Influence of Soret and Dufour effects on unsteady flow past a moving horizontal cylinder

[1]Bismeeta Buragohain, [2]B R Sharma

[1]Research Scholar, [2]Professor, Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India.

Abstract - The unsteady laminar flow due to a moving horizontal cylinder with Soret effect and Dufour effect is investigated numerically. The governing partial differential equations in cylindrical form are first transformed into dimensionless governing equations and then solved numerically by an iterative technique based on finite difference scheme. The effect of Soret number, Dufour number, Prandtl number and Schmidt number on temperature and concentration distributions are discussed graphically. The local Nusselt number is also shown graphically.

Keywords: *Soret effect, Dufour effect, unsteady flow, moving horizontal cylinder, finite difference method.*

I. INTRODUCTION

The unsteady laminar flow due to moving horizontal cylinder is important in extrusion and industrial processes. Takhar et. al. [1] discussed the combined heat and mass transfer along a vertical moving cylinder with a free stream. Ganesan and Langanathan [2] studied unsteady free convection flow over a moving vertical cylinder with heat and mass transfer. Saeid [3] studied analysis of free convection about a horizontal cylinder in a porous media using a thermal non-equilibrium model. N. Zainuddin et al. [4] discussed oscillatory free convection about a horizontal circular cylinder in presence of heat generation. Sharma and konwar [5] studied the thermal diffusion, radiation and chemical reaction effects about an axially moving cylinder in a binary fluid mixture.

The mass flux caused by temperature gradient is referred as the Soret effect. The Soret effect has been consumed for isotope division and in mixtures between light molecular weight of gases. The energy flux caused by concentration gradient is referred as the Dufour effect. The Dufour effect is created a significant magnitude for gases of medium molecular weight. Similarity solutions for double diffusion effects on free convection flow in a non-Darcy porous medium with Soret and Dufour effects has been obtained by Partha et al. [6]. El-Kabeir [7] discussed the Soret and Dufour effects on laminar flow over a permeable stretching cylinder in presence of chemical reaction. Ramzan et. al [8] studied the soret and Dufour effects on MHD stagnation point flow by a permeable stretching cylinder in presence of heat generation/ absorption and chemical reaction. Sharma and Borgohain [9] discussed chemical reaction, Soret and Dufour effects on unsteady MHD mixed convection flow in the forward stagnation region of a rotating sphere in a porous medium. Mahdy [10] discussed the Soret and Dufour effects on flow and heat transfer of a casson fluid due to a stretching cylinder.

II. FORMULATION OF THE PROBLEM

Consider a laminar flow which is unsteady past an impulsively started semi-infinite horizontal cylinder of radius R. The z'-axis is considered along the axis of the cylinder and the r'-axis is considered in the radial direction. It is assumed that at first the cylinder and the fluid are maintained at the same temperature T'_∞ and concentration level C'_∞ for all $t' \leq 0$.

At time $t' > 0$ the cylinder starts moving in the horizontal direction with a uniform velocity u'_0 and surface of cylinder is raised to a uniform temperature $T_w{}'$ and concentration $C_w{}'$.

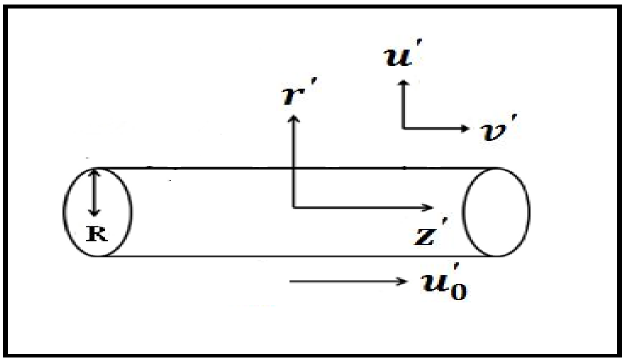

Figure 1. Physical configuration and coordinate system

The governing equations are

$$\frac{1}{r'}\frac{\partial (r'u')}{\partial r'} + \frac{\partial v'}{\partial z'} = 0, \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial r'} + v'\frac{\partial u'}{\partial z'} = -\frac{1}{\rho}\frac{\partial P'}{\partial r'} + \nu\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'} + \frac{\partial^2 u'}{\partial z'^2} - \frac{u'}{r'^2}\right), \qquad (2)$$

 DOI : 10.18231/2454-9150.2019.0020

$$\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial r'}+v'\frac{\partial v'}{\partial z'}=-\frac{1}{\rho}\frac{\partial P'}{\partial z'}+\nu\left(\frac{\partial^2 v'}{\partial r'^2}+\frac{1}{r'}\frac{\partial v'}{\partial r'}+\frac{\partial^2 v'}{\partial z'^2}\right), \quad (3)$$

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial r'}+v'\frac{\partial T'}{\partial z'}=\frac{k}{\rho C_P}\left(\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}+\frac{\partial^2 T'}{\partial z'^2}\right)+\frac{D_m k_t}{C_S C_P}\left(\frac{\partial^2 C'}{\partial r'^2}+\frac{1}{r'}\frac{\partial C'}{\partial r'}+\frac{\partial^2 C'}{\partial z'^2}\right), \quad (4)$$

$$\frac{\partial C'}{\partial t'}+u'\frac{\partial C'}{\partial r'}+v'\frac{\partial C'}{\partial z'}=D_m\left(\frac{\partial^2 C'}{\partial r'^2}+\frac{1}{r'}\frac{\partial C'}{\partial r'}+\frac{\partial^2 C'}{\partial z'^2}\right)+\frac{D_m k_t}{T_m}\left(\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}+\frac{\partial^2 T'}{\partial z'^2}\right), \quad (5)$$

where u' and v' are the velocity components in r' and z' directions respectively, ν is the kinematic viscosity, ρ is the fluid density, k is the thermal conductivity, D_m is mass diffusivity, C_P is the specific heat at constant pressure, C_S is concentration susceptibility, T_m is the mean fluid temperature, k_t is the thermal diffusivity ratio.

The appropriate boundary conditions for the problem are given by

$$\left.\begin{array}{l} u'=0,\ v'=0,\ T'=T_\infty',\ C'=C_\infty' \ \text{for}\ t'\le 0,\\ t'>0: \end{array}\right\} \quad (6)$$

$$\left.\begin{array}{l} u'=u_0',\ v'=0,\ T'=T_w',\ C'=C_w' \ \text{at}\ r'=R,\\ u'=0,\ T'=T_\infty',\ C'=C_\infty' \ \text{at}\ z'=0\\ u'\to 0,\ T'\to T_\infty',\ C'\to C_\infty' \ \text{as}\ r'\to\infty \end{array}\right\} \quad (7)$$

Introducing the following non dimensional quantities

$$\left.\begin{array}{l} u_0'=\frac{-2\nu}{R}u_0,\ u=\frac{u'}{u_0'},\ v=\frac{v'}{u_0'},\\ r'=rR,\ z'=zR,\ \mathrm{Sr}=\frac{D_m k_t}{T_m \nu}\frac{(T_w'-T_\infty')}{(C_w'-C_\infty')},\\ P'=\frac{4\nu^2 P}{R^2},\ \mathrm{Du}=\frac{D_m k_t}{C_S C_P}\frac{(C_w'-C_\infty')}{(T_w'-T_\infty')},\\ \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'},\ \phi=\frac{C'-C_\infty'}{C_w'-C_\infty'},\\ \mathrm{Sc}=\frac{\nu}{D_m},\ \mathrm{Pr}=\frac{\mu C_P}{k},\ t=\frac{t'u_0'}{R} \end{array}\right\} \quad (8)$$

where Pr is prandtl number, Sc is Schmidt number, Sr is Soret number, Du is Dufour number.

Substituting equation (8) into equations (1), (3), (4) and (5), we get the following equations:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial v}{\partial z}=0, \quad (9)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=\frac{-1}{2}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}\right), \quad (10)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial r}+v\frac{\partial \theta}{\partial z}=\frac{-1}{2}\left(\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}+\frac{\partial^2 \theta}{\partial z^2}\right)+\mathrm{Du}\left(\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{\partial^2 \phi}{\partial z^2}\right)\right), \quad (11)$$

$$\frac{\partial \phi}{\partial t}+u\frac{\partial \phi}{\partial r}+v\frac{\partial \phi}{\partial z}=\frac{-1}{2}\left(\frac{1}{Sc}\left(\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{\partial^2 \phi}{\partial z^2}\right)+\mathrm{Sr}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}+\frac{\partial^2 \theta}{\partial z^2}\right)\right), \quad (12)$$

The boundary condition (6), (7) and (8) becomes

$$u=0,\ v=0,\ \theta=0,\ \phi=0 \quad \text{at} \quad t\le 0$$

$t>0$:

$$\left.\begin{array}{l} u=1,\ v=0,\ \theta=1,\ \phi=1 \ \text{at}\ r=1,\\ u=0,\ \theta=0,\ \phi=0 \quad \text{at}\ z=0\\ u\to 0,\ \theta\to 0,\ \phi\to 0 \quad \text{as}\ r\to\infty \end{array}\right\} \quad (13)$$

Now, the pressure P can be determined from equation (2) in the form:

$$\frac{P}{\rho}=\frac{1}{2}\left(\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)-\int\frac{\partial u}{\partial t}dr\right)-u^2 \quad (14)$$

The Nusselt number Nu and Sherwood number Sh, which are defined as

$$\left.Nu=\frac{Rq_w}{k(T_w'-T_\infty')},\ Sh=\frac{Rq_m}{D_m(C_w'-C_\infty')}\right\} \quad (15)$$

The heat transfer q_w and the mass transfer q_m from the surface of the cylinder are given by

$$q_w=-k\left(\frac{\partial T'}{\partial r'}\right)_{r'=R},\ q_m=-D_m\left(\frac{\partial C'}{\partial r'}\right)_{r'=R} \quad (16)$$

Using equations (8) and (16) in equation (15) we have,

$$\mathrm{Nu}=-2\theta'(1),\ \mathrm{Sh}=-2\phi'(1)$$

III. METHOD OF SOLUTION

The dimensionless partial differential equations (9)-(12) subject to the initial and boundary conditions (13) are reduced to a system of difference equations using the following finite difference scheme $\frac{\partial v}{\partial y}=\frac{v^{i+1}{}_{j,n}-v^{i}{}_{j,n}}{\Delta y}$, $\frac{\partial^2 v}{\partial y^2}=\frac{v^{i+1}{}_{j,n}+v^{i-1}{}_{j,n}-2v^{i}{}_{j,n}}{(\Delta y)^2}$ and then the system of difference equations is solved numerically by an iterative method. The finite difference scheme of equations (9) – (12) is as follows:

$$\frac{u^{i+1}{}_{j,n}-u^{i}{}_{j,n}}{\Delta r}+\frac{u^{i}{}_{j,n}}{r}+\frac{v^{j+1}{}_{i,n}-v^{j}{}_{i,n}}{\Delta z}=0, \quad (17)$$

$$\frac{v^{n+1}{}_{i,j}-v^{n}{}_{i,j}}{\Delta t}+u^{i}{}_{j,n}\left(\frac{v^{i+1}{}_{j,n}-v^{i}{}_{j,n}}{\Delta r}\right)+v^{i}{}_{j,n}\left(\frac{v^{j+1}{}_{i,n}-v^{j}{}_{i,n}}{\Delta z}\right)=$$

 DOI : 10.18231/2454-9150.2019.0020

$$\frac{-1}{2}\left\{\left(\frac{v^{i+1}{}_{j,n}-2v^{i}{}_{j,n}+v^{i-1}{}_{j,n}}{(\Delta r)^2}\right)+\frac{1}{r}\left(\frac{v^{i+1}{}_{j,n}-v^{i}{}_{j,n}}{\Delta r}\right)+\left(\frac{v^{j+1}{}_{i,n}-2v^{j}{}_{i,n}+v^{j-1}{}_{i,n}}{(\Delta z)^2}\right)\right\}, \qquad (18)$$

$$\frac{\theta^{n+1}{}_{i,j}-\theta^{n}{}_{i,j}}{\Delta t}+u^{i}{}_{j,n}\left(\frac{\theta^{i+1}{}_{j,n}-\theta^{i}{}_{j,n}}{\Delta r}\right)+v^{i}{}_{j,n}\left(\frac{\theta^{j+1}{}_{i,n}-\theta^{j}{}_{i,n}}{\Delta z}\right)=$$
$$\frac{-1}{2Pr}\left\{\left(\frac{\theta^{i+1}{}_{j,n}-2\theta^{i}{}_{j,n}+\theta^{i-1}{}_{j,n}}{(\Delta r)^2}\right)+\frac{1}{r}\left(\frac{\theta^{i+1}{}_{j,n}-\theta^{i}{}_{j,n}}{\Delta r}\right)+\left(\frac{\theta^{j+1}{}_{i,n}-2\theta^{j}{}_{i,n}+\theta^{j-1}{}_{i,n}}{(\Delta z)^2}\right)\right\}-$$
$$\frac{Du}{2}\left\{\left(\frac{\phi^{i+1}{}_{j,n}-2\phi^{i}{}_{j,n}+\phi^{i-1}{}_{j,n}}{(\Delta r)^2}\right)+\frac{1}{r}\left(\frac{\phi^{i+1}{}_{j,n}-\phi^{i}{}_{j,n}}{\Delta r}\right)+\left(\frac{\phi^{j+1}{}_{i,n}-2\phi^{j}{}_{i,n}+\phi^{j-1}{}_{i,n}}{(\Delta z)^2}\right)\right\} \qquad (19)$$

$$\frac{\phi^{n+1}{}_{i,j}-\phi^{n}{}_{i,j}}{\Delta t}+u^{i}{}_{j,n}\left(\frac{\phi^{i+1}{}_{j,n}-\phi^{i}{}_{j,n}}{\Delta r}\right)+v^{i}{}_{j,n}\left(\frac{\phi^{j+1}{}_{i,n}-\phi^{j}{}_{i,n}}{\Delta z}\right)=$$
$$\frac{-1}{2Sc}\left\{\left(\frac{\phi^{i+1}{}_{j,n}-2\phi^{i}{}_{j,n}+\phi^{i-1}{}_{j,n}}{(\Delta r)^2}\right)+\frac{1}{r}\left(\frac{\phi^{i+1}{}_{j,n}-\phi^{i}{}_{j,n}}{\Delta r}\right)+\left(\frac{\phi^{j+1}{}_{i,n}-2\phi^{j}{}_{i,n}+\phi^{j-1}{}_{i,n}}{(\Delta z)^2}\right)\right\}-$$
$$\frac{Sr}{2}\left\{\left(\frac{\theta^{i+1}{}_{j,n}-2\theta^{i}{}_{j,n}+\theta^{i-1}{}_{j,n}}{(\Delta r)^2}\right)+\frac{1}{r}\left(\frac{\theta^{i+1}{}_{j,n}-\theta^{i}{}_{j,n}}{\Delta r}\right)+\left(\frac{\theta^{j+1}{}_{i,n}-2\theta^{j}{}_{i,n}+\theta^{j-1}{}_{i,n}}{(\Delta z)^2}\right)\right\} \qquad (20)$$

IV. RESULTS AND DISCUSSION

Numerical calculations are calculated for various values of the Dufour number *Du*, the Soret number *Sr*, the prandtl number *Pr*, time *t* and the Schmidt number *Sc*. From the process of numerical computation, the local Nusselt number and the local Sherwood number are proportional to $-\theta'(1)$ and $-\phi'(1)$ repectively, Here the local Nusselt number is shown graphically. To exhibit the Dufour effect and Soret effect on temperature and concentration distributions we have drawn the temperature profile in **Figures. 2-3** and concentration profile in **Figures. 4-5** against *r* for various values of Dufour number and Soret number. The effect of Prandtl number on the temperature profile is also shown in **Figure. 6**.

It is observed from **Figures. 2-3** that the temperature increases with increase in Dufour number and the temperature decreases with increase in Soret number. Thermal diffusivity increases with increase in Dufour number, the process results in increasing manner of energy transfer increase thermal boundary layer. Soret number represents the involvement of the temperature gradient to concentration gradient. It can be seen that increase in Soret number causes fall in temperature.

It is noticed from **Figures. 4-5** that the concentration decreases with increase in Dufour number and the concentration increases with increase in Soret number. Dufour number represents the involvement of the concentration gradient to temperature gradient. It can be seen that increase in Dufour number causes fall in concentration. Increase in Soret number causes rise in concentration as a result of greater mass diffusivity. It is seen from the **Figure. 6** that increase in Prandtl number causes fall in temperature as a result of lower thermal diffusivity.

It is observed from **Figure. 7** that the concentration decreases with increase in Schmidt number. With increase in Schmidt number mass diffusivity decreases causing a reduction in concentration. It is noticed from **Figure. 8** that temperature increases with the increase of time.

From **Figures.9-10** it can be noticed that the local Nusselt number increases with the decrease of Dufour number and increase of Prandtl number. The local Nusselt number decreases with the increase of Soret number and increase of Schmidt number.

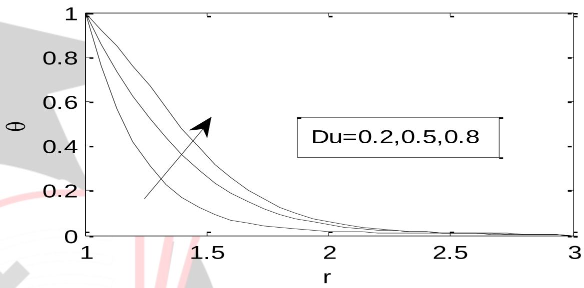

Figure 2. The temperature profile for Dufour number

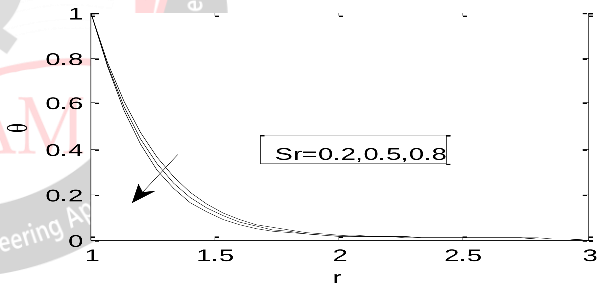

Figure 3. The temperature profile for Soret number

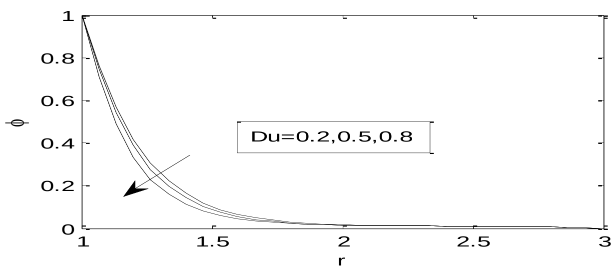

Figure 4. The concentration profile for Dufour number

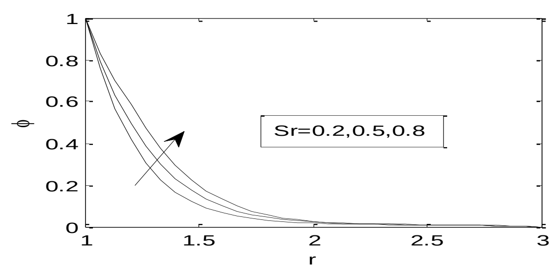

Figure 5. The concentration profile for Soret number

 DOI : 10.18231/2454-9150.2019.0020

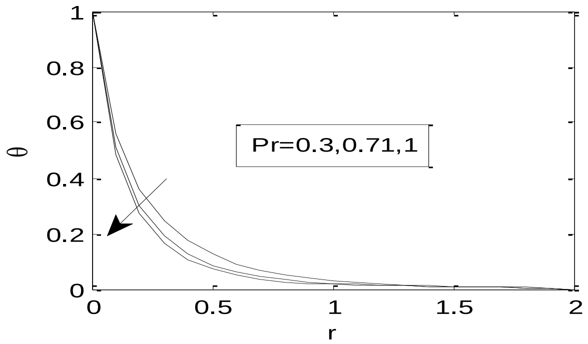

Figure 6. The temperature profile for Prandtl number

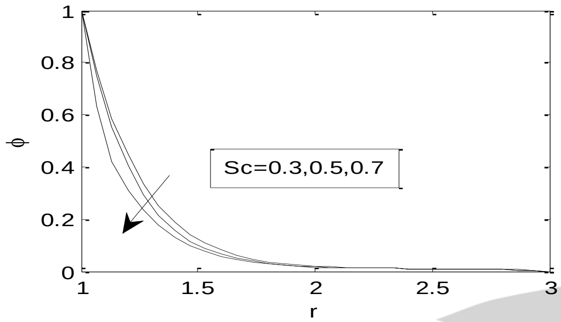

Figure 7. The concentration profile for Schmidt number

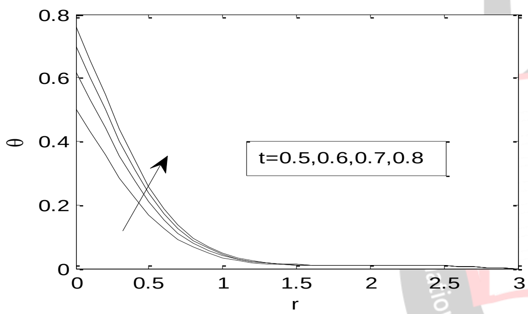

Figure 8. The temperature profile for time

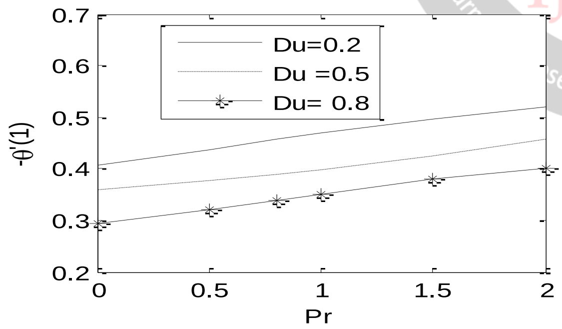

Figure 9. Effect of Dufour number on the Nusselt number with various values of Prandtl number

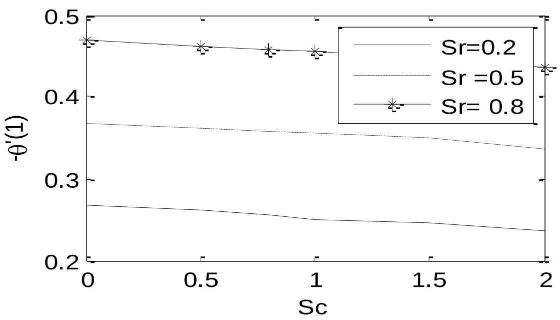

Figure 10. Effect of Soret number on the Nusselt number with various values of Schmidt number

V. CONCLUSION

- The temperature is decelerated but the concentration is accelerated with the increase of Soret number (*Sr*).
- The temperature is accelerated but the concentration is decelerated with the increase of Dufour number (*Du*).
- The temperature is decelerated with the increase of Prandtl number (*Pr*) and the concentration is decelerated with the increase of Schmidt number (*Sc*).
- The temperature is progressively accelerated with the increase of time (*t*) .
- The local Nusselt number decreases with the increase of Soret number (*Sr*) and increase of Schmidt number(*Sc*).
- The local Nusselt number increases with the decrease of Dufour number (*Du*) and increase of Prandtl number (*Pr*).

REFERENCES

[1] H. S. Takhar , A. J. Chamkha and G. Nath , " Combined heat and mass transfer along a vertical moving cylinder with a free stream" Heat and Mass transfer, 2000, Vol. 36, pp. 237-246.

[2] P. Ganesan and P. Langanathan, " Unsteady free convection flow over a moving vertical cylinder with heat and mass transfer " International journal of heat and mass transfer, 2001, Vol. 37, pp. 59-65.

[3] N. H. Saeid, " Analysis of free convection about a horizontal cylinder in a porous media using a thermal non-equilibrium model" International communications in heat and mass transfer, 2006 , Vol. 33, pp. 158-165.

[4] N. Zainuddin, M. Ismoen , R. Roslan and I. Hashim, "discussed oscillatory free convection about a horizontal circular cylinder in presence of heat generation " Mathematical Problems in Engineering , 2014, Vol. 2014, pp. 1-8.

[5] B. R. Sharma and H. Konwar, " MHD flow, heat and mass transfer due to axially moving cylinder in presence of thermal diffusion, radiation and chemical reactions in a binary fluid mixture" IJCA, 2015, Vol. 110, pp. 52-59.

[6] M.K. Partha, P.V.S.N. Murthy and G.P. Raja Sekhar, "Soret and Dufour Effects in a Non-Darcy Porous Medium" ASME J. Heat Transfer, 2006, Vol. 128, pp. 605-610.

[7] M. M. S. El – Kabeir, "Soret and Dufour effects on heat and mass transfer due to a stretching cylinder saturated porous medium with chemically reactive species" Latin American Applied Research, 2011, Vol. 41, pp. 331–337.

[8] M. Ramzan, M. Farooq and T. Hayat, " MHD stagnation point flow by a permeable stretching cylinder with Soret and Dufour effects" J. Cent. South Univ., 2015, Vol. 22, pp. 707–716.

[9] B. R. Sharma and D. Borgohain, " Effects of chemical reaction, Soret and Dufour number on unsteady MHD mixed convection flow in the forward stagnation region of a rotating sphere in a porous medium" IJSER, 2015, Vol. 6, pp. 1878-1887.

[10] A. Mathy, " Heat transfer and flow of a Casson fluid due to a stretching cylinder with Soret and Dufour effects" Journal of Engineering physics and Thermophysics, 2015, Vol. 88, pp. 928-936.

 DOI : 10.18231/2454-9150.2019.0020