x^*\\ S_y & S_y^*\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}}\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}\begin{bmatrix} z & z^3\end{bmatrix}dz,$$

VI. ANALYTICAL SOLUTION

Among all of the analytical methods available, the Navier solution technique is very simple and easy to use when the plate is of rectangular geometry (side dimensions = a and b; thickness = h) with simply supported edge conditions. This solution to different Kirchhoff plate problems of rectangular geometry is well documented in various texts.

Following are the boundary conditions used for two opposite infinite simply supported edges:

At edges x = 0 and x = a:

$v_0 = 0,\ w_0 = 0,\ \theta_y = 0,\ \theta_z = 0,\ M_x = 0,$

$v_0^* = 0,\ w_0^* = 0,\ \theta_y^* = 0,\ \theta_z^* = 0,\ M_x^* = 0,$

$N_x = 0,\ N_x^* = 0.$

At edges $y = 0$ and $y = b$:

$u_0 = 0,\ w_0 = 0,\ \theta_x = 0,\ \theta_z = 0,\ M_y = 0,$

$u_0^* = 0,\ w_0^* = 0,\ \theta_x^* = 0,\ \theta_z^* = 0,\ M_y^* = 0,$

$N_y = 0,\ N_y^* = 0.$

The generalized displacement field to satisfy the previous boundary conditions is expanded in double Fourier series as,

$$u_0=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}u_{0_{mn}}\cos\alpha x\sin\beta y,\ \ v_0=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}v_{0_{mn}}\sin\alpha x\cos\beta y,$$

$$w_0=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}w_{0_{mn}}\sin\alpha x\sin\beta y,\ \ \theta_x=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}\theta_{x_{mn}}\cos\alpha x\sin\beta y,$$

$$\theta_y=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}\theta_{y_{mn}}\sin\alpha x\cos\beta y,\ \ \theta_z=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}\theta_{z_{mn}}\sin\alpha x\sin\beta y,$$

$$u_0^*=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}u_{0_{mn}}^*\cos\alpha x\sin\beta y,\ \ v_0^*=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}v_{0_{mn}}^*\sin\alpha x\cos\beta y,$$

$$w_0^*=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}w_{0_{mn}}^*\sin\alpha x\sin\beta y,\ \ \theta_x^*=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}\theta_{x_{mn}}^*\cos\alpha x\sin\beta y,$$

$$\theta_y^*=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}\theta_{y_{mn}}^*\sin\alpha x\cos\beta y,$$

The thermal load is expressed as doubly sinusoidal loading at top of laminate as,

$$T(x,y,z)=\sum_{m=1,}^{\infty}\sum_{n=1,}^{\infty}T_{mn}\sin\alpha x\sin\beta y$$

where $\alpha = m\pi/a$, $\beta = n\pi/b$.

After following the standard steps for collecting the coefficients of the 11 displacement degrees of freedom in a 11x11 system of simultaneous equations, the Fourier amplitudes of the displacements are obtained in the following form.

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$$[X]_{11\times 11}\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_x \\ \theta_y \\ \theta_z \\ u_0^* \\ v_0^* \\ w_0^* \\ \theta_x^* \\ \theta_y^* \end{Bmatrix}_{11\times 1} = \{T_r\}_{11\times 1}$$

Where, [X] is stiffness matrix and {Tr} is temperature vector. The coefficients of matrix [X] and matrix {Tr} are given in Appendix A and Appendix B respectively.

VII. NUMERICAL EXAMPLES AND RESULTS

A homogeneous, orthotropic simply supported plate [18] subjected to thermal load has been considered to study the effect of the two different temperature variations through thickness.

Following two thermal load cases are considered.

1. CASE A - Equal temperature rise of the bottom and the top surface of the plate with sinusoidal inplane variations:

$$\Delta \mathrm{T}(\mathrm{x},\mathrm{y},\pm \mathrm{h}/2) = \mathrm{T}_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

2. CASE B - Equal rise and fall of temperature of the top and bottom surface of the plate with sinusoidal inplane variations:

$$\Delta \mathrm{T}(\mathrm{x},\mathrm{y},\mathrm{h}/2) = -\Delta \mathrm{T}(\mathrm{x},\mathrm{y},-\mathrm{h}/2) = \mathrm{T}_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

Rohwers [14] material properties are considered as follows,

$$E_1 = 150.0GPa,\ E_2 = 10.0GPa,\ E_3 = 10.0GPa,$$
$$\nu_{12} = 0.30,\ \nu_{13} = 0.30,\ \nu_{23} = 0.30,$$
$$\mathrm{G}_{12} = 5.0GPa, \mathrm{G}_{13} = 5.0GPa,\ \mathrm{G}_{23} = 3.378GPa,$$
$$\alpha_1 = 0.139\times 10^{-6},\ \alpha_2 = 9.0\times 10^{-6},\ \alpha_3 = 9.0\times 10^{-6}$$

The following normalization have been used in example considered here,

$$S = \frac{a}{h},\ \bar{u},\bar{v} = \frac{1}{h\alpha_1 T_0 s^3}(\mathrm{u};\mathrm{v}),\ \bar{w} = \frac{h^3 w}{\alpha_1 T_0 a^4},\ \bar{\sigma}_z = \frac{\sigma_z}{E_2\alpha_1 T_0},$$
$$(\bar{\sigma}_x;\bar{\sigma}_y;\bar{\tau}_{xy}) = \frac{1}{\mathrm{E}_2\alpha_1 \mathrm{T}_0 \mathrm{s}^2}(\sigma_x;\sigma_y;\tau_{xy}),\ (\bar{\tau}_{xz};\bar{\tau}_{yz}) = \frac{1}{\mathrm{E}_2\alpha_1\mathrm{T}_0\mathrm{s}}(\tau_{xz};\tau_{yz})$$

The normalized maximum stresses $(\overline{\sigma_x}, \overline{\sigma_y}, \overline{\tau_{xy}})$ and displacement $(\bar{u}, \bar{v}, \bar{w})$ for different aspect ratios are presented in Table 1.0 and Table 2.0 for both type of thermal variations. Also, the graphical representation of results for an aspect ratio of 5 are shown in figures 2.0 to 7.0, for thermal load case A and case B. Semi analytical results of in-plane normal stress $(\overline{\sigma_x})$ and transverse displacement $(\bar{w})$ by Kant, et. al. [18] are plotted on same graph for comparison with the present solution results. This comparison clearly indicates that the present results are very close to the semi analytical solution.

VIII. CONCLUSION

The HOSNT11 theory has been successfully applied for the thermal analysis of the simply supported composite laminated plates with constant and linear temperature variation through thickness. This model considers the effect of transverse shear deformation hence eliminates the need of shear correction factor. Results of stresses and displacements are presented for different aspect ratios. The result obtained shows the excellent agreement with the semi-analytical model available in literature which demonstrates the accuracy of the present HOSNT11 model.

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Table 1.0 Maximum stresses and the displacement of square homogeneous orthotropic plate under thermal load, CASE A: $\Delta T(x, y, \pm h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$

aspect ratio	$\overline{\sigma_x}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\overline{\sigma_y}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\overline{\tau_{xy}}\,(0,0,\pm\frac{h}{2})$		$\overline{w}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\bar{u}\,(0,\frac{b}{2},\pm\frac{h}{2})$	$\bar{v}\,(\frac{a}{2},0,\pm\frac{h}{2})$
	Semi-analytical model	Present	Semi-analytical model	Present	Semi-analytical model	Present	Semi-analytical model	Present	Present	Present
10	-0.1261	-0.1294 [2.64]	-0.2092	-0.2083 [-0.41]	-0.2233	-0.2230 [-0.15]	± 0.378	±0.3584 [-5.18]	-0.1736	-14.0204
20	-0.0484	-0.0486 [0.41]	0.0538	-0.0537 [-0.11]	-0.0547	-0.0547 [-0.05]	± 0.0236	±0.0224 [-5.06]	-0.0105	-3.4702

[ ]% Error = (Present – Semi-analytical) x 100/ Semi-analytical

Table 2.0 Maximum stresses and the displacement of square homogeneous orthotropic plate under thermal load, CASE B: $\Delta T(x, y, h/2) = -\Delta T(x, y, -h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$

aspect ratio	$\overline{\sigma_x}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\overline{\sigma_y}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\overline{\tau_{xy}}\,(0,0,\pm\frac{h}{2})$		$\overline{w}\,(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})$		$\bar{u}\,(0,\frac{b}{2},\pm\frac{h}{2})$	$\bar{v}\,(\frac{a}{2},0,\pm\frac{h}{2})$
	Semi-analytical model	Present	Semi-analytical model	Present	Semi-analytical model	Present	Semi-analytical model	Present	Present	Present
10	±0.4845	± 0.4823[-0.45]	-/+ 0.5638	-/+ 0.5638[-0.01]	-/+0.638	-/+ 0.6363[-0.26]	1.4042	1.3840[-1.44]	-1.7001	-2.3508
20	±0.1198	± 0.1197[-0.11]	-/+ 0.1448	-/+ 0.1448[-0.02]	-/+0.1405	-/+0.1404[-0.08]	0.2916	0.2903[-0.44]	-0.4256	-0.4681

[ ]% Error = (Present – Semi-analytical) x 100/ Semi-analytical

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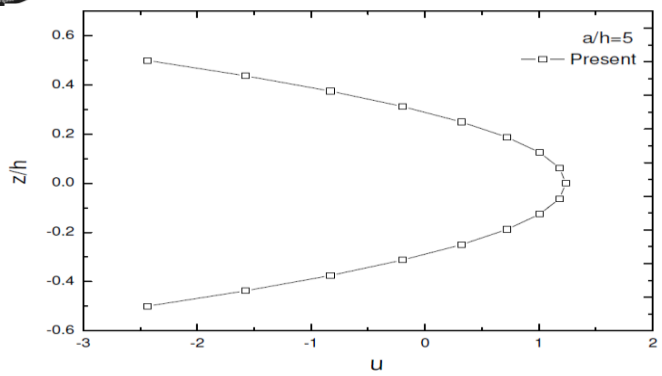

Fig 2.0 (a)

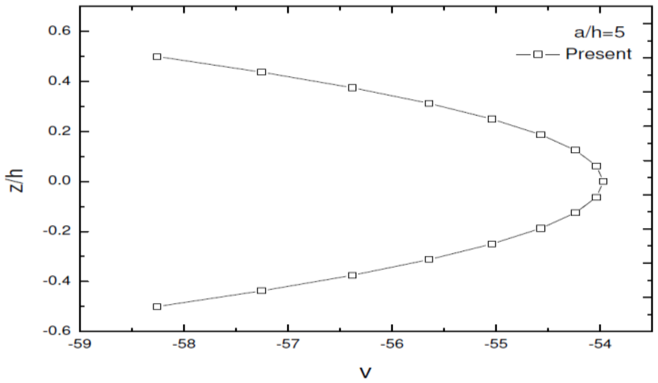

Fig 2.0 (b)

Figure 2.0 Variation of in-plane displacement (a) *u*; (b) *v* along the thickness direction of homogeneous orthotropic plate subjected to thermal load,

$$\Delta T(x, y, \pm h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case A.

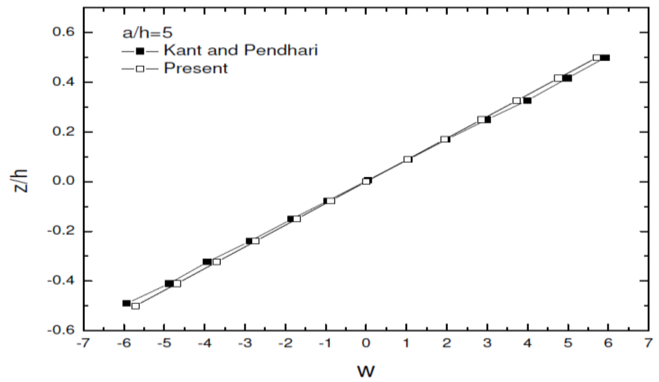

Fig 3.0 (a)

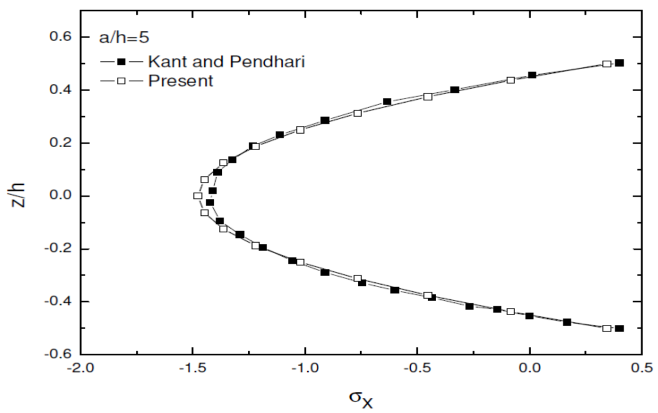

Fig 3.0 (b)

Figure 3.0 Variation of (a) transverse displacement w; (b) in-plane normal stress σ_x along the thickness direction of homogeneous orthotropic plate subjected to thermal load,

$$\Delta T(x, y, \pm h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case A.

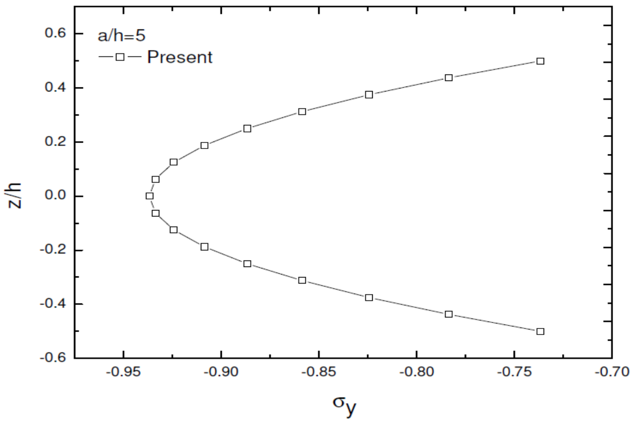

Fig 4.0 (a)

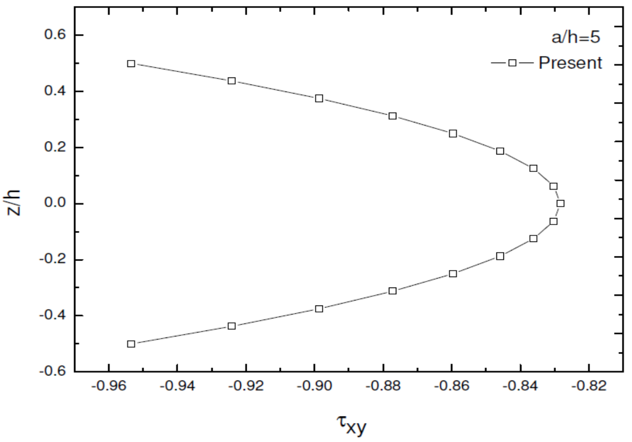

Fig 4.0 (b)

Figure 4.0 Variation of (a) inplane normal stress σ_y; (b) in-plane shear stress τ_{xy} along the thickness direction of homogeneous orthotropic plate subjected to thermal load,

$$\Delta T(x, y, \pm h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case A.

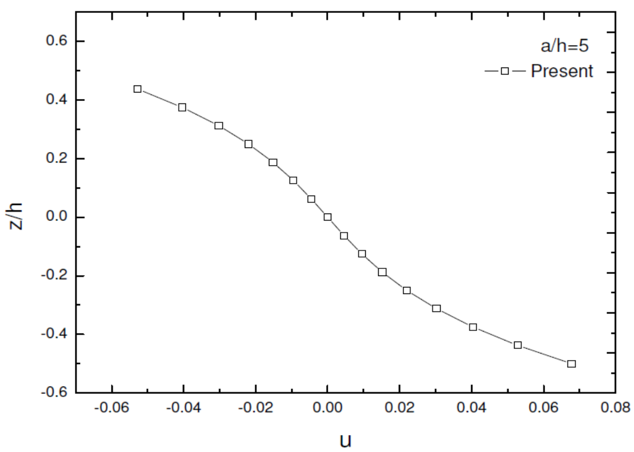

Fig 5.0 (a)

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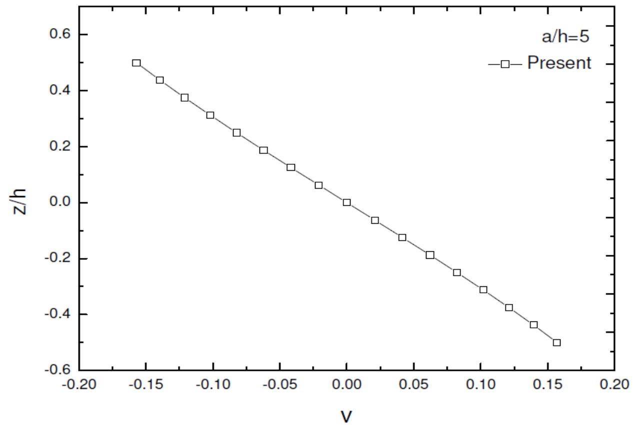

Fig 5.0 (b)

Figure 5.0 Variation of in-plane displacement (a) *u*; (b) *v* along the thickness direction of homogeneous orthotropic plate subjected to thermal load

$$\Delta T(x, y, h/2) = -\Delta T(x, y, -h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case B.

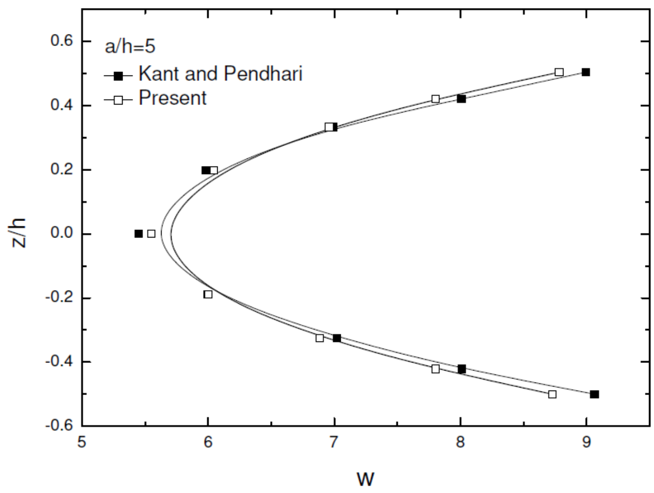

Fig 6.0 (a)

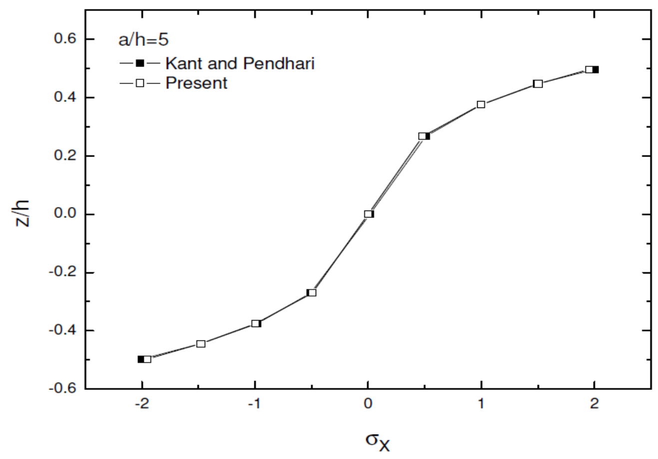

Fig 6.0 (b)

Figure 6.0 Variation of (a) transverse displacement w; (b) in-plane normal stress σ_x along the thickness direction of homogeneous orthotropic plate subjected to thermal load,

$$\Delta T(x, y, h/2) = -\Delta T(x, y, -h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case B.

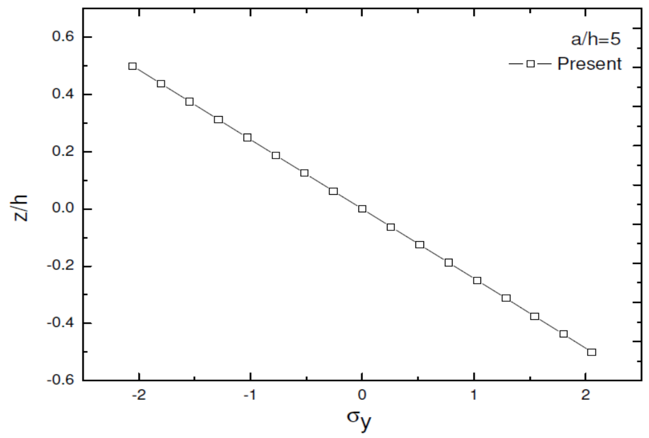

Fig 7.0 (a)

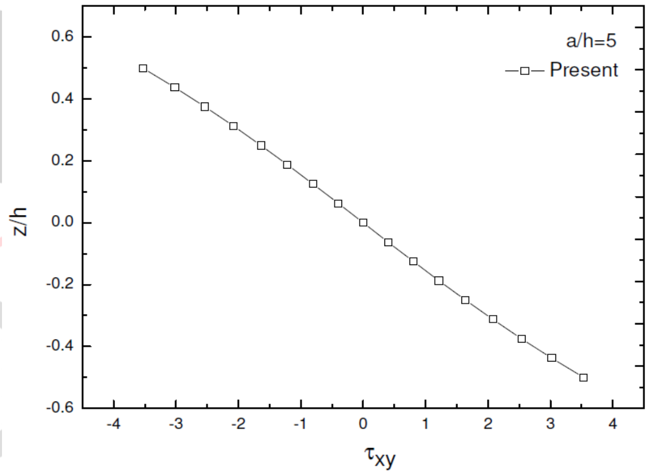

Fig 7.0 (b)

Figure 7.0 Variation of (a) inplane normal stress σ_y ; (b) in-plane shear stress τ_{xy} along the thickness direction of homogeneous orthotropic plate subjected to thermal load,

$$\Delta T(x, y, h/2) = -\Delta T(x, y, -h/2) = T_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$, Case B.

APPENDIX A: Coefficients of [*X*] matrix

$$X_{1,1} = -\frac{h\pi(12b^2\pi Q_{11}+12a^2\pi Q_{44})}{12a^2b^2}$$

$$X_{1,2} = -\frac{h\pi(12ab\pi Q_{12}+12ab\pi Q_{44})}{12a^2b^2}$$

$X_{1,3} = 0$

$X_{1,4} = 0$

$X_{1,5} = 0$

$$X_{1,6} = \frac{h\pi Q_{13}}{a}$$

$$X_{1,7} = -\frac{h\pi(b^2h^2\pi Q_{11}+a^2h^2\pi Q_{44})}{12a^2b^2}$$

$$X_{1,8} = -\frac{h\pi(abh^2\pi Q_{12}+abh^2\pi Q_{44})}{12a^2b^2}$$

$X_{1,9} = 0$

$X_{1,10} = 0$

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$X_{1,11} = 0$

$X_{2,2} = -\frac{h\pi(12a^2\pi Q_{22}+12b^2\pi Q_{44})}{12a^2b^2}$

$X_{2,3} = 0$

$X_{2,4} = 0$

$X_{2,5} = 0$

$X_{2,6} = \frac{h\pi Q_{23}}{b}$

$X_{2,7} = -\frac{h\pi(abh^2\pi Q_{12}+abh^2\pi Q_{44})}{12a^2b^2}$

$X_{2,8} = -\frac{h\pi(a^2h^2\pi Q_{22}+b^2h^2\pi Q_{44})}{12a^2b^2}$

$X_{2,9} = 0$

$X_{2,10} = 0$

$X_{2,11} = 0$

$X_{3,3} = -\frac{h\pi(12a^2\pi Q_{55}+12b^2\pi Q_{66})}{12a^2b^2}$

$X_{3,4} = -\frac{h\pi Q_{66}}{a}$

$X_{3,5} = -\frac{h\pi Q_{55}}{b}$

$X_{3,6} = 0$

$X_{3,7} = 0$

$X_{3,8} = 0$

$X_{3,9} = -\frac{h\pi(a^2h^2\pi Q_{55}+b^2h^2\pi Q_{66})}{12a^2b^2}$

$X_{3,10} = -\frac{h^3\pi Q_{66}}{4a}$

$X_{3,11} = -\frac{h^3\pi Q_{55}}{4b}$

$X_{4,4} = -\frac{h(a^2h^2\pi^2Q_{44}+b^2(h^2\pi^2Q_{11}+12a^2Q_{66}))}{12a^2b^2}$

$X_{4,5} = -\frac{h(20abh^2\pi^2Q_{12}+20abh^2\pi^2Q_{44})}{240a^2b^2}$

$X_{4,6} = 0$

$X_{4,7} = 0$

$X_{4,8} = 0$

$X_{4,9} = -\frac{h(-40ab^2h^2\pi Q_{13}+20ab^2h^2\pi Q_{66})}{240a^2b^2}$

$X_{4,10} = -\frac{h(a^2h^4\pi^2Q_{44}+b^2(h^4\pi^2Q_{11}+20a^2h^2Q_{66}))}{80a^2b^2}$

$X_{4,11} = -\frac{h(3abh^4\pi^2Q_{12}+3abh^4\pi^2Q_{44})}{240a^2b^2}$

$X_{5,5} = -\frac{h(20a^2h^2\pi^2Q_{22}+20b^2h^2\pi^2Q_{44}+240a^2b^2Q_{55})}{240a^2b^2}$

$X_{5,6} = 0$

$X_{5,7} = 0$

$X_{5,8} = 0$

$X_{5,9} = -\frac{h(-40a^2bh^2\pi Q_{23}+20a^2bh^2\pi Q_{55})}{240a^2b^2}$

$X_{5,10} = -\frac{h(3abh^4\pi^2Q_{12}+3abh^4\pi^2Q_{44})}{240a^2b^2}$

$X_{5,11} = -\frac{h(3a^2h^4\pi^2Q_{22}+3b^2h^4\pi^2Q_{44}+60a^2b^2h^2Q_{55})}{240a^2b^2}$

$X_{6,6} = \frac{h(-12a^2b^2Q_{33}-a^2h^2\pi^2Q_{55}-b^2h^2\pi^2Q_{66})}{12a^2b^2}$

$X_{6,7} = \frac{h(ab^2h^2\pi Q_{13}-2ab^2h^2\pi Q_{66})}{12a^2b^2}$

$X_{6,8} = \frac{h(a^2bh^2\pi Q_{23}-2a^2bh^2\pi Q_{55})}{12a^2b^2}$

$X_{6,9} = 0$

$X_{6,10} = 0$

$X_{6,11} = 0$

$X_{7,7} = -\frac{h^3(3b^2h^2\pi^2Q_{11}+3a^2h^2\pi^2Q_{44}+80a^2b^2Q_{66})}{240a^2b^2}$

$X_{7,8} = -\frac{h^3(3abh^2\pi^2Q_{12}+3abh^2\pi^2Q_{44})}{240a^2b^2}$

$X_{7,9} = 0$

$X_{7,10} = 0$

$X_{7,11} = 0$

$X_{8,8} = -\frac{h^3(3a^2h^2\pi^2Q_{22}+3b^2h^2\pi^2Q_{44}+80a^2b^2Q_{55})}{240a^2b^2}$

$X_{8,9} = 0$

$X_{8,10} = 0$

$X_{8,11} = 0$

$X_{9,9} = -\frac{h^3(80a^2b^2Q_{33}+3a^2h^2\pi^2Q_{55}+3b^2h^2\pi^2Q_{66})}{240a^2b^2}$

$X_{9,10} = -\frac{h^3(-6ab^2h^2\pi Q_{13}+9ab^2h^2\pi Q_{66})}{240a^2b^2}$

$X_{9,11} = -\frac{h^3(-6a^2bh^2\pi Q_{23}+9a^2bh^2\pi Q_{55})}{240a^2b^2}$

$X_{10,10} = -\frac{h^3(5a^2h^4\pi^2Q_{44}+b^2(5h^4\pi^2Q_{11}+252a^2h^2Q_{66}))}{2240a^2b^2}$

$X_{10,11} = -\frac{h^3(5abh^4\pi^2Q12+5abh^4\pi^2Q_{44})}{2240a^2b^2}$

$X_{11,11} = -\frac{h^3(5a^2h^4\pi^2Q_{22}+5b^2h^4\pi^2Q_{44}+252a^2b^2h^2Q_{55})}{2240a^2b^2}$

APPENDIX B: Coefficients of $[T_r]$ matrix

$T_{r\,1,1} = \frac{h\pi(12ab^2Q_{13}\alpha 2+12ab^2Q_{11}\alpha x0+12ab^2Q_{12}\alpha y0)}{12a^2b^2}$

$T_{r\,2,1} = \frac{h\pi(12a^2bQ_{23}\alpha 2+12a^2bQ_{12}\alpha x0+12a^2bQ_{22}\alpha y0)}{12a^2b^2}$

$T_{r\,3,1} = 0$

$T_{r\,4,1} = 0$

$T_{r\,5,1} = 0$

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$T_{r\,6,1} = -\frac{h(12a^2b^2Q_{33}\alpha 2+12a^2b^2Q_{13}\alpha x0+12a^2b^2Q_{23}\alpha y0)}{12a^2b^2}$

$T_{r\,7,1} = \frac{h^3(20ab^2\pi Q_{13}\alpha 2+20ab^2\pi Q_{11}\alpha x0+20ab^2\pi Q_{12}\alpha y0)}{240a^2b^2}$

$T_{r\,8,1} = \frac{h^3(20a^2b\pi Q_{23}\alpha 2+20a^2b\pi Q_{12}\alpha x0+20a^2b\pi Q_{22}\alpha y0)}{240a^2b^2}$

$T_{r\,9,1} = 0$

$T_{r\,10,1} = 0$

$T_{r\,11,1} = 0$

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