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On s-k Bi-EP Bi-matrices

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Abstract - The concept of bi-EP bi-matrix and s-k bi-EP bi-matrix as a generalization of EP bi-matrices are introduced in this paper. Also, some properties of bi-EP bi-matrices and s-k bi-EP bi-matrices are obtained.

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Keywords: EP bi-matrix, Bi-EP bi-matrix, s-k bi-EP bi-matrix.

I. INTRODUCTION

The very common procedure of an every scholar of linear algebra is, that starting their studies in detail about the matrices over the fields of complex and real as a main class of algebra. And this is defined through the property of being matrix in the main diagonal [7, 8]. In 1860, the concept of matrices is discovered by the French mathematician Cayley. Matrices have applied in many branches of Science and Applied Mathematics.

Several special types of matrices exists namely, normal matrix, unitary, skew hermitian, hermitian, skew symmetric, symmetric etc. The matrices which play an important role in the spectral theory of rectangular matrices and in the theory of generalized inverses.

Toeplitz introduced the concept of a normal matrix in 1918 with entries from the complex field [18]. He also gave the necessary and sufficient conditions for a complex matrix satisfies to be normal. Since then many researchers have developed the concept and many generalizations of normality were studied [4, 10, 12].

As a generalization of normality Schwerdtfeger introduced the EP matrices concept over the complex field [17]. The class of complex EP matrices includes the class of all non singular matrices, hermitian and normal matrices.

If the range spaces of matrix A and $A^{\dagger}$ are equal means then that complex matrix A is called Ep. The complex matrix A is of order n. Range hermitian matrix is termed as an Ep matrix by Grevile [9]. Pearl [15] has proved that A is an EP matrix if and only if A commutes with $A^{\dagger}$, ie, $AA^{\dagger} = A^{\dagger}A$. The four generalized inverse equations [16] yields an unique solution called $A^{\dagger}$. Since $AA^{\dagger}$ and $A^{\dagger}A$ are the orthogonal projectors on to the space $R(A)$ and $R(A^{*})$, here A is termed as an equiprojector matrix (or) in short 'EP' matrix.

Every square matrix A over a field is similar to its transpose A^{T}, that is, there exists a symmetric matrix B such that $A^{T} = B^{-1}AB$. For instance, let $'V'$ be a permutation matrix having units in the secondary diagonal, then the above similarity transformation defines the matrices being symmetric with respect to the secondary diagonal. Anna Lee [1, 2] has defined secondary symmetric (s-symmetric), secondary skew symmetric (s-skew symmetric) and secondary orthogonal (s-orthogonal) matrices and to derive their properties [6].

The secondary transpose of A is defined as $A^{S} = VA^{T}V$ where V is the permutation matrix and a unit is in the secondary diagonal [2]. Also, there exists a secondary symmetric matrix B such that $A^{S} = B^{-1}AB$ is established [3].

Recently Hill and Waters [11] have developed a theory for k-real and k-hermitian matrices, where 'k' is a fixed product of disjoint transposition in S_n, the set of all permutations on {1, 2, . . . , n}. As we know that the is bi involuntary, the verification can be done by the properties which is satisfied by the associated permutation matrix and its given below.

$$\left[K_1 \cup K_2\right] = \left[K_1^{T} \cup K_2^{T}\right] = \left[K_1^{-1} \cup K_2^{-1}\right] \quad \text{and} \quad K_1 \cup K_2 \quad \left[K_1^{2} \cup K_2^{2}\right] = I_1 \cup I_2 \qquad \textbf{(1.1)}$$

$$K_1 \cup K_2 \qquad \left[V_1 \cup V_2\right] = \left[V_1^{T} \cup V_2^{T}\right] = \left[V_1^{-1} \cup V_2^{-1}\right] \text{and} \left[V_1^{2} \cup V_2^{2}\right] = I_1 \cup I_2$$

(1.2)

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$$(KVA)^{\dagger} = A^{\dagger}VK \text{ and } (AVK)^{\dagger} = KVA^{\dagger} \text{ for } A \in C_n \qquad \textbf{(1.3)}$$

In [19, 20], W.B.Vasantha Kandasamy et. al. introduced the concept of bimatrices and analyses its properties. Bimatrices play a powerful and an advanced tool which can handle more than one linear model at a time. Bimatrices will be useful when time bound comparisons are needed in the analysis of the model.

Definition 1.1 [19]

$A_B = A_1 \cup A_2$ [$\cup$ is not operation only a symbol] is Bi matrix

Definition 1.2 [17]

A Matrix $A \in C_{n\times n}$ is EP-Matrix $AA^{\dagger} = A^{\dagger}A$

Definition 1.3

A Bi Matrix $A_B \in C_{n\times n}$ is EP-Bi Matrix $A_B A_B^{\dagger} = A_B^{\dagger} A_B$

$$[A_1 \cup A_2][A_1^{\dagger} \cup A_2^{\dagger}] = [A_1^{\dagger} \cup A_2^{\dagger}][A_1 \cup A_2]$$

$$[A_1A_1^{\dagger} \cup A_2A_2^{\dagger}] = [A_1^{\dagger}A_1 \cup A_2^{\dagger}A_2]$$

Lemma 1.4 [13]

A matrix $A \in C_{n\times n}$ is k-EP $\Leftrightarrow$ AK is EP $\Leftrightarrow$ KA is EP.

Lemma 1.5 [14]

A matrix $A \in C_{n\times n}$ is s-EP $\Leftrightarrow$ AV is EP $\Leftrightarrow$ VA is EP.

In this paper, the concept of Bi-EP Bi-matrix and s-k bi-EP bi-matrix are introduced and analyzed its properties.

II. S-K BI-EP BI-MATRICES

In this section we study the characterizations of bi-EP bi-matrix and s-k bi-EP bi-matrix as a generalization of EP bi-matices.

Definition 2.1

A Matrix $A \in C_{n\times n}$ is Bi EP-Matrix $AA^{\dagger}A^{\dagger}A = A^{\dagger}AAA^{\dagger}$

Definition 2.2

A Bi Matrix $A_B \in C_{n\times n}$ is Bi EP-Bi Matrix $A_B A_B^{\dagger} A_B^{\dagger} A_B = A_B^{\dagger} A_B A_B A_B^{\dagger}$

$$[A_1 \cup A_2][A_1^{\dagger} \cup A_2^{\dagger}][A_1^{\dagger} \cup A_2^{\dagger}][A_1 \cup A_2] = [A_1^{\dagger} \cup A_2][A_1 \cup A_2][A_1 \cup A_2][A_1^{\dagger} \cup A_2^{\dagger}]$$

$$[A_1A_1^{\dagger}A_1^{\dagger}A_1] \cup [A_2A_2^{\dagger}A_2^{\dagger}A_2] = [A_1^{\dagger}A_1A_1A_1^{\dagger}] \cup [A_2^{\dagger}A_2A_2A_2^{\dagger}]$$

Example 2.3

Let a Bi Matrix $A_B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is Bi EP-Bi Matrix.

Definition 2.4

A Bi Matrix $A_B \in C_{n\times n}$ is *s*- *k*- Bi EP-Bi Matrix $K_B V_B A_B A_B^{\dagger} A_B^{\dagger} A_B = A_B^{\dagger} A_B A_B A_B^{\dagger} V_B K_B$

$$[K_1 \cup K_2][V_1 \cup V_2][A_1 \cup A_2][A_1^{\dagger} \cup A_2^{\dagger}][A_1^{\dagger} \cup A_2^{\dagger}][A_1 \cup A_2]$$

$$= [A_1^{\dagger} \cup A_2^{\dagger}][A_1 \cup A_2][A_1 \cup A_2][A_1^{\dagger} \cup A_2^{\dagger}][V_1 \cup V_2][K_1 \cup K_2]$$

$$[K_1V_1A_1A_1^{\dagger}A_1^{\dagger}A_1] \cup [K_2V_2A_2A_2^{\dagger}A_2^{\dagger}A_2] = [A_1^{\dagger}A_1A_1A_1^{\dagger}V_1K_1] \cup [A_2^{\dagger}A_2A_2A_2^{\dagger}V_2K_2]$$

Example 2.5

Let a Bi Matrix $A_B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is s-k-Bi EP-Bi Matrix

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Theorem 2.6

For $A_B \in C_{n\times n}$ the following are equivalent.

1. A_B is *s*-*k*-Bi EP-Bi Matrix
2. $K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is Bi EP-Bi Matrix
3. $A_BA_B^{\dagger}A_B^{\dagger}A_BK_BV_B$ is Bi EP-Bi Matrix
4. $A_BA_B^{\dagger}A_B^{\dagger}A_BV_BK_B$ is Bi EP-Bi Matrix
5. $V_BK_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is Bi EP-Bi Matrix
6. $K_BV_BA_B^{\dagger}A_BA_BA_B^{\dagger}$ is Bi EP-Bi Matrix
7. $A_B^{\dagger}A_BA_BA_B^{\dagger}K_BV_B$ is Bi EP-Bi Matrix
8. $V_BK_BA_B^{\dagger}A_BA_BA_B^{\dagger}$ is Bi EP-Bi Matrix
9. $A_B^{\dagger}A_BA_BA_B^{\dagger}V_BK_B$ is Bi EP-Bi Matrix
10. $K_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is s-Bi EP-Bi Matrix
11. $A_BA_B^{\dagger}A_B^{\dagger}A_BK_B$ is s-Bi EP-Bi Matrix
12. $K_BA_B^{\dagger}A_BA_BA_B^{\dagger}$ is s-Bi EP-Bi Matrix
13. $A_B^{\dagger}A_BA_BA_B^{\dagger}K_B$ is s-Bi EP-Bi Matrix
14. $V_BA_B^{\dagger}A_BA_BA_B^{\dagger}$ is k-Bi EP-Bi Matrix
15. $A_B^{\dagger}A_BA_BA_B^{\dagger}V_B$ is k-Bi EP-Bi Matrix
16. $V_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is k-Bi EP-Bi Matrix
17. $A_BA_B^{\dagger}A_B^{\dagger}A_BV_B$ is k-Bi EP-Bi Matrix
18. $A_B^{\dagger}A_BA_BA_B^{\dagger}$ is s-k-Bi EP-Bi Matrix
19. $A_BA_B^{\dagger}A_B^{\dagger}A_B = V_BK_BA_B^{\dagger}A_BA_BA_B^{\dagger}V_BK_BH_B$ for non singular $n\times n$ matrix H_B
20. $A_BA_B^{\dagger}A_B^{\dagger}A_B = H_BK_BV_BA_B^{\dagger}A_BA_BA_B^{\dagger}K_BV_B$ for non singular $n\times n$ matrix H_B

Proof

The Proof of equivalence of (1), (2), (3) and (11) are as follows

A_B is s-k-Bi EP-Bi Matrix.

$\Leftrightarrow K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B = A_B^{\dagger}A_BA_BA_B^{\dagger}V_BK_B$ (by definition 2.4)

$\Leftrightarrow K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B = \left[K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B\right]^{\dagger}$ (by (1.3))

$= \left[K_B^{\dagger}V_B^{\dagger}A_BA_B^{\dagger}A_B^{\dagger}A_B\right]$ is Bi EP-Bi Matrix. (by definition of Bi EP-Bi Matrix)

$\Leftrightarrow K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is Bi EP-Bi Matrix

$\Leftrightarrow [V_BK_B]\left[K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B\right][V_BK_B]^{\dagger}$ is Bi EP-Bi Matrix (by Lemma 3 of [5])

$\Leftrightarrow V_BK_BK_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_BK_BV_B$ is Bi EP-Bi Matrix (by (1.3))

$\Leftrightarrow A_BA_B^{\dagger}A_B^{\dagger}A_BK_BV_B$ is Bi EP-Bi Matrix

$\Leftrightarrow A_BA_B^{\dagger}A_B^{\dagger}A_BK_B$ is s-Bi EP-Bi Matrix (by Lemma 1.5)

$(2) \Leftrightarrow (15) \Leftrightarrow (18)$

$K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B$ is Bi EP-Bi Matrix $\Leftrightarrow \left(K_BV_BA_BA_B^{\dagger}A_B^{\dagger}A_B\right)^{\dagger}$ is Bi EP-Bi Matrix

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$\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger V_B K_B$ is Bi EP-Bi Matrix $\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger V_B$ k- Bi EP-Bi Matrix

(by Lemma 1.4)

$\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger$ is s-k- Bi EP-Bi Matrix (by Lemma 1.5)

(2) $\Leftrightarrow$ (16)

$K_B V_B A_B A_B^\dagger A_B^\dagger A_B$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B A_B A_B^\dagger A_B^\dagger A_B$ is k- Bi EP-Bi Matrix (by Lemma 1.4)

(9) $\Leftrightarrow$ (8) $\Leftrightarrow$ (12)

$\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger V_B K_B$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B \left(A_B^\dagger A_B A_B A_B^\dagger V_B K_B\right)\left(V_B K_B\right)^\dagger$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B A_B^\dagger A_B A_B A_B^\dagger V_B K_B K_B V_B$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B A_B^\dagger A_B A_B A_B^\dagger$ is Bi EP-Bi Matrix

$\Leftrightarrow K_B A_B^\dagger A_B A_B A_B^\dagger$ is s -Bi EP-Bi Matrix (by Lemma 1.5)

(5) $\Leftrightarrow$ (4) $\Leftrightarrow$ (17)

$V_B K_B A_B A_B^\dagger A_B^\dagger A_B$ is Bi EP-Bi Matrix

$\Leftrightarrow \left(V_B K_B\right)^\dagger \left(V_B K_B A_B A_B^\dagger A_B^\dagger A_B\right)\left(V_B K_B\right)$ is Bi EP-Bi Matrix

$\Leftrightarrow K_B V_B V_B K_B A_B A_B^\dagger A_B^\dagger A_B V_B K_B$ Bi EP-Bi Matrix

$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B V_B K_B$ is Bi EP-Bi Matrix

$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B V_B$ is k -Bi EP-Bi Matrix

(4) $\Leftrightarrow$ (5) $\Leftrightarrow$ (10)

$A_B A_B^\dagger A_B^\dagger A_B V_B K_B$ is Bi EP-Bi Matrix

$\Leftrightarrow \left(V_B K_B\right)\left(A_B A_B^\dagger A_B^\dagger A_B V_B K_B\right)\left(V_B K_B\right)^\dagger$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B A_B A_B^\dagger A_B^\dagger A_B V_B K_B V_B K_B$ Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B A_B A_B^\dagger A_B^\dagger A_B$ is Bi EP-Bi Matrix $\Leftrightarrow K_B A_B A_B^\dagger A_B^\dagger A_B$ is s -Bi EP-Bi Matrix

(6) $\Leftrightarrow$ (7) $\Leftrightarrow$ (13)

$K_B V_B A_B^\dagger A_B A_B A_B^\dagger$ is Bi EP-Bi Matrix

$\Leftrightarrow \left(K_B V_B\right)^\dagger \left(K_B V_B A_B^\dagger A_B A_B A_B^\dagger\right)\left(K_B V_B\right)$ is Bi EP-Bi Matrix

$\Leftrightarrow V_B K_B K_B V_B A_B^\dagger A_B A_B A_B^\dagger K_B V_B$ is Bi EP-Bi Matrix

$\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger K_B V_B$ is Bi EP-Bi Matrix $\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger K_B$ is s -Bi EP-Bi Matrix

(7) $\Leftrightarrow$ (6) $\Leftrightarrow$ (14)

$A_B^\dagger A_B A_B A_B^\dagger K_B V_B$ is Bi EP-Bi Matrix

$\Leftrightarrow \left(K_B V_B\right)\left(A_B^\dagger A_B A_B A_B^\dagger K_B V_B\right)\left(K_B V_B\right)^\dagger$ is Bi EP-Bi Matrix

$\Leftrightarrow K_B V_B A_B^\dagger A_B A_B A_B^\dagger K_B V_B V_B K_B$ is Bi EP-Bi Matrix

$\Leftrightarrow K_B V_B A_B^\dagger A_B A_B A_B^\dagger$ is Bi EP-Bi Matrix

(2) $\Leftrightarrow$ (19)

$K_B V_B A_B A_B^\dagger A_B^\dagger A_B$ is Bi EP-Bi Matrix

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$$\Leftrightarrow \left(K_B V_B A_B A_B^\dagger A_B^\dagger A_B\right)^\dagger = \left(K_B V_B A_B A_B^\dagger A_B^\dagger A_B\right) H_B$$ for a non-singular nxn matrix H_B

$$\Leftrightarrow A_B^\dagger A_B A_B A_B^\dagger V_B K_B = \left(K_B V_B A_B A_B^\dagger A_B^\dagger A_B\right) H_B$$

$$\Leftrightarrow V_B K_B A_B^\dagger A_B A_B A_B^\dagger V_B K_B = A_B A_B^\dagger A_B^\dagger A_B\, H_B$$

$$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B = V_B K_B A_B^\dagger A_B A_B A_B^\dagger V_B K_B H_B^{-1}$$

$$H_B = H_B^{-1}$$

$$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B = V_B K_B A_B^\dagger A_B A_B A_B^\dagger V_B K_B\, H_B$$

$(9) \Leftrightarrow (20)$

$A_B^\dagger A_B A_B A_B^\dagger V_B K_B$ is Bi EP-Bi Matrix.

$$\Leftrightarrow \left(A_B^\dagger A_B A_B A_B^\dagger V_B K_B\right)^\dagger = H_B \left(A_B^\dagger A_B A_B A_B V_B K_B\right)$$ for a non-singular n×n matrix H_B.

$$\Leftrightarrow K_B V_B A_B A_B^\dagger A_B^\dagger A_B = H_B \left(A_B^\dagger A_B A_B A_B^\dagger V_B K_B\right)$$

$$\Leftrightarrow K_B V_B A_B A_B^\dagger A_B^\dagger A_B K_B V_B = H_B\, A_B A_B^\dagger A_B^\dagger A_B$$

$$\Leftrightarrow H_1\, A_B A_B^\dagger A_B^\dagger A_B = K_B V_B A_B^\dagger A_B A_B A_B^\dagger K_B V_B$$

$$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B = K_B V_B A_B^\dagger A_B A_B A_B^\dagger K_B V_B\, H_B^{-1}$$

$$H_B = H_B^{-1}$$

$$\Leftrightarrow A_B A_B^\dagger A_B^\dagger A_B = H\, K_B V_B A_B^\dagger A_B A_B A_B^\dagger K_B V_B$$

Theorem 2.7

Let $A_B \in C_{n\times n}$. Then any two of the following conditions imply the other one.

1. A_B is Bi EP-Bi Matrix
2. A_B is s-k-Bi EP-Bi Matrix.
3. $K_B V_B A_B A_B^\dagger A_B^\dagger A_B = A_B^\dagger A_B A_B A_B^\dagger V_B K_B$

Proof

First we Prove that whenever (1) holds, then (2) and (3) are equivalent suppose (1) hold, then A_B is Bi EP implies

$$A_B A_B^\dagger A_B^\dagger A_B = A_B^\dagger A_B A_B A_B^\dagger$$

Now by Theorem 2.6, A_B is s-k-Bi EP-Bi Matrix

$$\left[K_B V_B A_B A_B^\dagger A_B^\dagger A_B\right]^\dagger = A_B^\dagger A_B A_B A_B^\dagger V_B K_B$$

Therefore A_B is s-k-Bi EP $\Leftrightarrow K_B V_B A_B A_B^\dagger A_B^\dagger A_B$

This completes the Proof of $[(1)\, and\, (2)] \Rightarrow (3)$ and $[(1)\, and\, (3)] \Rightarrow (2)$

Now Let us Prove $[(2)\, and\, (3)] \Rightarrow (1)$

A_B is s-k-Bi EP $\Leftrightarrow \left[K_B V_B A_B A_B^\dagger A_B^\dagger A_B\right]^\dagger = A_B^\dagger A_B A_B A_B^\dagger V_B K_B$ by using (3) we have

$$K_B V_B A_B A_B^\dagger A_B^\dagger A_B = A_B^\dagger A_B A_B A_B^\dagger V_B K_B$$

$$= A_B^\dagger A_B A_B A_B^\dagger$$

$$A_B A_B^\dagger A_B^\dagger A_B = A_B^\dagger A_B A_B A_B^\dagger$$

$\Rightarrow A_B$ is Bi EP-Bi Matrix

Corollary 2.8

If A_B is Bi EP-Bi Matrix and $A_B A_B^\dagger$ is s-k-Bi EP-Bi Matrix then A_B is s-k-Bi EP-Bi Matrix.

Proof

Since A_B is Bi EP-Bi Matrix,

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We have $AA^{\dagger}A^{\dagger}A = A^{\dagger}AAA^{\dagger}$

$\left[AA^{\dagger}A^{\dagger}A\right]\left[A^{\dagger}AAA^{\dagger}\right]^{\dagger}$ is s-k-Bi EP-Bi Matrix

$$\Leftrightarrow \left[AA^{\dagger}A^{\dagger}A\right]\left[A^{\dagger}AAA^{\dagger}\right]^{\dagger} = K_B V_B \left[\left[A_B A_B^{\dagger} A_B^{\dagger} A_B\right]\left[A_B^{\dagger} A_B A_B A_B^{\dagger}\right]\right]^{\dagger}$$

$$\Leftrightarrow \left[AA^{\dagger}A^{\dagger}A\right]\left[A^{\dagger}AAA^{\dagger}\right]^{\dagger} = K_B V_B \left[A_B A_B^{\dagger} A_B^{\dagger} A_B\right]\left[A_B^{\dagger} A_B A_B A_B^{\dagger}\right]^{\dagger}$$

$$\Leftrightarrow K_B V_B A_B A_B^{\dagger} A_B^{\dagger} A_B = A_B^{\dagger} A_B A_B A_B^{\dagger} V_B K_B$$

$\Leftrightarrow A_B$ is a s-k-Bi EP-Bi Matrix [by Theorem 2.7]

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