International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-12, Mar 2019

Recursive form of B-spline collocation Method for Numerical Solution of the one dimensional heat equation

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Abstract - In this paper, recursive form of quadratic B-spline Collocation method is proposed for calculating numerical solution of one dimensional heat equation. Recursive form of B-spline is used for the spatial coordinates whereas forward difference scheme is applied for the time derivative. The performance of the method is tested at different time level. The numerical result shows that the present method is a successful numerical technique to find solution for time dependent problems.

Keywords : One Dimensional Heat Equation, B-spline, Collocation Method, Forward difference scheme

I. INTRODUCTION

Heat transfer problems play the important role in boilers, condensers, air pre-heaters, economizers, electric motors, generators and transformers and in chemical reactions. The study of heat transfer problems gives the solutions to optimize the usage of materials and their performance in different systems [7]. Many numerical methods are developed to solve such type of Heat transfer problems. Widely used such numerical methods are finite volume method and finite element method [1, 2, 3, 4]. These methods depend on discretization of domains, conversion of strong form into weak form of the governing equation and evaluation of integrals to obtain linear system of equations. This process leads to errors like geometrical error, reduction in the continuity requirement of the approximating function which demands the fine mesh for an acceptable solution. In addition to these errors, mesh generation is more time consuming and costly.

Owing to the above mentioned difficulties in the mesh based methods, various point wise approximation techniques are developed like smoothed particle hydrodynamics (SPH) method, The element free Galerkin (EFG), Collocation methods etc. In collocation method, an approximating function is defined based on the nodal distribution only. In this method, many inter mediatory evaluations such as conversion of strong form differential equation to weak form, evaluation of integrals can be avoided.The use of B-Spline basis functions in collocation method improves the smoothness and accuracy of the solution. **The efficiency of the method in explicit form has been proved by many researchers [8]-[14]**. In this manuscript, a methodology is developed as in the form of recursive form of B-slpine for the solution of one dimensional Heat problems using quadratic B-spline Collocation method. Applicability and convergence of the present method is tested by considering numerical heat transfer problem.

One dimensional heat equation is given as

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \quad a \le x \le b, \quad t \ge 0 \qquad \ldots\ldots \quad (1)$$

subject to the initial condition

$$u(x,0)=f(x), \qquad x\in[a,b] \qquad \ldots\ldots \quad (2)$$

and the boundary conditions

$$u(a,t)=g_0(t), \quad u(b,t)=g_1(t), \qquad t\in[0,T] \qquad \ldots\ldots \quad (3)$$

II. B-SPLINE COLLOCATION METHOD

The solution domain $a \le x \le b$ is partitioned into a mesh of uniform length $h = x_{j+1} - x_j$, where j=0,1,2,…, N-1,N . Such that $a= x_0 < x_1 < x_2 \ldots\ldots < x_{n-1} < x_n=b$.

In the cubic B-spline collocation method the approximate solution is written as the linear combination of cubic B-spline basis functions for the approximation space under consideration. The proposed numerical solution for solving Eq. (1) using the collocation method with cubic B-spline is to find an

 DOI : 10.18231/2454-9150.2019.0102

approximation solution $U^k(x,t)$ at k[th] time level to the exact solution $U(x,t)$ in the form:

$$U^k(x,t) = \sum_{i=-3}^{n+3} C^k{}_i(t) N_{i,p}(x) \quad (4)$$

where $C_i(t)$'s are time dependent quantities to be determined from the initial and boundary conditions and collocation from the differential equation.

A zero degree and other than zero degree B-spline basis functions [5, 6] are defined at x_i recursively over the knot vector space $X = \{x_1, x_2, x_3 \ldots\ldots\ldots x_{n-1}, x_n\}$ as

i) if $p = 0$

$$N_{i,p}(x) = 1 \qquad if \;\; x \in (x_i, x_{i+i})$$

$$N_{i,p}(x) = 0 \qquad if \;\; x \notin (x_i, x_{i+i})$$

ii) if $p \geq 1$

$$N_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N_{i,p-1}(x) + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N_{i+1,p-1}(x) \quad \ldots\ldots\ldots\ldots \quad (5)$$

where p is the degree of the B-spline basis function and x is the parameter belongs to X. When evaluating these functions, ratios of the form 0/0 are defined as zero.

Derivatives of B-splines

If p=3, we have

$$N'_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N'_{i,p-1}(x) + \frac{N_{i,p-1}(x)}{x_{i+p} - x_i} + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N'_{i+1,p-1}(x) - \frac{N_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}}$$

$$N^{iii}{}_{i,p}(x) = 3\frac{N^{ii}{}_{i,p-1}(x)}{x_{i+p} - x_i} - 3\frac{N^{ii}{}_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}} \quad \ldots\ldots\ldots \quad (6)$$

$$(U^k)^{ii}(x) = \sum_{i=-3}^{n+3} C^k{}_i N^{ii}{}_{i,p}(x) \quad \ldots\ldots \quad (7)$$

III. SOLUTION OF HEAT EQUATION

We discrete the time derivative of Eq.(1) by a first order accurate forward difference formula and

$$\frac{u^{k+1} - u^k}{\Delta t} = \frac{1}{2}\left(\frac{\partial^2 u^{k+1}}{\partial x^2} + \frac{\partial^2 u^k}{\partial x^2}\right)$$

$$2 * u^{k+1} - \Delta t * \frac{\partial^2 u^{k+1}}{\partial x^2} = 2 * u^k + \Delta t * \frac{\partial^2 u^k}{\partial x^2}$$

……. (8)

Substituting the Eq.(2) in Eq.(8) then we have

$$2 * \sum_{i=-3}^{n+3} C_i^{k+1}(t) N_{i,p}(x) - \Delta t * \sum_{i=-3}^{n+3} C_i^{k+1}(t) N_i''{}_{,p}(x) =$$

$$2 * \sum_{i=-3}^{n+3} C_i^{k}(t) N_{i,p}(x) + \Delta t * \sum_{i=-3}^{n+3} C_i^{k}(t) N_i''{}_{,p}(x)$$

………. (9)

Evaluating Eq.(9) at the nodal points $x = x_0, x_1, x_2 \ldots\ldots x_{N-1}, x_N$. These are also collocation points.(n+1) equations are obtained in (n+3) unknowns. Two more equations are obtained by using the boundary conditions. Finally, (n+3) equations are obtained in (n+3) unknowns C_i^{k+1}'s at (k+1)[th] time level. These unknowns are determined by using the above obtained (n+3) equations. Substituting all the C_i^{k+1}' in equation (4), the solution is known to find values $U(x,t)$ at any time in the given domain of x.

Initial vector

Initial vector C_i^{0}' is obtained by using the initial condition $u(x,0) = f(x)$ for $x = x_0, x_1, x_2 \ldots\ldots x_{N-1}, x_N$ and by the boundary conditions $u_x(x_0, 0) = f'(x)$, $u_x(x_N, 0) = f'(x)$. This yields the (n+3) equations in (n+3) unknowns.

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IV. NUMERICAL EXAMPLE

One dimensional heat equation is given with initial and boundary conditions as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad , \; 0 < x < 1 \, , \, t \ge 0 \; \ldots\ldots \qquad (1)$$

subject to the initial condition

$$u\,(x,0) = \sin \pi \, x \qquad x \in [0,1] \qquad \ldots\ldots \qquad (2)$$

and the boundary conditions

$$u\,(0,t) = 0 \, , \;\; u\,(1,t) = 0 \qquad \ldots\ldots. \quad (3)$$

The exact solution is known to be

$$u\,(x,t) = e^{-\pi^2 t} \sin \pi \, x$$

Cubic B-spline collocation is applied to the numerical example and presented maximimu Relative error inTable 1. It is noticed from the Table1 that the values which are obtained by present method is almost equal to the values of exact values for different size of time.This shows that the present method is successfully demonstrated to find the numerical solution of one dimensional heat equation.

Table1: Maximum Absolute Relative Error obtained for the problem

Δt (time difference)	Present method Maximum Absolute Relative Error
.0100	8.1199e-004
.0200	$1.5X10^{-3}$
.0300	$2X10^{-3}$
0400	$2.4X10^{-3}$

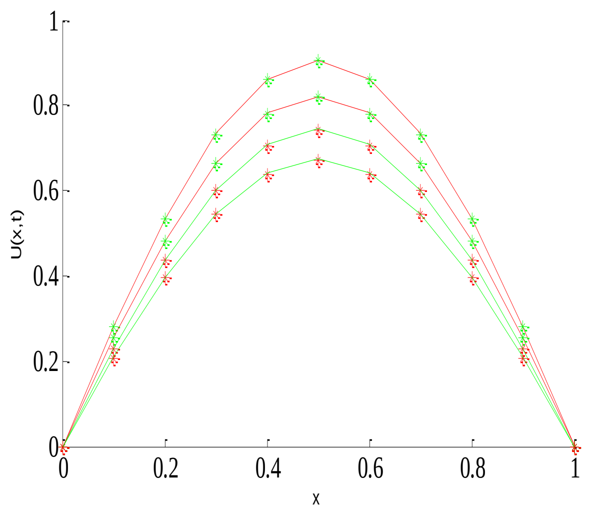

Figure1: Comparison of cubic B-spline collocation solution with exact solution for different time levels

V. CONCLUSIONS

The B-spline basis functions defined recursively are incorporated in the collocation method and applied the same to time dependent two point boundary value problem. The effectiveness of the proposed method is illustrated by considering one numerical example.

The second degree B-spline basis function is used in collocation method and forward difference scheme is applied to discrtize the time derivative function. Numerical results got by the present method are good agreement with the analytical solution values. By using this method, amount of work time is reduced majorly. This method may be applied to different types of some more time dependent boundary value problems for its efficient.

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 DOI : 10.18231/2454-9150.2019.0102