ON CAYLEY-SYMMETRIC Γ-SEMIGROUPS

Dr.S.V.B.Subrahmanyeswara Rao, #T.Srinivasa Rao, $N.Rama Krishna

Professor, #Assistant Professor, Ramachandra College of Engineering,Eluru, W.G. Dist,AP

manyam4463@gmail.com, thota90@gmail.com, nandigam.mrk@gmail.com

ABSTRACT. The concept of Cayley-symmetric Γ-semigroups is introduced, and many equivalent conditions of a Cayley-symmetric Γ-semigroups are given. It is proved that a strong semilattice of self-decomposable Γ-semigroups \( S_a \) is Cayley-symmetric if and only if each \( S_a \) is Cayley-symmetric.

Key words: Generalized Cayley graphs, Cayley-Symmetric Γ-semigroup, strong semilattice of Γ-semigroups, self-decomposable.

I. INTRODUCTION

Based on the research papers on Cayley graphs of semigroups, Yongwen Zhu first introduced the concept of generalized Cayley graphs of semigroups in which some fundamental properties of generalized Cayley graphs of semigroups were studied. Based on the works on Cayley graphs of semigroups, we introduced the concept of Cayley-symmetric Γ-semigroups. Several equivalent conditions of a Cayley-symmetric Γ-semigroup are presented in this paper and established a necessary and sufficient condition for a semilattices of Γ-semigroups to be Cayley-symmetric.

II. PRELIMINARIES

Definition 2.1. Let \( T \) be an ideal extension of a semigroup \( S \) and \( \rho \subseteq T^1 \times T^1 \), \( T^1 \) is a semigroup \( T \) with identity adjoined. Then Cayley graph \( \text{Cay}(S, \rho) \) of \( S \) relative to \( \rho \) is defined as the graph with vertex set \( S \) and edge set \( E(\text{Cay}(S, \rho)) \) consisting of those ordered pairs \( (a, b) \), where \( x \rho y = b \) for some \( (x, y) \in \rho \). Also we call these defined Cayley graphs as generalized Cayley graphs.

Notation 2.1. If \( S \) is a semigroup and \( a \in S \), then \( P(a) = S^1 a S^1 \), \( L(a) = S^1 a \), \( R(a) = a S^1 \) are the principal, left, right ideals generated by \( a \) resp. where \( S^1 \) is a semigroup with identity adjoined.

Definition 2.2. Let \( S_a = S^1 \times \{1\} \), \( S_R = \{1\} \times S^1 \), \( S^1_a = S^1 \times S^1 \) be the left, right and the universal relations on \( S^1 \), then the generalized Cayley graphs \( \text{Cay}(S, S_a), \text{Cay}(S, S_R), \text{Cay}(S, S^1) \) are called the left universal, right universal and universal Cayley graphs of \( S \) resp.

Definition 2.3. A semigroup \( S \) is called Cayley-symmetric if \( \text{Cay}(S, S_a) = \text{Cay}(S, S_R) \)

Definition 2.4. Let \( T \) be an ideal extension of a semigroup \( S \). If \( \text{Cay}(S, T_a) = \text{Cay}(S, T_R) \), then we say that \( S \) is Cayley-symmetric in \( T \).
(1) $S$ is Cayley-symmetric in $T$

(2) $L_T(a) = R_{T}(a)$ for every $a \in S$

Proof. Suppose that $S$ is Cayley-symmetric in $T$

From the definition of generalized Cayley graphs, we have $(a, b) \in E(Cay(S, T_L))$

$\iff b = xaa$ for some $x \in T^1, a \in \Gamma$

$\iff b \in T^1\Gamma a = L_T(a)$

Again

$(a, b) \in E(Cay(S, T_R))$

$\iff b = a\beta y$ for some $y \in T^1, \beta \in \Gamma$

$\iff b \in a\Gamma T^1 = R_T(a)$

If $Cay(S, T_L) = Cay(S, T_R)$, then for any $a, b \in S, (a, b) \in E(Cay(S, T_L))$ if and only if $(a, b) \in E(Cay(S, T_R))$. Thus for all $a, b \in S$ we have $b \in L_T(a)$ if and only if $b \in R_T(a)$.

As $S$ is an ideal of $T$, $L_T(a) = T^1\Gamma a \subseteq S$ and $R_T(a) = a\Gamma T^1 \subseteq S$ for every $a \in S$.

Therefore $L_T(a) = R_T(a)$ for every $a \in S$.

Conversely, suppose that $L_T(a) = R_T(a)$ for every $a \in S$.

Then for all $a, b \in S$, we have $b \in L_T(a)$ if and only if $b \in R_T(a)$.

Hence

$(a, b) \in E(Cay(S, T_L))$ if and only if $(a, b) \in E(Cay(S, T_R))$.

i.e., $Cay(S, T_L) = Cay(S, T_R)$ which implies $S$ is Cayley-symmetric in $T$.

IV. CAYLEY-SYMMETRY OF STRONG SEMILATTICES OF $\Gamma$-SEMGROUPS

First we define strong semilattice of $\Gamma$-semigroups as follows:

Let $Y$ be a semilattice and a set of $\Gamma$-semigroups indexed by $Y$, and suppose that, for all $a, b \in S$, we have $b \in L_T(a)$ and only if $b \in R_T(a)$.

Hence

$(a, b) \in E(Cay(S, T_L))$ if and only if $(a, b) \in E(Cay(S, T_R))$.

i.e., $Cay(S, T_L) = Cay(S, T_R)$ which implies $S$ is Cayley-symmetric in $T$.

Theorem 4.3. Suppose that $S = [Y; S_\alpha : \varnothing_{a,\beta}]$, where each $S_\alpha$ is self-decomposable. Then $S$ is Cayley-symmetric if and only if for every $\alpha \in \Upsilon, S_\alpha$ is Cayley-symmetric.

Proof. Since for each $\alpha$, $S_\alpha$ is self-decomposable we have $S = [Y; S_\alpha : \varnothing_{a,\beta}]$ is self-decomposable.

Necessary. Suppose that $S$ is Cayley-symmetric. For $a, b \in S_\alpha$ with $\alpha \in \Upsilon$, by Lemma 4.2, there exist $\beta \in \Upsilon$ and $c \in S_\beta$ such that $aa, b = ca_\alpha a_\beta : a_\alpha, a_\beta \in \Gamma$. Since $aab \in S_\alpha$, we have $\beta \geq \alpha$. Now $S_\alpha$ is self-decomposable, there exists $u \in S_\alpha$ such that $a = u a_\alpha a_\beta a_\alpha$. It gives that $aa, b = ca_\alpha a_\beta = ca_\alpha (ua_\beta a) = (ca_\alpha u)a_\beta a_\alpha$,

where $ca_\alpha u \in S_\alpha$.

Therefore $a\Gamma S_\alpha \subseteq S_\alpha \Gamma a$. Similarly we can show that $S_\alpha \Gamma a \subseteq a\Gamma S_\alpha$, which means that $S_\alpha \Gamma a = a\Gamma S_\alpha$. By Lemma 5.2, $S_\alpha$ is Cayley-symmetric.

Sufficient. Suppose that $S_\alpha$ is Cayley-symmetric for all $\alpha$. Let $a \in S_\alpha$ and $b \in S_\beta$ with $\alpha, \beta \in \Upsilon$.

Take $a \beta = \gamma \in \Upsilon$, then $a, \beta \gamma \leq \gamma$. Since $aab \in S_\gamma$, $\sigma \in \Upsilon$ and $S_\gamma$ is self-decomposable, there exists $x, y \in S_\gamma$ such that $x = (x\alpha a)(a\beta y); \gamma, \varphi \in \Upsilon$.

Therefore $aab = x(aab)y = (x\alpha a)(a\beta y)$, where $x = (x\alpha a)\gamma \in S_\gamma$. Since $S_\gamma$ is Cayley-symmetric, from Lemma 5.2 there exists $z \in S_\gamma$ such that $(x\alpha a)(a\beta y) = z\alpha(a\beta y) = z\alpha(x\alpha a) = (z\alpha x)\alpha a$.

This implies $aab = (z\alpha x)\alpha a \in S_\alpha$. We have proved that $a\Gamma S \subseteq S_\alpha \Gamma a$.Similarly we can prove the inverse conclusion. Hence $a\Gamma S = S_\alpha \Gamma a$. Again by Lemma 5.2, $S$ is Cayley-symmetric.

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REFERENCES


