Difference cordial labeling of some special graphs and related to fan graphs

*A.Sugumaran, #V.Mohan

*,#Department of mathematics, Government Arts College, Thiruvannamalai, Tamilnadu, India.

*sugumaranaruna@gmail.com, #vmb5685@gmail.com

Abstract. A *difference cordial labeling* of a graph G is an injective function f from V(G) to {1, 2, 3, ⋯ , |V(G)|} such that if each edge uv is assigned the label 1 if |f(u) – f(v)| = 1, and 0 otherwise. Then the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1. A graph with difference cordial labeling is called a *difference cordial graph*. In this paper we proved that, the graphs obtained by switching of a pendant vertex in P_n, switching of an apex vertex in CH_n, the graph obtained by duplication of each vertex of path by an edge, the graph obtained by barycentric subdivision of crown graph $C_n \Theta K_1$, path union of r copies of fan P(r. F_n), cycle union of r copies of fan C(r. F_n) and open star of r copies of fan S(r. F_n) graph are difference cordial graphs.

Mathematics subject classification. 05C78

Keywords. ***Barycentric subdivision, Cycle union, Difference cordial labeling, Difference cordial graph, Duplication of a vertex by an edge, Open star of a graph, Path union, Switching of a vertex.***

I. INTRODUCTION

In this paper, we deal with finite, simple and undirected graphs. Graph labeling connects many branches of mathematics and is consider as one of the important blocks of graph theory. Labeled graphs plays an important in communication network addressing and models for constraint programming over finite domains [2]. Cordial labeling was first introduced in 1987 by Cahit [1], and then there has been a major effort in this area made this topics growing steadily and widely [2].

Initially the difference cordial labeling was stated in [4]. R. Ponraj et al. studied difference cordiality of some types of graphs such as path, cycle, complete graph, complete bipartite graph, star, helm, sunflower graph, lotus inside a circle, pyramid, permutation graphs, graphs which are obtained from triangular snake, quadrilateral snake, book with n pentagonal pages, double fan and some more standard graphs were investigated in [4, 5, 6, 7, 8, 9, 10, 11]. In [14], A. Sugumaran and V. Mohan have studied the difference cordial labeling behavior of the P(r. P_n^2), C(r. P_n^2), S(r. P_n^2) and related graphs. In [12, 13], M. A. Seoud and Shakir M. Salman studied, the graph obtained by duplication of a vertex by an edge, bow graph, butterfly graph, ladder, triangular ladder, step ladder, two sided step ladder, grid are difference cordial graphs. Also they discussed some families of graphs which may be difference cordial or not, such as diagonal ladder and some types of one point union graphs. For standard terminology and notations we follow Harary [3].

Definition 1.1. A *difference cordial labeling* of a graph G is an injective function f from V(G) to {1, 2, 3, ⋯ , |V(G)|} such that if each edge uv is assigned the label 1 if |f(u) – f(v)| = 1, and 0 otherwise. Then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with difference cordial labeling is called a *difference cordial graph*. We denote, $e_f(0)$ and $e_f(1)$ are the number of edges labeled with 0 and the number of edges labeled with 1 respectively.

Definition 1.2. Let G be a graph and let $G_1, G_2, G_3, \cdots, G_r$, $r \geq 2$ be r copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} ($1 \leq i \leq r-1$) is called a *path union of G* and is denoted by P(r. G).

Definition 1.3. Let G be a graph and let $G_1, G_2, G_3, \cdots, G_r$, $r \geq 2$ be r copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} ($1 \leq i \leq r-1$) and G_r to G_1 is called a *cycle union of G* and is denoted by C(r. G).

Definition 1.4. A *fan graph* $F_n = P_n + K_1$, is obtained from a cycle P_n, by attaching a pendant edge at each vertex of the P_n.

Definition 1.5. The *barycentric subdivision* of the graph G is obtained by inserting a vertex of degree 2 into every edge of original graph G.

Definition 1.6. A *crown graph* $C_n \Theta K_1$, is obtained from a cycle C_n, by attaching a pendant edge at each vertex of the C_n.

Definition 1.7. A *vertex switching* G_v of a graph G is obtained by taking a vertex v of G, removing the entire

 DOI : 10.18231/2454-9150.2019.0099

edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.8. A *helm* H_n, n ≥ 3 is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the rim of the wheel W_n.

Definition 1.9. A *closed helm* CH_n, n ≥ 3 is the graph obtained from the helm H_n and adding edges between the pendant vertices.

Definition 1.10. *Duplication of a vertex* v_j by a new edge e = $v_j'v_j''$ in a graph G produces a new graph G' such that N(v_j') = $\{v_j, v_j''\}$ and N(v_j'') = $\{v_j, v_j'\}$.

II. Main Results

Theorem 2.1. The switching of a pendant vertex in path P_n produces a difference cordial graph.

Proof. Let G = P_n and v_1, v_2, v_3, ⋯ , v_n be the successive vertices of path P_n and G_{v_n} denotes the vertex switching of G with respect to the pendant vertex v_n. It is obvious that |V(G_{v_n}) |= n and |E(G_{v_n})| = 2n – 2. We define a vertex labeling function f : V(G_{v_n}) ⟶{1, 2, 3, ⋯ , n}as follows:

f(v_i) = i, for $1 \le i \le n$.

Now we observe that $e_f(0) = e_f(1)$ = n - 1. Hence | $e_f(0) - e_f(1)$| = 0. Thus, the switching of the pendant vertex in path P_n produces a difference cordial graph. The difference cordial labeling of switching of the pendant vertex in path P_5 is shown in Figure 1.

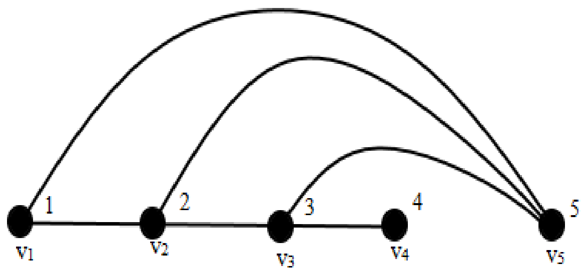

Figure 1. The switching of the pendant vertex in path P_5.

Theorem 2. 2. The switching of an apex vertex in closed helm CH_n produces a difference cordial graph.

Proof. Let G = CH_n. Let u_1, u_2, u_3, ⋯ , u_n and let v_1, v_2, v_3, ⋯ , v_n be the inner and outer cycle vertices of closed helm CH_n respectively. Let v be the apex vertex of G and let G_v denotes the vertex switching of G with respect to the apex vertex v. It is obvious that |V(G_v)| = 2n+1 and |E(G_v)| = 4n.

We define vertex labeling function f : V(G_v) ⟶{1, 2, 3, ⋯ , 2n+1}as follows:

f(u_i) = i, for $1 \le i \le n$,

f(v_i) = n+1+i, for $1 \le i \le n-1$,

f(v_n) = n+1,

f(v) = 2n+1.

Now we observe that $e_f(0) = e_f(1)$ = 2n. Hence | $e_f(0) - e_f(1)$| = 0. Thus, switching of an apex vertex in closed helm CH_n produces a difference cordial graph. The difference cordial labeling of the switching of an apex vertex in closed helm CH_4 is shown in Figure 2.

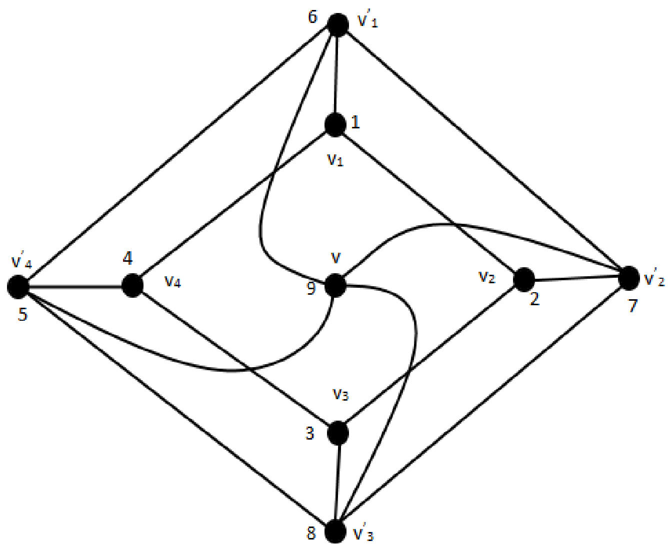

Figure 2. The difference cordial labeling of the switching of an apex vertex in CH_4.

Theorem 2.3. The graph obtained by duplication of each vertex of path by an edge is a difference cordial graph.

Proof. Let v_1, v_2, v_3, ⋯ , v_n be the successive vertices of path P_n and let G be the graph obtained by duplication of each vertex v_j by an edge $v_j'v_j''$. We note that |V(G)| = 3n and |E(G)| = 4n – 1. We define vertex labeling function f : V(G) ⟶{1, 2, 3, ⋯ , 3n}as follows:

f(v_i) = i, for $1 \le i \le n$,

f(v_j') = (n-1) + 2i, for $1 \le i \le n$,

f(v_j'') = n + 2i, for $1 \le i \le n$.

Now we conclude that $e_f(0)$ = 2n and $e_f(1)$ = 2n - 1. Hence | $e_f(0) - e_f(1)$| = 1. Thus, the graph obtained by duplication of each vertex of path by an edge is a difference cordial graph. The difference cordial labeling of the graph obtained by duplication of each vertex of path P_5 by an edge is shown in Figure 3.

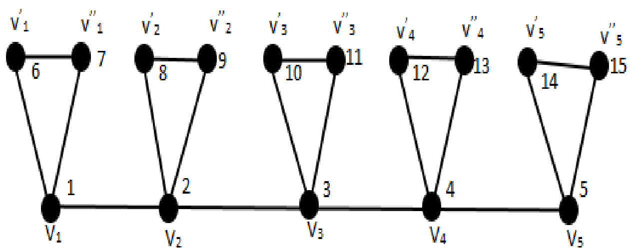

Figure 3. The graph obtained by duplication of an each vertex of path P_5 by an edge.

 DOI : 10.18231/2454-9150.2019.0099

Theorem 2.4. The graph obtained by barycentric subdivision of crown graph $C_n\Theta K_1$ is a difference cordial graph.

Proof. Consider a crown graph $C_n\Theta K_1$. Let $v_1, v_2, v_3, \cdots, v_n$ be the successive vertices in cycle C_n of $C_n\Theta K_1$. Let $u_1, u_2, u_3, \cdots, u_n$ be the pendant vertices of crown graph $C_n\Theta K_1$. Let G be the barycentric subdivision of $C_n\Theta K_1$. To obtain barycentric subdivision, we insert new vertex x_i to each edge v_iv_{i+1} $(1 \le i \le n-1)$ and vertex x_n to the edge v_nv_1. Similarly, we insert new vertex y_i to each edge v_iu_i $(1 \le i \le n)$. We note that |V(G)| = 4n and |E(G)| = 4n. We define a vertex labeling function f : V(G) →{1, 2, 3, ⋯, 4n}as follows:

$f(v_i) = 2i - 1$, for $1 \le i \le n$,

$f(x_i) = 2i$, for $1 \le i \le n$,

$f(y_i) = 2n + i$, for $1 \le i \le n$,

$f(u_i) = 3n + i$, for $1 \le i \le n$.

Finally we interchange the labels of the vertices u_1 and u_n. Now we observe that $e_f(0) = e_f(1) = 2n$. Hence $|e_f(0) - e_f(1)| = 0$. Thus, the graph obtained by barycentric subdivision of crown graph $C_n\Theta K_1$ is a difference cordial graph. The difference cordial labeling of the barycentric subdivision of crown $C_4\Theta K_1$ is shown in Figure 4.

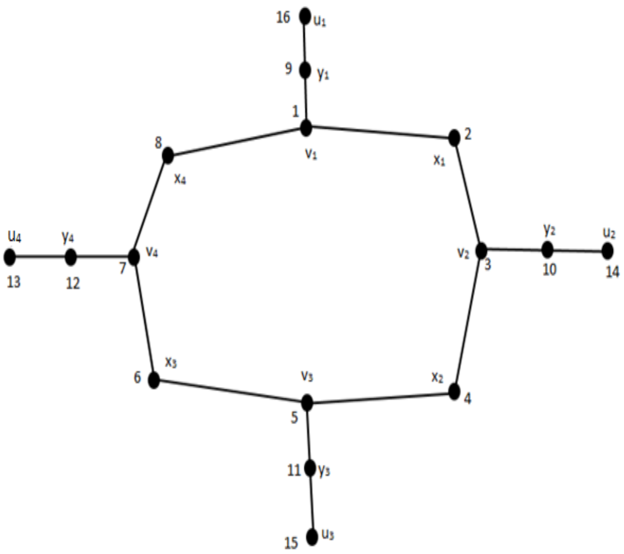

Figure 4. The difference cordial labeling of the graph obtained by barycentric subdivision of crown graph $C_4\Theta K_1$.

Theorem 2.5. The path union of finite r copies of fan Fn is a difference cordial graph.

Proof. Consider a fan graph $F_n = P_n + K_1$. Let $G = P(r. F_n)$. Let u_i $(1 \le i \le r)$ be the apex vertex of i[th] copy of F_n. Let $u_{i,j}$ be the j[th] vertex in P_n of i[th] copy of F_n. We note that |V(G)| = r (n + 1) and |E(G)| = 2nr - 1. We define a vertex labeling function f : V(G) →{1, 2, 3, ⋯, r (n + 1)}as follows:

$f(u_i) = 5i - 4$, for $1 \le i \le r$,

$f(u_{i,j}) = f(u_i) + j$, for $1 \le i \le r$, $1 \le j \le n$.

In view of the above labeling pattern f, we have $|e_f(0) - e_f(1)| \le 1$. Hence, the path union of finite r copies of fan graph is a difference cordial graph. The difference cordial labeling of path union of finite 3 copies of fan graph F_4 is shown in Figure 5.

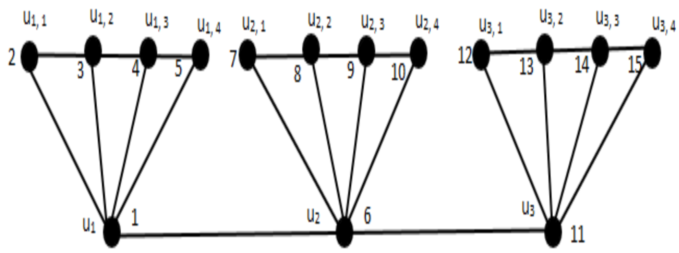

Figure 5. The difference cordial labeling of path union of 3 copies of fan graph F_4.

Theorem 2.6. The cycle union of finite r copies of fan graph is a difference cordial graph.

Proof. Consider a fan graph $F_n = P_n + K_1$. Let $G = C(r. F_n)$. Let u_i $(1 \le i \le r)$ be the apex vertex of i[th] copy of F_n. Let $u_{i,j}$ $(1 \le i \le r, 1 \le j \le n)$ be the j[th] vertex in P_n of i[th] copy of F_n. We note that |V(G)| = r(n + 1) and |E(G)| = 2nr. We define a vertex labeling function f : V(G) →{1, 2, 3, ⋯, r (n + 1)}as follows:

$f(u_i) = 5i - 4$, for $1 \le i \le r$,

$f(u_{i,j}) = f(u_i) + j$, for $1 \le i \le r$, $1 \le j \le n$.

In view of the above labeling pattern f, we observe that $e_f(0) = e_f(1) = nr$. Hence $|e_f(0) - e_f(1)| = 0$. Hence, the cycle union of finite r copies of fan graph is a difference cordial graph. The difference cordial labeling of cycle union of 4 copies of fan graph F_4 is shown in Figure 6.

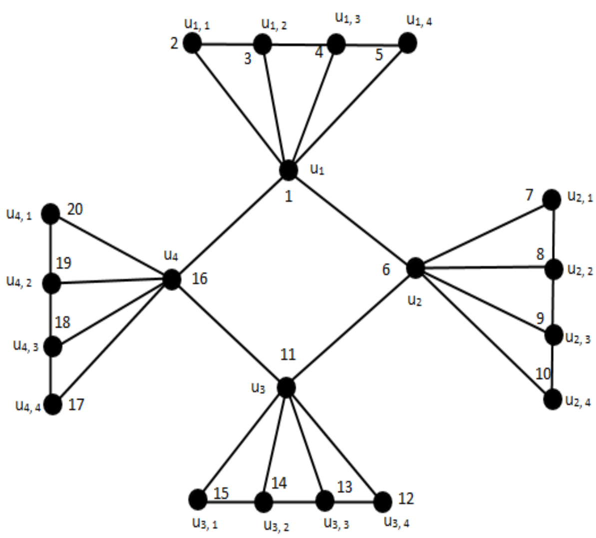

Figure 6. The difference cordial labeling of cycle union of 4 copies of fan graph F_4.

 DOI : 10.18231/2454-9150.2019.0099

Theorem 2.7. The open star of finite r copies of fan graph is a difference cordial graph.

Proof. Consider a fan graph $F_n = P_n + K_1$. Let $G = S(r.\ F_n)$. Let u_i $(1 \leq i \leq r)$ be the apex vertex of i^{th} copy of F_n. Let $u_{i,j}$ $(1 \leq i \leq r,\ 1 \leq j \leq n)$ be the j^{th} vertex in P_n of i^{th} copy of F_n. Let u be the apex vertex of the star graph $K_{1,\ r}$. We note that $|V(G)| = r(n + 1) + 1$ and $|E(G)| = 2nr$. We define a vertex labeling function $f : V(G) \rightarrow \{1, 2, 3, \cdots, r\,(n + 1) + 1\}$ as follows:

$f(u) = 1,$

$f(u_i) = 5i - 3$ for $1 \leq i \leq r,$

$f(u_{i,j}) = f(u_i) + j$ for $1 \leq i \leq r,\ 1 \leq j \leq n.$

Finally, we interchange the labels of the vertices u and u_1. In view of the above labeling pattern f, we observe that $e_f(0) = e_f(1) = nr$. Hence $|\ e_f(0) - e_f(1)| = 0$. Hence, the open star of finite r copies of fan graph is a difference cordial graph. The difference cordial labeling of open star of 4 copies of fan graph F_4 is shown in Figure 7.

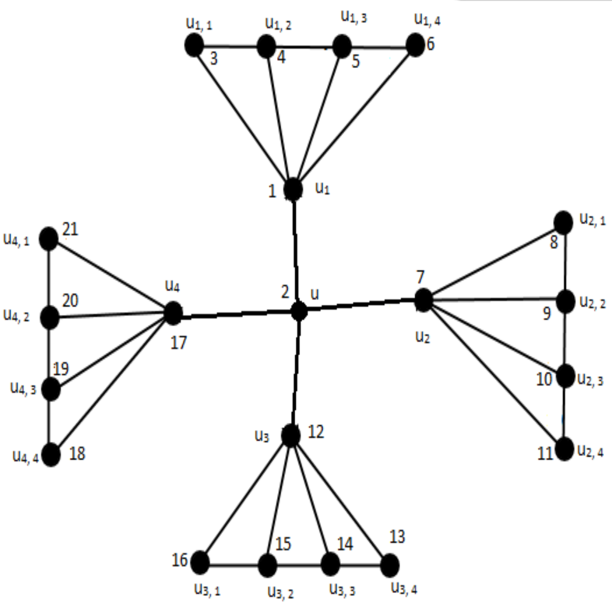

Figure 7. The difference cordial labeling of open star of 4 copies of fan graph F_4.

III. CONCLUSION

In this paper, we proved that, the graphs such as, the graph obtained by switching of a pendant vertex in P_n, the graph obtained by switching of an apex vertex in CH_n, the graph obtained by duplication of each vertex of path by an edge, the graph obtained by barycentric subdivision of crown graph $C_n \Theta K_1$, path union of fan $P(r.\ F_n)$ graph, cycle union of fan $C(r.\ F_n)$ graph and open star of fan $S(r.\ F_n)$ graph are difference cordial graphs.

REFERENCES.

[1] I. Cahit, *Cordial graph: A weaker version of graceful and harmonious graph*, Ars Combinatorial, 23(1987), 201-207.

[2] J. A. Gallian, *A dynamic survey of graph labeling*, The Electronic Journal of Combinatories, 17(2015), #DS6.

[3] F. Harary, *Graph theory*, Narosa Publishing House, (2001).

[4] R. Ponraj, S. Sathish Narayanan and R. Kala, *Difference cordial labeling of graphs*, Global Journal of Mathematical Sciences: Theory and Practical, 5(2013), 185-196.

[5] R. Ponraj, S. Sathish Narayanan, *Difference cordial labeling of some derived graphs*, International Journal of Mathematical Combinatorics, 4(2014), 37-48.

[6] R. Ponraj, S. Sathish Narayanan and R. Kala, *Difference cordial labeling of subdivision of snake graphs*, Universal Journal of Applied Mathematics, 2(1)(2014), 40-45.

[7] R. Ponraj, S. Sathish Narayanan and R. Kala, *A note on difference cordial graphs*, Palestine Journal of Mathematics, 4(1)(2015), 189-197.

[8] R. Ponraj, S. Sathish Narayanan and R. Kala, *Difference cordial labeling of some special graphs*, Palestine Journal of Mathematics, 6(11)(2017), 135-140.

[9] R. Ponraj, S. Sathish Narayanan, *Further results on difference cordial labeling of corona graphs*, The Journal of the Indian Acadamy of Mathematics, 3(2013), 217-235.

[10] R. Ponraj, S. Sathish Narayanan and R. Kala, *Difference cordial labeling of graphs obtained from double snakes*, International Journal of Mathematics Research, 5(3)(2013), 317-322.

[11] R. Ponraj, S. Sathish Narayanan, *Difference cordial labeling of some graphs obtained from double alternate snake graphs*, Global Journal of Mathematical Science: Theory and Practical, 5(2013), 167-175.

[12] M. A. Seoud and Shakir M. Salman, *On Difference cordial graphs*, Mathematica Aeterna, 5(1)(2015), 105-124.

[13] M. A. Seoud and Shakir M. Salman, *Some results and examples on difference cordial graphs*, Turkish Journal of Mathematics, 40(2016), 417-427.

[14] A. Sugumaran and V. Mohan, *Difference cordial labeling of graphs related to square graphs*, Journal of Emerging Technologies and Innovative Research, 5(11)(2018), 589-594.

 DOI : 10.18231/2454-9150.2019.0099