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Some Expansion Formulae Involving A Basic Analogue Of Aleph $(\aleph)$ - Function

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Abstract - In the present paper an expansion formulae for a basic analogue Aleph $(\aleph)$ -function have been derived by the applications of the q -Leibniz rule for the type q -derivatives of a product of two functions. Expansion formulae involving a basic analogue of Fox's H -function, Meijer's G -function and MacRobert's E -function have been derived as special cases of the main results.

Key words: q -Leibniz rule, Weyl fractional, q -integral operator, Fox's H -function, $(\aleph)$ -function.

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I. INTRODUCTION

"Yadav and Purohit [10]" introduced a new q -extension of the lebniz rule for the derivatives of a product of two basic functions in terms of a finite q -series involving Weyl type q -derivatives of the functions in the following manner:

$$ {}_zD^{\alpha}_{\infty,q}\left\{U(z)V(z)\right\}=\sum_{r=0}^{\alpha}\frac{(-1)^r q^{r(r+1)/2}\left(q^{-\alpha};q\right)_r}{(q;q)_r}\,{}_zD^{\alpha-r}_{\infty,q}\left\{U(z)\right\}\,{}_zD^{\alpha}_{\infty,q}\left\{V\left(zq^{\alpha-r}\right)\right\}, \qquad (1.1) $$

Where $U(z)$ and $V(z)$ are two functions and the fractional q -differential operator ${}_zD^{\alpha}_{\infty,q}(.)$ of Weyl type is given by

$$ {}_zD^{\alpha}_{\infty,q}\left\{f(z)\right\}=\frac{q^{-\alpha(1+\alpha)/2}}{\Gamma_q(-\alpha)}\int_z^{\infty}(t-z)_{-\alpha-1}f(tq^{1+\alpha})d(t;q), \qquad (1.2) $$

Where $\mathrm{Re}(\alpha)<0$ and

$$ (x-y)_v=x^v\prod_{n=0}^{\infty}\left[\frac{1-\left(y/x\right)q^n}{1-\left(y/x\right)q^{v+n}}\right], \qquad (1.3) $$

The basic integration cf. "Gasper and Rehman [2]", is defined as:

$$ \int_z^{\infty}f(t)d(t;q)=z(1-q)\sum_{k=1}^{\infty}q^{-k}f\left(zq^{-k}\right). \qquad (1.4) $$

In view of the relation (1.4), operator (1.2) can be expressed as:

$$ {}_zD^{\alpha}_{\infty,q}\left\{f(z)\right\}=\frac{q^{\alpha(1-\alpha)/2}z^{-\alpha}(1-q)}{\Gamma_q(-\alpha)}\sum_{k=0}^{\infty}q^{\alpha k}(1-q^{k+1})_{-\alpha-1}f\left(zq^{\alpha-k}\right), \qquad (1.5) $$

Where $\mathrm{Re}(\alpha)<0$.

In particular, for $f(z)=z^{-p}$, the equation (1.5) yields to

$$ {}_zD^{\alpha}_{\infty,q}\left\{z^{-p}\right\}=\frac{\Gamma_q(p+\alpha)}{\Gamma_q(p)}q^{-\alpha p+\alpha(1-\alpha)/2}z^{-p-\alpha}, \qquad (1.6) $$

Where $\mathrm{Re}(\alpha)<0$.

We shall make use of the following notations and definitions in the sequel:

For real or complex a and $|q|<1$, the q -shifted factorial is defined as:

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$$(a;q)_n = \begin{cases} 1, & if\ n=0 \\ (1-a)(1-aq)\ldots(1-aq^{n-1}), & if\ n\in N \end{cases} \tag{1.7}$$

In terms of the q -gamma function, (1.7) can be expressed as

$$(a;q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)}, n>0 \tag{1.8}$$

Where the q -gamma function cf. Gasper and Rahman, is given by

$$\Gamma_q(a) = \frac{(q;q)_\infty}{(q^a;q)_\infty (1-q)^{a-1}}, \tag{1.9}$$

Where $a \neq 0,-1,-2,\ldots$.

The $\aleph$ - function introduced by Suland et.al. [8] defined and represented in the following form:

$$\aleph[z] = \aleph^{m,n}_{p_i,q_i;\tau_i;r}[z] = \aleph^{m,n}_{p_i,qi;\tau_i;r}\left[z \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix} \right] = \frac{1}{2\pi\omega}\int_L \theta(s) z^s ds \tag{1.10}$$

Where $\omega = \sqrt{-1}$;

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s\right\}} \tag{1.11}$$

We shall use the following notation:

$A^* = (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}, B^* = (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}$

The basic analogue of the $\aleph$ -function in terms of Mellin-Barnes type basic contour integral is in the following manner:

$$\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[z;q\,|^{A^*}_{B^*}\right] = \frac{1}{2\pi w}\int_C \frac{\prod_{j=1}^{m}G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n}G\left(q^{1-a_j+\alpha_j s}\right)\pi z^s}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=n+1}^{p_i}G\left(q^{b_{ji}-\beta_{ji}s}\right)\prod_{j=m+1}^{q_i}G\left(q^{1-a_{ji}+\alpha_{ji}s}\right)\right\}}ds \tag{1.12}$$

Where

$$G(q^\alpha) = \left\{\prod_{n=0}^{\infty}\left(1-q^{\alpha+n}\right)\right\}^{-1} = \frac{1}{(q^\alpha;q)_\infty} \tag{1.13}$$

And $0 \le m \le q_i, 0 \le n \le p_i; \alpha_j$ and β_j are all positive integers. The contour C is a line parallel to $\mathrm{Re}(ws)=0$, with indentations, if necessary, in such a manner that all the poles of $G\left(q^{b_j-\beta_j s}\right), 1 \le j \le m$, are to the right, and those of $G\left(q^{1-a_j+\alpha_j s}\right), 1 \le j \le n$ to the left of C. The integral converges if $\mathrm{Re}[s\log(z) - \log\sin\pi s] < 0$ for large values of $|s|$ on the contour C. That is, if $|\left\{\arg(z) - w_2 w_1^{-1}\log|z|\right\}| < \pi$ where $|q|<1, \log q = -w = -(w_1+iw_2), w, w_1, w_2$ are definite quantities. w_1 and w_2 being real.

For $\tau_i = 1$ $r=1$, the $(\aleph)$ -function reduces to Fox's H -function and eq. (1.12) reduces to the q -analogue of the Fox's H -function due to Saxena et. al. [6], namely

$$H^{m,n}_{p,q}\left[z;q\left|^{(a,\alpha)}_{(b,\beta)}\right.\right] = \frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m}G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n}G\left(q^{1-a_j-\alpha_j s}\right)\pi z^s}{\prod_{j=m+1}^{q}G\left(q^{1-b_j-\beta_j s}\right)\prod_{j=n+1}^{p}G\left(q^{a_j+\alpha_j s}\right)\sin\pi s}ds, \tag{1.14}$$

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Where $0 \le m \le q, 0 \le n \le p$ and $\operatorname{Re}[s\log(z) - \log\sin\pi s] < 0$.

For $\alpha_j = \beta_j = 1, j = 1,...,q$ the definition (1.14) reduces to the q-analogue of the Meijer's G-function due to Saxena et. al. [6], namely

$$H_{p,q}^{m,n}\left[z;q\middle|^{(a,1)}_{(b,1)}\right] = G_{p,q}^{m,n}\left[z;q\middle|^{a_1,...,a_p}_{b_1,...,b_q}\right] = \frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m} G\left(q^{b_j-s}\right)\prod_{j=1}^{n} G\left(q^{1-a_j-s}\right)\pi z^s}{\prod_{j=m+1}^{q} G\left(q^{1-b_j-s}\right)\prod_{j=n+1}^{p} G\left(q^{a_j+s}\right)\sin\pi s} ds \tag{1.15}$$

Where $0 \le m \le q, 0 \le n \le p$ and $\operatorname{Re}[a\log(z) - \log\sin\pi s] < 0$.

Further, if we set $n = 0$ and $m = q$ in the equation (1.15), we get the basic analogue of MacRobert's E-function due to Agarwal [1], namely

$$G_{p,q}^{m,0}\left[z;q\middle|^{a_1,...,a_A}_{b_1,...,b_B}\right] = E_q\left[q;b_j:p;a_j:z\right] = \frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{q} G\left(q^{b_j-s}\right)\pi z^s}{\prod_{j=1}^{p} G\left(q^{a_j-s}\right)G\left(q^{b_j-s}\right)\sin\pi s} ds, \tag{1.16}$$

Where $\operatorname{Re}[s\log(z) - \log\sin\pi s] < 0$.

II. MAIN RESULTS

In this section, the author will establish certain results associated with the basic analogue of $(\aleph)$-function by assigning suitable values to the function $U(z), V(z)$ and α in the q-Leibniz rule (1.1). The main results to be established are as under:

$$\aleph_{p_i+1,q_i+1:\tau_i r}^{m+1,n}\left[\rho\left(zq^{\mu}\right)^k;q\middle|^{A^*,(\lambda,k)}_{(\mu+\lambda,k),B^*}\right] = \sum_{R=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$
$$\aleph_{p_i+1,q_i+1:\tau_i r}^{m+1,n}\left[\rho\left(zq^{\mu}\right)^k;q\middle|^{A^*,(0,k)}_{(R,k),B^*}\right], \tag{2.1}$$

Where $0 \le m \le q_i, 0 \le n \le p_i, \operatorname{Re}[s\log(z) - \log\sin\pi s] < 0, k \ge 0$ and ρ being any complex quantity.

$$\aleph_{p_i+1,q_i+1:\tau_i:r}^{m,n+1}\left[\rho\left(zq^{\mu}\right)^k;q\middle|^{(1-\mu-\lambda,-k),A^*}_{B^*,(1-\lambda,-k)}\right] = \sum_{R=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$
$$\aleph_{p_i+1,q_i+1:\tau_i:r}^{m,n+1}\left[\rho\left(zq^{\mu}\right)^k;q\middle|^{(1-R,-k),A^*}_{B^*,(1,-k)}\right] \tag{2.2}$$

Where $0 \le m \le q_i, 0 \le n \le p_i, \operatorname{Re}[s\log(z) - \log\sin\pi s] < 0, k < 0$ and ρ being any complex quantity.

Proof : To prove the result (2.1) and (2.2), we begin with $U(z) = z^{-\lambda}$ and

$$V(z) = \aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\rho z^k;q\middle|^{A^*}_{B^*}\right]$$

In equation (1.1.) to obtain

$${}_zD_{\infty,q}^{\mu}\left\{z^{-\lambda}\aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\rho z^k;q\middle|^{A^*}_{B^*}\right]\right\} = \sum_{r=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2}\left(q^{-\mu};q\right)_R}{(q;q)_R}\,{}_zD_{\infty,q}^{\mu-R}\left\{z^{-\lambda}\right\}$$
$${}_zD_{\infty,q}^{\alpha}\left\{\aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\rho(z^{\mu-R})^k;q\middle|^{A^*}_{B^*}\right]\right\} \tag{2.3}$$

n view of the definition (1.12), the left hand side of equation (2.3) becomes

$${}_zD_{\infty,q}^{\mu}\left\{z^{-\lambda}\aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\rho z^k;q\middle|^{A^*}_{B^*}\right]\right\}$$

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$$= \frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m} G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n} G\left(q^{1-a_j+\alpha_j s}\right)\pi\rho^s}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i} G\left(q^{1-b_{ji}+\beta_{ji}s}\right)\prod_{j=n+1}^{p_i} G\left(q^{a_{ji}-\alpha_{ji}s}\right)G\left(q^{1-s}\right)\sin\pi s\right\}}\, {}_zD^{\mu}_{\infty,q}\left\{z^{-(\lambda-ks)}\right\}ds\,. \quad (2.4)$$

On making use of fractional q-derivative formula (1.6) in the above equation (2.4), we obtain following interesting transformation for the $\aleph_q(.)$ function after certain simplifications:

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda}\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\rho z^k;q\Big|^{A*}_{B*}\right]\right\} = \frac{z^{-\lambda-\mu}q^{-\mu\lambda+\mu(1-\mu)/2}}{(1-q)^{\mu}}\aleph^{m+1,n}_{p_i+1,q_i+1;\tau_i;r}\left[\rho(zq^{\mu})^k q;\Big|^{A*,(\lambda,k)}_{(\mu+\lambda,k),B*}\right], \quad (2.5)$$

Where $k \geq 0$.

Again, if we take $k<0$, we obtain the following fractional q-derivative formula for the $\aleph_q(.)$ function, namely

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda}\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\rho z^k;q\Big|^{A*}_{B*}\right]\right\} = \frac{z^{-\lambda-\mu}q^{-\mu\lambda+\mu(1-\mu)/2}}{(1-q)^{\mu}}\aleph^{m,n+1}_{p_i+1,q_i+1;\tau_i;r}\left[\rho(zq^{\mu})^k;q\Big|^{(1-\mu-\lambda,-k),A*}_{B*,(1-\lambda,-k)}\right]. \quad (2.6)$$

We now substitute and replace μ by R and then z by $zq^{\mu-R}$ respectively, in equation (2.5) to obtain the following transformation for the $\aleph_q(.)$ function:

$${}_zD^{R}_{\infty,q}\left\{\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\rho\left(zq^{\mu-R}\right)^k;q\Big|^{A*}_{B*}\right]\right\} = \frac{z^{-R}q^{\frac{R(R+1)}{2}-\mu R}}{(1-q)^{R}}\aleph^{m+1,n}_{p_i+1,q_i+1;\tau_i;r}\left[\rho(zq^{\mu})^k;q\Big|^{A*,(0,k)}_{(R,k),B*}\right]. \quad (2.7)$$

Further, in view of the result (1.6), one can easily obtain the following relation

$${}_zD^{\mu-R}_{\infty,q}\left\{z^{-\lambda}\right\} = \frac{\Gamma_q(\lambda+\mu-R)}{\Gamma_q(\lambda)}q^{(\mu-R)(1-\mu-R-2\lambda)/2}z^{-\lambda-\mu+R}. \quad (2.8)$$

On substituting the values of various expressions involved in the equation (2.3), from equations (2.5), (2.7) and (2.8), we arrive at the main result (2.1).

The proof of the result (2.2) follows similarly when $k<0$ and by the usages of the transformation formula (2.6) and the relation (2.8).

III. SPECIAL CASES

In this section, we shall consider some special cases of the main results and deduce certain expansion formulae involving the basic analogue of Fox's H-function, basic analogue of Meijer's G-function and basic analogue of MacRobert's E-function.

If we set $r=1=\tau_i$, in the main result (2.1), we obtain the following expansion formula involving Fox's H-function, namely

$$H^{m+1,n}_{p+1,q+1}\left[\rho\left(zq^{\mu}\right)^k;q\Big|^{(a_j,\alpha_j)_{1,p},(\lambda,k)}_{(\mu+\lambda,k),(b_j,\beta_j)_{1,q}}\right] = \sum_{R=0}^{\mu}\frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$
$$H^{m+1,n}_{p+1,q+1}\left[\rho(zq^{\mu})^k;q\Big|^{(a_j,\alpha_j)_{1,p},(0,k)}_{(R,k),(b_j,\beta_j)_{1,q}}\right], \quad (3.1)$$

Where $0\leq m\leq q, 0\leq n\leq p, \mathrm{Re}[s\log(z)-\log\sin\pi s]<0, k\geq 0$ and ρ being any complex quantity.

Similarly, for $r=1=\tau_i$ and $k=-1$, the main result (2.1) reduces to yet another expansion formula associated with the basic analogue of Fox's H-function, namely

$$H^{m,n+1}_{p+1,q+1}\left[\rho(zq^{\mu})^k;q\Big|^{(1-\mu-\lambda,-k),(a_j,\alpha_j)_{1,p}}_{(b_i,\beta_i)_{1,q},(1-\lambda,-k)}\right] = \sum_{R=0}^{\mu}\frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R(q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$H^{m,n+1}_{p+1,q+1}\left[\rho(zq^{\mu})^k;q\Big|^{(1-R,-k),(a_j,\alpha_j)_{1,p}}_{(b_i,\beta_i)_{1,q},(1,-k)}\right], \quad (3.2)$$

Where $0\leq m\leq q, 0\leq n\leq p, \mathrm{Re}[s\log z-\log\sin\pi s]<0, k<0$ and ρ being any complex quantity.

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If we set $\alpha_j = \beta_i = 1, j = 1,..., p; i = 1,..., q$ and $k = 1$, in (3.1), we obtain the following expansion formula involving Meijer's $G_q(.)$ function, namely

$$G^{m+1,n}_{p+1,q+1}\left[\rho z q^{\mu}; q\left|\begin{smallmatrix}(a_j,1)_{1,p},(\lambda,1)\\(\mu+\lambda,1),(b_i,1)_{1,q}\end{smallmatrix}\right.\right] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R (q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$G^{m+1,n}_{p+1,q+1}\left[\rho z q^{\mu}; q\left|\begin{smallmatrix}(a_j,1)_{1,p},(0,1)\\(R,1),(b_i,1)_{1,q}\end{smallmatrix}\right.\right], \qquad (3.3)$$

Where $0 \le m \le q, 0 \le n \le p, \mathrm{Re}[s \log z - \log \sin \pi s] < 0$ and ρ being any complex quantity.

Similarly, for $\alpha_j = \beta_i = 1, j = 1,..., p; i = 1,..., q$ and $k = -1$, the result (3.2) reduces to yet another expansion formula associated with the basic analogue of Meijer's $G_q(.)$ function, namely

$$G^{m,n+1}_{p+1,q+1}\left[\rho/(z q^{\mu}); q\left|\begin{smallmatrix}(1-\mu-\lambda,1),(a_j,1)\\(b_i,1)_{1,q},(1-\lambda,1)\end{smallmatrix}\right.\right] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R (q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$G^{m,n+1}_{p+1,q+1}\left[\rho/(z q^{\mu}); q\left|\begin{smallmatrix}(1-R,1),(a_j,1)_{1,p}\\(b_i,1)_{1,q},(1,1)\end{smallmatrix}\right.\right], \qquad (3.4)$$

Where $0 \le m \le q, 0 \le n \le p, \mathrm{Re}[s \log z - \log \sin \pi s] < 0$ and ρ being any complex quantity.

Finally, if we set $n = 0$ and $m = q$, the result (3.3), yields to an expansion formula involving MacRobert's $E_q(.)$ function, namely

$$E_q[q+1; b_j, \mu+\lambda : p+1; a_j, \lambda : \rho z q^{\mu}] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R (q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$E_q[q+1; b_j, R : p+1, a_j, 0 : \rho z q^{\mu}], \qquad (3.5)$$

Where $\mathrm{Re}[s \log z - \log \sin \pi s] < 0$ and ρ being any complex quantity.

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