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$f-\gamma-$ PSg $-$Closed sets in Fine Topological Spaces

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Abstract: In this paper, we have defined a new class of sets called $f\gamma-P_S-$ generalized closed sets using $f\gamma-P_S-$open set and $f\gamma-P_S-$closure of a set in a fine topological space. Also, we have defined some new functions namely $f\gamma-P_S-g-$continuous, $f\gamma-P_S-g-$closed and $f\gamma-P_S-g-$open. Some properties of these functions are explored.

AMS Subject Classification: 54XX, 54CXX.

Key Words: Fine-open sets, $f\gamma-P_S-$open set, $f\gamma-P_S-$closed set, $f\gamma-P_S-g-$continuous function.

I. INTRODUCTION

Kasahara [8] defined the concept of $\alpha-$closed graphs of an operation on the topology τ defined over X. Later, Ogata [11] renamed the operation α as γ operation on τ. He defined $\gamma-$open sets and introduced the notion of τ_γ which is the class of all $\gamma-$open sets in a topological space (X,τ). Further study by Krishnan and Balachandran ([9],[10]) defined two types of sets called $\gamma-$preopen and $\gamma-$semiopen sets. The notion of $\alpha-\gamma-$open sets have been defined by Kalaivani and Krishnan[7]. Meanwhile, Basu, Afsan and Ghosh [5] defined $\gamma-\beta-$open sets by using the operation γ on τ. Carpintero, Rajesh and Rosas [6] introduced another notion of $\gamma-$open set called $\gamma-b-$open sets of a topological space (X,τ). Asaad, Ahmad and Omar [4] defined the notion of γ-regular-open sets which lies strictly between the classes of $\gamma-$open set and $\gamma-$clopen set. They introduced a new class of sets called $\gamma-P_S-$ open sets, and also defined $\gamma-P_S-$operations and their properties. They also introduced a new class of sets called $\gamma-P_S-$generalized closed set using $\gamma-P_S-$open set and $\tau_\gamma-P_S-$closure of a set and then investigate some of its properties.

Powar P.L. and Rajak K.[12] have introduced fine-topological space which is a special case of generalized topological space. This new class of fine-open sets contains all $\alpha-$open sets, $\beta-$ open sets, semi-open sets, pre-open sets, regular open sets etc. and fine-irresolute mapping include pre-continuous function, semi-continuous function, $\alpha-$ continuous function, $\beta-$continuous function, $\alpha-$ irresolute and $\beta-$irresolute function.

In this paper, we have defined a new class of sets called $f\gamma-P_S-$generalized closed sets using $f\gamma-P_S-$open set and $f\gamma-P_S-$closure of a set in a fine topological space. Also, we have defined some new functions namely $f\gamma-P_S-g-$ continuous, $f\gamma-P_S-g-$ closed and f $f\gamma-P_S-g-$open. Some properties of these functions have been investigated.

II. PRELIMINARIES

Throughout this paper, spaces (X,τ) and (Y,σ) always mean topological spaces

on which no separation axioms assumed unless explicitly defined.

Definition 2.1. An operation γ on the topology τ on X is a mapping $\gamma:\tau\to P(X)$ such that $U\subseteq\gamma(U)$ for each $U\in\tau$, where P(X) is the power set of X and $\gamma(U)$ denotes the value of γ at U.

Definition 2.2. Let (X,τ) be a topological space and γ be an operation on τ. A subset A of X is said to be:

1. $\gamma-$open set if for each $x\in A$ there exist an open set U such that $x\in U$ and $\gamma(U)\subseteq A$. τ_γ denotes the set of all $\gamma-$open sets in (X,τ). The complement of $\gamma-$open set is called a $\gamma-$closed set.
2. $\gamma-$regular-open if $A=\tau_\gamma-int(\tau_\gamma-cl(A))$ [4].
3. $\gamma-$preopen if $A\subseteq\tau_\gamma-int(\tau_\gamma-cl(A))$[9].
4. $\gamma-$semiopen if $A\subseteq\tau_\gamma-cl(\tau_\gamma-int(A))$ [10].
5. $\alpha-\gamma-$ open if $A\subseteq\tau_\gamma-int(\tau_\gamma-cl(\tau_\gamma-int(A)))$ [7].
6. $\gamma-b-$ open if $A\subseteq\tau_\gamma-cl(\tau_\gamma-int(A))\cup\tau_\gamma-int(\tau_\gamma-cl(A))$ [6].

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7. $\gamma-\beta-$ open if $A \subseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$ [5].
8. $\gamma-$clopen if it is both $\gamma-$open and $\gamma-$closed.
9. $\gamma-$dense if $\tau_\gamma - cl(A) = X$.

Definition 2.3. [7] The complement of $\gamma-$regular-open, $\gamma-$preopen, $\gamma-$semiopen $\alpha-\gamma-$open, $\gamma-b-$open and $\gamma-\beta-$ open set is said to be $\gamma-$ regular-closed, $\gamma-$ preclosed, $\gamma-$ semiclosed, $\alpha-\gamma-$ closed, $\gamma-b-$closed and $\gamma-\beta-$closed, respectively.

Definition 2.4. [1] A $\gamma-$ preopen subset A of a topological space (X,τ) is called $\gamma-P_S-$open if for each $x \in A$, there exists $\gamma-$semiclosed set F such that $x \in F \subseteq A$. The complement of a $\gamma-P_S-$open set is called a $\gamma-P_S-$closed.

The class of all $\gamma-P_S-$ open and $\gamma-P_S-$ closed subsets of a topological space (X,τ) are denoted by $\tau_\gamma - P_SO(X)$ and $\tau_\gamma - P_SC(X)$, respectively.

Definition 2.5. [1] Let A be any subset of a topological space (X,τ) and γ be anoperation on τ. Then:

1. The $\tau_\gamma - P_S-$ interior of A is defined as the union of all $\gamma-P_S-$open sets of X contained in A and it is denoted by $\tau_\gamma - P_S\ int(A)$.

2. The $\tau_\gamma - P_S-$closure of A is defined as the intersection of all $\gamma-P_S-$closed sets of X contained A and it is denoted by $\tau_\gamma - P_S\ cl(A)$.

3. $\tau_\gamma-$preclosure and $\tau_{\alpha-\gamma}-$closure of A is defined as the intersection of all $\gamma-$preclosed and $\alpha-\gamma-$closed sets of X containing A and it is denoted by $\tau_\gamma - pcl(A)$ and $\tau_{\alpha-\gamma} - cl(A)$, respectively.

Remark 2.1. [2] Let (X,τ) be a topological space and γ be an operation on τ. For any subset A of a space X. The following statements hold:

1. A is $\gamma-P_S-$closed if and only if $\tau_\gamma - P_Scl(A) = A$.

2. A is $\gamma-P_S-$open if and only if $\tau_\gamma - P_Sint(A) = A$.

3 $\tau_\gamma - P_Scl(X\backslash A) = X\backslash(\tau_\gamma - P_Sint(A))$ and $\tau_\gamma - P_Sint(X\backslash A) = X\backslash(\tau_\gamma - P_Scl(A))$.

Definition 2.6.[2] Let (X,τ) be a topological space and γ be an operation on τ. A subset A of X is called:

1. $\gamma-$pre-generalized closed ($\gamma-$pre g-closed) if $\tau_\gamma - pcl(A) \subseteq G$ whenever $A \subseteq G$and G is a $\gamma-$preopen set in X.

2. $\alpha-\gamma-$ generalized closed ($\alpha-\gamma-g-$ closed) if $\tau_{\alpha-\gamma} - cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $\alpha-\gamma-$open set in X.

Definition 2.7. [2] Let (X,τ) and (Y,σ) be two topological spaces and γ be an operation on τ. A functiof: $(X,\tau) \rightarrow (Y,\sigma)$ is calle $\gamma-P_S-$continuous if the pre-image of every closed set in Y is $\gamma-P_S-$closed set in X.

Definition 2.8.[2] Let A be any subset of a topological space (X,τ) with an operation γ on τ is called $\gamma-P_S-$generalized closed ($\gamma-P_S-g-$closed) if $\tau_\gamma - P_Scl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $\gamma-P_S-$open set in X.

The class of all $\gamma-P_S-g-$closed sets of X is denoted by $\tau_\gamma - P_SGC(X)$ and the class of all $\gamma-P_S-g-$open sets of X is denoted by $\tau_\gamma - P_SGO(X)$.

A set A is said to be $\gamma-P_S-$generalized open ($\gamma-P_S-g-$ open) if its complement $\gamma-P_S-g-$closed. Or equivalently, a set A is $\gamma-P_S-g-$open if $F \subseteq \tau_\gamma - P_Sint(A)$ whenever $F \subseteq A$ and F is a $\gamma-P_S-$closed set in X.

Definition 2.9. [2] Let (X,τ) and (Y,σ) be two topology spaces and γ be an operation on τ. A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called $\gamma-P_S-g-$continuous if the pre-image of every closed set in Y is $\gamma-P_S-g-$closed set in X.

Definition 2.10. [12] Let (X,τ) be a topological space we define $\tau(A_\alpha) = \tau_\alpha$ (say) $= \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where J is the index set. $\}$ Now, define $\tau_f = \{\phi, X\} \cup_\alpha \{\tau_\alpha\}$. The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X,τ,τ_f) is said to be the fine space X generated by the topology τ on X.

Definition 2.11.[12] A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open set of X is called the fine-closed set of X and we denote the collection by F_f.

Remark 2.2. Let (X,τ,τ_f) be a fine space the arbitrary union of fine open set in X is fine open in X.

Remark 2.3. The intersection of two fine-open sets need not be a fine-open set.

Definition 2.12. [12] A fine-open set S of a space (X,τ,τ_f) is called:

1. $f\alpha-$open if S is $\alpha-$open subset of a topological space (X,τ).

2. $fs-$open if S is a semi open subset of a topological space (X,τ).

3. $fp-$open if S is a pre-open subset of a topological space (X,τ).

4. $f\beta-$open if S is a $\beta-$open subset of a topological space (X,τ).

5. $fr-$open if S is a regular-open subset of a topological space (X,τ).

6. $f-$clopen (fine-clopen) if S is both fine-open and fine-closed subset of a topological space (X,τ).

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Definition 2.13. Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by $f_{int}(A)$.

Definition 2.14. Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by $f_{cl}(A)$.

Definition 2.15. A function $f:(X,\tau,\tau_f)\to(Y,\tau',\tau_f')$ is called fine-irresolute if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y.

Definition 2.16. A function $f:(X,\tau,\tau_f)\to(Y,\tau',\tau_f')$ is called fine-irresolute ($f-$irresolute) homeomorphism if

(1) f is one-one and onto.

(2) Both the function f and inverse function $f^{-1}:(Y,\tau',\tau_f')\to(X,\tau,\tau_f)$ are $f-$irresolute.

III. $f\gamma-P_S-$GENERALIZED CLOSED SETS

In this section, we define a new class of sets called $f\gamma-P_S-$generalized closed sets using $f\gamma-P_S-$open set and $f\gamma-P_S-$closure of set. We also study some of the basic properties of there sets.

Definition 3.1. Let (X,τ,τ_f) be a fine topological space, an operation γ on the fine topology τ_f is a mapping from τ_f on to the power set P(X) of X such that $U\subseteq\gamma(U)$ for each $U\in\tau_f$, where $\gamma(U)$ denotes the value of γ at U.

Example 3.1. Let $X=\{a,b,c\}$ with the topology $\tau=\{\phi,\{b\},X\}$ and $\tau_f=\{\phi,\{b\},\{b,c\},\{a,b\},X\}$. Define an operation $\gamma:\tau_f\to P(X)$ by $\gamma(A)=cl(A)$ for all $A\in\tau_f$. Then $A\subseteq\gamma(A)$ for all $A\in\tau_f$.

Definition 3.2. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is said to be $f\gamma-$ open set if for each $x\in A$ there exist an fine-open set U such that $x\in U$ and $\gamma(U)\subseteq A$. $f\tau_\gamma$ denotes the set of all $f\gamma-$open sets in (X,τ,τ_f). Complement of $f\gamma-$open set is $f\gamma-$ closed set and the collection is denoted by $F\tau_\gamma$.

Example 3.2. Let $X=\{a,b,c\}$ with the topology $\tau=\{\phi,\{b\},X\}$ and $\tau_f=\{\phi,\{b\},\{b,c\},\{a,b\},X\}$. Define an operation $\gamma:\tau_f\to P(X)$ by $\gamma(A)=A$ for all $A\in\tau_f$. $f\tau_f=\{\phi,\{b\},\{b,c\},\{a,b\},X\}$, $F\tau_\gamma=\{\phi,\{a,c\},\{a\},\{c\},X\}$.

Definition 3.3. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. Then, $f\tau_f-$interior of A is defined as the union of all $f\gamma-$open sets contained in A and it is denoted $f\tau_\gamma-int(A)$. That is $f\tau_\gamma-int(A)=\cup\{U: U \text{ is a } f\gamma-\text{open set and } U\subseteq A\}$.

Definition 3.4. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. Then, $f\tau_f-$closure of A is defined as the intersection of all $f\gamma-$closed sets containing A and it is denoted $f\tau_\gamma-cl(A)$. That is $f\tau_\gamma-cl(A)=\cap\{F: F \text{ is a } f\gamma-\text{closed set and } A\subseteq F\}$.

Example 3.3. Let $X=\{a,b,c\}$ with the topology $\tau=\{\phi,\{b\},X\}$ and $\tau_f=\{\phi,\{b\},\{b,c\},\{a,b\},X\}$. Define an operation $\gamma:\tau_f\to P(X)$ by $\gamma(A)=A$ for all $A\in\tau_f$. $f\tau_f=\{\phi,\{b\},\{b,c\},\{a,b\},X\}$, $F\tau_\gamma=\{\phi,\{a,c\},\{a\},\{c\},X\}$. If $S=\{b,c\}\subseteq X$, then $f\tau_\gamma-int(S)=\{b\}$ and $f\tau_\gamma-cl(S)=X$.

Definition 3.5. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is said to be:

1. $f\gamma-$regular-open if $A=f\tau_\gamma-int(f\tau_\gamma-cl(A))$.
2. $f\gamma-$preopen if $A\subseteq f\tau_\gamma-int(f\tau_\gamma-cl(A))$.
3. $f\gamma-$semiopen if $A\subseteq f\tau_\gamma-cl(f\tau_\gamma-int(A))$.
4. $f\alpha-\gamma-$ open if $A\subseteq f\tau_\gamma-int(f\tau_\gamma-cl(f\tau_\gamma-int(A)))$.
5. $f\gamma-b-$ open if $A\subseteq f\tau_\gamma-cl(f\tau_\gamma-int(A))\cup f\tau_\gamma-int(f\tau_\gamma-cl(A))$.
6. $f\gamma-\beta-$ open if $A\subseteq f\tau_\gamma-cl(f\tau_\gamma-int(f\tau_\gamma-cl(A)))$.
7. $f\gamma-$ clopen if it is both $f\gamma-$ open and $f\gamma-$closed.
8. $f\gamma-$dense if $f\tau_\gamma-cl(A)=X$.

Example 3.4. Let $X=\{a,b,c\}$, with topology $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau_f=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$ define an operation γ on τ_f such that

$$\gamma(A)=\begin{cases}A & \text{if } A=\{a\}\\ A\cup\{c\} & \text{if } A\neq\{a\}\end{cases}$$

for every $A\in\tau_f$. Set of all $f\gamma-$ open sets $f\tau_\gamma=\{\phi,X,\{a\},\{b,c\},\{a,c\}\}$ and set of all $f\gamma-$ closed sets $F\tau_\gamma=\{\phi,X,\{b,c\},\{a\},\{b\}\}$. If $A=\{a,b\}$ then $f\tau_\gamma-int\left(f\tau_\gamma-cl(A)\right)=X\Rightarrow A\subseteq f\tau_\gamma-int(f\tau_\gamma-cl(A))$ hence, A is $f\gamma-$preopen and also $f\gamma-\beta-$open and $f\gamma-$dense set. If $A=\{a\}$ or $\{b,c\}$ both are $f\gamma-$clopen because they are both $f\gamma-$open and $f\gamma-$closed.

Remark 3.1. Every $\gamma-$ open, $\gamma-$ regular open, $\gamma-$preopen is $f\gamma-$open, $f\gamma-$regular open, $f\gamma-$preopen respectively, but converse is not necessarily true.

Example 3.5. Let $X=\{a,b,c\}$, with topology $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau_f=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$ define an operation γ on τ_f such that

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$$\gamma(A)=\begin{cases}A & \text{if } A=\{a\}\\ A\cup\{c\} & \text{if } A\neq\{a\}\end{cases}$$

for every $A\in\tau_f$. Set of all $f\gamma-$ open sets $f\tau_\gamma=\{\phi,X,\{a\},\{b,c\},\{a,c\}\}$ and set of all $f\gamma-$ closed sets $F\tau_\gamma=\{\phi,X,\{b,c\},\{a\},\{b\}\}$. Set of all fine $\gamma-$preopen sets $f\tau_\gamma-PO(X)=\{\phi,X,\{a\},\{b,c\},\{a,c\},\{c\},\{a,b\}\}$ and $f\tau_\gamma-RO(X)=\{\phi.X,\{a\},\{b,c\}\}$.

- Set of all $\gamma-$ open sets $\tau_\gamma=\{\phi,X,\{a\}\}$ then, every $\gamma-$open sets are $f\gamma-$open but, converse is not true.
- Set of all $\gamma-$ preopen sets $\tau_\gamma-PO(X)=\{\phi,X,\{a\},\{a,c\},\{a,b\}\}$ then, every $\gamma-$preopen sets are $f\gamma-$preopen but, converse is not true.
- Set of all $\gamma-$regular open sets $\tau_\gamma-RO(X)=\{\phi,X\}$ then, every $\gamma-$regular open sets are $f\gamma-$regular open, but converse is not true.

Definition 3.6. The complement of $f\gamma-$regular-open, $f\gamma-$preopen, $f\gamma-$semiopen, $f\alpha-\gamma-$open, $f\gamma-b-$open and $f\gamma-\beta-$open set is said to be $f\gamma-$regular-closed, $f\gamma-$ preclosed, $f\gamma-$ semiclosed, $f\alpha-\gamma-$ closed, $f\gamma-b-$closed and $f\gamma-\beta-$closed, respectively.

Definition 3.7. A $f\gamma-$ preopen subset A of a fine topological space (X,τ,τ_f) is called $f\gamma-P_S-$open if for each $x\in A$, there exists $f\gamma-$ semiclosed set F such that $x\in F\subseteq A$. The complement of a $f\gamma-P_S$ $-$open set is called a $f\gamma-P_S$ $-$closed.

The class of all $f\gamma-P_S-$ open and $f\gamma-P_S$ $-$closed subsets of a fine topological space (X,τ,τ_f) are denoted by $f\tau_\gamma-P_SO(X)$ and $f\tau_\gamma-P_SC(X)$, respectively.

Definition 3.8. Let A be a subset of a fine space X, we say that a point $x\in X$ is a $f\gamma-P_S-$limit point of A if every $f\gamma-P_S-$open set of X containing x must contains at least one point of A other than x.

Example 3.6. Let $X=\{a,b,c\}$, $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\}\}$ and fine space $\tau_f=\{\phi,X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,a\}\}$ define an operation γ on τ_f such that

$$\gamma(A)=\begin{cases}A & \text{if } a\in A\\ cl(A) & \text{if } a \text{ not in } A\end{cases}$$

for every $A\in\tau_f$. $f\tau_\gamma=f\tau_\gamma-PO(X)=f\tau_\gamma-P_SO(X)=P(X)$.

Example 3.7. Let $X=\{a,b,c\}$, with topology $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau_f=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$ define an operation γ on τ_f such that

$$\gamma(A)=\begin{cases}A & \text{if } A=\{a\}\\ A\cup\{c\} & \text{if } A\neq\{a\}\end{cases}$$

for every $A\in\tau_f$. Set of all $f\gamma-$open sets $f\tau_\gamma=\{\phi,X,\{a\},\{b,c\},\{a,c\}\}$ and set of all $f\gamma-$closed sets $F\tau_\gamma=\{\phi,X,\{b,c\},\{a\},\{b\}\}$. Set of all fine $\gamma-$ preopen sets $f\tau_\gamma-PO(X)=\{\phi,X,\{a\},\{b,c\},\{a,c\},\{c\},\{a,b\}\}$. Set of all fine $\gamma-$ semiopen sets $f\tau_\gamma-SO(X)=\{\phi.X,\{a\},\{b,c\},\{a,c\}\}$ and $f\tau_\gamma-SC(X)=\{\phi,X,\{b,c\},\{a\},\{b\}\}$. Then the class of all $f\gamma-P_S-$open sets $f\tau_\gamma-P_SO(X)=\{\phi.X,\{a\},\{b,c\}\}$ and the class of $f\gamma-P_S-$ closed sets $f\tau_\gamma-P_SC(X)=\{\phi.X,\{a\},\{b,c\}\}$.

Definition 3.9. Let A be any subset of a fine topological space (X,τ,τ_f) and γ be anoperation on τ_f. Then:

1. The $f\tau_\gamma-P_S-$ interior of A is defined as the union of all $f\gamma-P_S$ $-$open sets of X contained in A and it is denoted by $f\tau_\gamma-P_S\,int(A)$.
2. The $f\tau_\gamma-P_S$ $-$ closure of A is defined as the intersection of all $f\gamma-P_S$ $-$closed sets of X contained A and it is denoted by $f\tau_\gamma-P_S\,cl(A)$.
3. $f\tau_\gamma-$preclosure and $f\tau_{\alpha-\gamma}-$closure of A is defined as the intersection of all $f\gamma-$preclosed and $f\alpha-\gamma-$closed sets of X containing A and it is denoted by $f\tau_\gamma-pcl(A)$ and $f\tau_{\alpha-\gamma}-cl(A)$, respectively.

Theorem 3.1. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. For any subset A of a space X. The following statements hold:

1. A is $f\gamma-P_S$ $-$closed if and only if $f\tau_\gamma-P_Scl(A)=A$.
2. A is $f\gamma-P_S$ $-$open if and only if $f\tau_\gamma-P_Sint(A)=A$.
3. $f\tau_\gamma-P_Scl(X\backslash A)=X\backslash(f\tau_\gamma-P_Sint(A))$ and $f\tau_\gamma-P_Sint(X\backslash A)=X\backslash(f\tau_\gamma-P_Scl(A))$.

Proof:

1. Let A be a $f\gamma-P_S$ $-$closed and $x\in A$) $\Rightarrow x\in f\tau_\gamma-P_S\,cl(A)\Rightarrow A\subseteq f\tau_\gamma-P_Scl(A)$. If $x\in f\tau_\gamma-P_Scl(A)$ such that x is $f\gamma-P_S-$limit point, since A is $f\gamma-P_S$ $-$closed set then $x\in A\Rightarrow f\tau_\gamma-P_Scl(A)\subseteq A$. Hence $f\tau_\gamma-P_Scl(A)=A$.

Conversely, if $f\tau_\gamma-P_Scl(A)=A$ then obviously A is $f\gamma-P_S-$closed.

2. Let A be a $f\gamma-P_S-$open and $f\tau_\gamma-P_Sint(A)$ is defined as the union of all $f\gamma-P_S-$ open subsets. Therefore $A\subseteq f\tau_\gamma-P_Sint(A)$ and $f\tau_\gamma-P_Sint(A)\subseteq A$ obvious. Hence $f\tau_\gamma P_Sint(A)=A$.

Conversely, if $f\tau_\gamma-P_Sint(A)=A$ then obviously A is $f\gamma-P_S-$open set.

3. Since $f\tau_\gamma-P_Sint(A)\subseteq A$ and let $x\in X\backslash(f\tau_\gamma-P_Sint(A))\Rightarrow$ x not belongs in $f\tau_\gamma-P_Sint(A)\Rightarrow x\in f\tau_\gamma-P_Scl(X\backslash A)\Rightarrow X\backslash f\tau_\gamma-P_Sint(A)\subseteq f\tau_\gamma-P_Scl(X\backslash A)$. If $x\in f\tau_\gamma-P_Scl(X\backslash A)\Rightarrow$ x not belongs in $f\tau_\gamma-P_Sint(A)\Rightarrow x\in X\backslash(f\tau_\gamma-P_Sint(A))\Rightarrow f\tau_\gamma-P_Scl(X\backslash A)\subseteq X\backslash(f\tau_\gamma-P_Sint(A))$. Hence $f\tau_\gamma-P_Scl(X\backslash A)=X\backslash(f\tau_\gamma-P_Sint(A))$. To prove $f\tau_\gamma-P_Sint(X\backslash A)=X\backslash(f\tau_\gamma-P_Scl(A)$, let $B=X\backslash A$. Then $X\backslash(f\tau_\gamma-P_Sint(B))=f\tau_\gamma-P_Scl(X\backslash B)$ $\Rightarrow X\backslash(f\tau_\gamma-P_Sint(X\backslash A)=f\tau_\gamma-P_Scl(X\backslash(X\backslash A))=$

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$f\tau_\gamma - P_S cl(A)$. Hence $f\tau_\gamma - P_S int(X \setminus A) = X \setminus (f\tau_\gamma - P_S cl(A))$.

Definition 3.10. Let (X, τ, τ_f) be a fine topological space and γ be an operation on τ_f. A subset A of X is called:

1. $f\gamma-$ pre-generalized closed ($f\gamma -$ pre g -closed) if $f\tau_\gamma - pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\gamma -$preopen set in X.
2. $f\alpha - \gamma -$generalized closed $(f\alpha - \gamma - g -$closed) if $f\tau_{\alpha-\gamma} - cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\alpha - \gamma -$open set in X.

Example 3.8. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, X\}$ and $\tau_f = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$. Define an operation $\gamma: \tau_f \to P(X)$ by $\gamma(A) = A$ for all $A \in \tau_f$. $f\tau_f = \{\phi, \{b\}, \{b, c\}, \{a, b\}, X\}$, $F\tau_\gamma = \{\phi, \{a, c\}, \{a\}, \{c\}, X\}$ and $f\tau_f - PO(X) = \{\phi, \{a\}, \{a, c\}, \{a, b\}, X\}$, $f\tau_f - P_S O(X) = \{\phi, X\}$, $f\tau_f - P_S C(X) = \{\phi, X\}$ and $f\tau_f - P_S GC(X) =$ all subsets of X.

Theorem 3.2. Given a topological space (X, τ), consider a fine space (X, τ, τ_f) generated by τ. Then following hold:

1. Every $f\gamma -$preclosed set is $f\gamma - pre - g -$closed.
2. Every $f\alpha - \gamma -$closed set is $f\alpha - \gamma - g -$closed.

Proof:

1. Given A be any $f\gamma -$preclosed set in a fine space X and $A \subseteq G$ where G is a $f\gamma -$preopen set in X.

claim: Set A is a $f\gamma -$pre$-g -$closed set.

Since by the definition, set A is said $f\gamma -$pre-generalized closed ($f\gamma -$pre$g$-closed) if $f\tau_\gamma - pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\gamma -$ preopen set in X. Then $f\tau_\gamma - pcl(A) \subseteq G$ since A is $f\gamma -$preclosed set. Therefore, A is $f\gamma -$pre$-g -$closed.

2. Given A be any $f\alpha - \gamma -$closed set in a fine space X and $A \subseteq G$ where G is a $f\alpha - \gamma -$open set in X.

claim: Set A is a $f\alpha - \gamma - g -$closed set.

Since by the definition, set A is said $f\alpha - \gamma -$generalized closed ($f\alpha - \gamma - g -$ closed) if $f\tau_{\alpha-\gamma} - cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\alpha - \gamma -$open set in X. Then $f\tau_{\alpha-\gamma} - cl(A) \subseteq G$ since A is $f\alpha - \gamma -$ closed set. Therefore, A is $f\alpha - \gamma - g -$closed.

Theorem 3.3. Given a topological space (X, τ), consider a fine space (X, τ, τ_f) generated by τ. Then following hold:

1. Every $\gamma -$open set is fine-open set.
2. Every $\gamma -$pre$-$open set is fine-open set.
3. Every $\gamma -$semi$-$open set is fine-open set.
4. Every $\alpha - \gamma -$open set is fine-open set.
5. Every $\gamma -$regular$-$open set is fine-open set.
6. Every $f\gamma -$open set is fine-open set.
7. Every $f\gamma -$regular$-$open set is fine-open set.
8. Every $f\gamma -$semi$-$open set is fine-open set.

Proof:

1. Let (X, τ, τ_f) be the fine space with respect to the topological space (X, τ). Consider A be a non-empty $\gamma -$open set then, for each $x \in A$ there exists an open set U such that $x \in U$, and $\gamma(U) \subseteq A$. Since each $x \in A$ belong in U_α, so $A \subseteq U_\alpha$ for all $\alpha \in J$. Since U is an open set and $A \cap U \neq \phi$ for $U \in \tau$ and $A \neq \phi \Rightarrow A \in \tau_\alpha = \{A: A \cap U_\alpha \neq \phi$ for $U_\alpha \in \tau$ and $A \neq \phi\} \Rightarrow \tau_f = \{\phi, X\} \cup \tau_\alpha \Rightarrow A \in \tau_f$.
2. Let A be a non-empty subset of X and $A \notin \tau_f$.

   Claim: A is not $\gamma -$pre$-$open set in X.

   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \ \forall \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow \tau_\gamma - cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and $\tau_\gamma - cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap \tau_\gamma - cl(A) = \phi \Rightarrow \tau_\gamma - int(\tau_\gamma - cl(A) = \phi \Rightarrow A$ is not subset of $\tau_\gamma - int(\tau_\gamma - cl(A)$. Hence A is not $\gamma -$ pre$-$ open set.
3. Let A be a non-empty subset of X and $A \notin \tau_f$.

   Claim: A is not $\gamma -$semi$-$open set in X.

   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \ \forall \alpha \in J \Rightarrow \tau_\gamma - int(A) = \phi \Rightarrow \tau_\gamma - cl\left(\tau_\gamma - int(A)\right) = \phi \Rightarrow A$ is not subset of $\tau_\gamma - cl\left(\tau_\gamma - int(A)\right)$. Hence A is not $\gamma -$semi –open set.
4. Let A be a non-empty subset of X and $A \notin \tau_f$.

   Claim: A is $\alpha - \gamma -$open set in X.

   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \ \forall \alpha \in J \Rightarrow \tau_\gamma - int(A) = \phi \Rightarrow \tau_\gamma - cl\left(\tau_\gamma - int(A)\right) = \phi \Rightarrow \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A))) = \phi \Rightarrow A$ is not subset of $\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$. Hence A is not $\alpha - \gamma -$semi –open set.
5. Let A be a non-empty subset of X and $A \notin \tau_f$.

   Claim: A is not $\gamma -$regular$-$open set in X.

   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \ \forall \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow \tau_\gamma - cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and $\tau_\gamma - cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap \tau_\gamma - cl(A) = \phi \Rightarrow \tau_\gamma - int(\tau_\gamma - cl(A) = \phi \Rightarrow A \neq \tau_\gamma - int(\tau_\gamma - cl(A)$. Hence A is not $\gamma -$ regular$-$ open set.
6. Let (X, τ, τ_f) be the fine space with respect to the topological space (X, τ). Consider A be a non-empty $f\gamma -$open set then, for each $x \in A$ there exists an $f -$open set U such that $x \in U$, and $\gamma(U) \subseteq A$. Since each $x \in A$ belong in U_α, so $A \subseteq U_\alpha$ for all $\alpha \in J$. Since U is an $f -$open set and $A \cap U \neq \phi$ for $U \in \tau_f$ and $A \neq \phi \Rightarrow A \in \tau_\alpha = \{A: A \cap U_\alpha \neq \phi$ for $U_\alpha \in \tau_f$ and $A \neq \phi\} \Rightarrow \tau_f = \{\phi, X\} \cup \tau_\alpha \Rightarrow A \in \tau_f$.
7. Let A be a non-empty subset of X and $A \notin \tau_f$.

   Claim: A is not $f\gamma -$regular$-$open set in X.

   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi \ \forall \alpha \in J \Rightarrow A \subseteq X - G_\alpha \Rightarrow f\tau_\gamma - cl(A) \subseteq X - G_\alpha$. Since $G_\alpha \cap X - G_\alpha = \phi$ and $f\tau_\gamma - cl(A) \subseteq X - G_\alpha \Rightarrow G_\alpha \cap f\tau_\gamma - cl(A) = \phi \Rightarrow f\tau_\gamma - int(f\tau_\gamma - cl(A) = \phi \Rightarrow$

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$A \neq f\tau_\gamma - int(f\tau_\gamma - cl(A))$. Hence A is not $f\gamma-$ regular$-$ open set.

8. Let A be a non-empty subset of X and $A \notin \tau_f$.
   Claim: A is not $f\gamma-$semi$-$open set in X.
   Since, it is given that $A \notin \tau_f \Rightarrow G_\alpha \cap A = \phi\ \forall\, \alpha \in J \Rightarrow f\tau_\gamma - int(A) = \phi \Rightarrow f\tau_\gamma - cl\left(f\tau_\gamma - int(A)\right) = \phi \Rightarrow A$ is not subset of $f\tau_\gamma - cl\left(f\tau_\gamma - int(A)\right)$. Hence A is not $f\gamma-$semi –open set.

Definition 3.11. Let (X,τ,τ_f) and (Y,σ,σ_f) be two fine topological spaces and γ be an operation on τ_f. A function $f{:}\,(X,\tau,\tau_f) \to (Y,\sigma,\sigma_f)$ is called $f\gamma - P_S-$continuous if the pre-image of every $f-$closed set in Y is $f\gamma - P_S$ $-$closed set in X.

Example 3.9. In example 3.7., we have class of all all $f\gamma - P_S-$ open sets $f\tau_\gamma - P_SO(X) = \{\phi.X,\{a\},\{b,c\}\}$ and the class of $f\gamma - P_S-$closed sets $f\tau_\gamma - P_SC(X) = \{\phi.X,\{a\},\{b,c\}\}$. Let $Y = \{b,c\}$ and $\sigma = \{\phi,Y,\{c\}\}, \sigma_f = \{\phi,Y,\{c\}\}$ and $F\sigma_f = \{\phi,Y,\{b\}\}$, define a mapping $f{:}\,(X,\tau,\tau_f) \to (Y,\sigma,\sigma_f)$ by $f(a) = b, f(b) = c, f(c) = c$. It is clear that pre image of each $f-$closed set in Y is $f\gamma - P_S-$closed set in X.

Definition 3.12. Let A be any subset of a fine topological space (X,τ,τ_f) with an operation γ on τ_f is called $f\gamma - P_S$ $-$generalized closed ($f\gamma - P_S - g-$closed) if $f\tau_\gamma - P_Scl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\gamma - P_S$ $-$open set in X.

The class of all $f\gamma - P_S$ $-g-$closed sets of X is denoted by $f\tau_\gamma - P_SGC(X)$ and the class of all $f\gamma - P_S$ $-g-$open sets of X is denoted by $f\tau_\gamma - P_SGO(X)$.

A set A is said to be $f\gamma - P_S$ $-$generalized open ($f\gamma - P_S$ $- g -$ open) if its complement $f\gamma - P_S$ $- g -$ closed. Or equivalently, a set A is $f\gamma - P_S$ $- g-$open if $F \subseteq f\tau_\gamma - P_Sint(A)$ whenever $F \subseteq A$ and F is a $f\gamma - P_S$ $-$closed set in X.

Example 3.10. Let $X = \{a,b,c\}$ with topology $\tau = \{\phi,X,\{b\}\}$ and fine space $\tau_f = \{\phi,X,\{b\},\{a,b\},\{b,c\}\}$ define an operation γ on τ_f such that $\gamma(A) = \begin{cases} A & if\ c \in A \\ cl(A) & if\ c \notin A \end{cases}$ for every $A \in \tau_f$. Set of all fine $-\gamma-$ open sets $f\tau_f = \{\phi,X,\{b,c\}\} = f\tau_f - SO(X)$ and $f\tau_f - P_SO(X) = \{\phi,X\}, f\tau_\gamma - P_SGO(X) = P(X)$.

Lemma 3.1. Every $f\gamma - P_S-$closed set is $f\gamma - P_S - g-$closed.

Proof : Given A be any $f\gamma - P_S-$closed set in a fine space X and $A \subseteq G$ where G is a $f\gamma - P_S-$open set in X.

claim: Set A is a $f\gamma - P_S - g-$closed set.

Since by the definition, set A is said $f\gamma - P_S-$generalized closed ($f\gamma - P_S - g-$closed) if $f\tau_\gamma - P_Scl(A) \subseteq G$ whenever $A \subseteq G$ and G is a $f\gamma - P_S-$open set in X. Then $f\tau_\gamma - P_Scl(A) \subseteq G$ since A is $f\gamma - P_S-$closed set. Therefore, A is $f\gamma - P_S - g-$closed.

Theorem 3.4. Let (x,τ,τ_f) be a fine topological space and γ be an operation on τ_f. If a subset A of X is $f\gamma - P_S - g-$ closed and $f\gamma - P_S-$ open, then A is $f\gamma - P_S-$closed.

Proof: Given A is $f\gamma - P_S - g-$ closed and $f\gamma - P_S-$open in X, then by Definition 3.12, $f\tau_\gamma - P_Scl(A) \subseteq A$ and $A \subseteq f\tau_\gamma - P_Scl(A)$ is obvious. Hence A is $f\gamma - P_S-$closed set.

Theorem 3.5. Let (X,τ,τ_f) be a fine topological space and γ be an operation on τ_f. If a subset A of X is $f\gamma - P_S - g-$ closed and $f\gamma - P_S-$ open and F is $f\gamma - P_S-$closed, then $A \cap F$ is $f\gamma - P_S-$closed.

Proof: Let A be $f\gamma - P_S - g-$closed and $f\gamma - P_S-$open in X. Then by Theorem 3.4, if A is $f\gamma - P_S - g-$closed and $f\gamma - P_S-$open, then A is $f\gamma - P_S-$closed and F is also $f\gamma - P_S-$ closed that means A and F both are $f\gamma - P_S-$closed, then $A \cap F$ is $f\gamma - P_S-$closed.

Corollary 3.1. If $A \subseteq X$ is both $f\gamma - P_S - g-$closed and $f\gamma - P_S-$open and F is $f\gamma - P_S-$closed, then $A \cap F$ is $f\gamma - P_S-$closed.

Proof: Since every $f\gamma - P_S-$ closed set is $f\gamma - P_S - g-$ closed. F is $f\gamma - P_S - g-$ closed then F is also $f\gamma - P_S-$ closed set. That means A and F both are $f\gamma - P_S-$closed set, then $A \cap F$ is $f\gamma - P_S - g-$closed.

Theorem 3.6. Let A be a subset of fine topological space (X,τ,τ_f) and γ be an operation on τ_f. Then A is $f\gamma - P_S - g-$closed if and only if $f\gamma - P_Scl(A)\backslash A$ does not contain any non-empty $f\gamma - P_S-$closed set.

Proof: Let F be a non-empty $f\gamma - P_S-$closed set in fine space X such that $F \subseteq f\gamma - P_Scl(A)\backslash A$. Then it is clear that $F \subseteq X\backslash A$ implies $A \subseteq X\backslash F$. Since F is $f\gamma - P_S-$closed set implies $X\backslash F$ is $f\gamma - P_S-$open set and A is $f\gamma - P_S - g-$ closed set, then $f\tau_\gamma - P_Scl(A) \subseteq X\backslash F$. That is $F \subseteq X\backslash(f\tau_\gamma - P_Scl(A))$. Hence $F \subseteq X\backslash(f\tau_\gamma - P_Scl(A)) \cap (f\tau_\gamma - P_Scl(A))\backslash A = \phi$. This implies that $F \subseteq \phi$ and $\phi \subseteq F$ obvious, consequently $F = \phi$. This is contradiction. Therefore, F is not subset of $f\gamma - P_Scl(A)\backslash A$.

Conversely, let $A \subseteq G$ and G is $f\gamma - P_S-$open set in X. So $X\backslash G$ is $f\gamma - P_S-$closed set in X. Suppose that $f\tau_\gamma - P_Scl(A)$ not subset of G, then $f\tau_\gamma - P_Scl(A) \cap X\backslash G$ is a non-empty $f\gamma - P_S-$ closed set such that $f\tau_\gamma - P_Scl(A) \cap X\backslash G \subseteq f\tau_\gamma - P_Scl(A))\backslash A$. Contradiction

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of hypothesis. Hence $f\tau_{\gamma} - P_S cl(A) \subseteq G$ and so A is $f\gamma - P_S - g$ −closed set.

IV. $f\gamma - P_S - g$ − CONTINUOUS FUNCTIONS

In this section, we have introduced a new class of functions called $f\gamma - P_S - g -$ continuous by using $f\gamma - P_S - g$ −closed set. Some theorems and properties for this function are studied.

Definition 4.1. Let (X, τ, τ_f) and (Y, σ, σ_f) be two fine topological spaces and γ be an operation on τ_f. A function $f: (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is called $f\gamma - P_S - g$ −continuous if the pre- image of every f −closed set in Y is $f\gamma - P_S - g$ −closed set in X.

Example 4.1. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, X, \{b\}\}$ and fine space $\tau_f = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$ define an operation γ on τ_f such that $\gamma(A) = \begin{cases} A & if\ c \in A \\ cl(A) & if\ c \notin A \end{cases}$ for every $A \in \tau_f$. Set of all fine $-\gamma-$ open sets $f\tau_f = \{\phi, X, \{b, c\}\} = f\tau_f - SO(X)$ and $f\tau_f - P_S O(X) = \{\phi, X\}, f\tau_\gamma - P_S GO(X) = P(X)$. Suppose that $Y = \{1,2,3\}$ and $\sigma = \{\phi, Y, \{1\}, \{1,3\}\}$, $\sigma_f = \{\phi, Y, \{1\}, \{1,2\}, \{1,3\}, \{3\}, \{2,3\}\}$. Let $f: (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be a function defined by $f(a) = 1, f(b) = 2, f(c) = 3$. Then f if $f\gamma - P_S - g$ −continuous, but f is not $f\gamma - P_S -$ continuous since $\{2,3,\}$ is $f-$ closed in (Y, σ, σ_f), but $f^{-1}(\{2,3\}) = \{b, c\}$ is not $f\gamma - P_S$ −closed set in (X, τ, τ_f).

Theorem 4.1. Every $f\gamma - P_S -$ continuous function is $f\gamma - P_S - g$ −continuous.s

Proof: Let $f: (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ be any $f\gamma - P_S -$ continuous function. Then pre-image of each f −closed set in Y is $f\gamma - P_S$ −closed set in X. By lemma 3.1, every $f\gamma - P_S -$closed set is $f\gamma - P_S - g -$closed. Then pre-image of each f −closed set in Y is $f\gamma - P_S - g$ −closed set in X. Then f is $f\gamma - P_S - g$ −continuous.

Converge of the theorem is not true see Example 4.1, f is $f\gamma - P_S - g -$ continuous but not $f\gamma - P_S$ −continuous.

The following result holds directly:

Theorem 4.2. Let γ be an operation on the fine-topological space (X, τ, τ_f). If the functions $f: (X, \tau, \tau_f) \to (Y, \sigma, \sigma_f)$ is $f\gamma - P_S - g$ −continuous and $g: (Y, \sigma, \sigma_f) \to (Z, \rho, \rho_f)$ is continuous. Then the composition function $g \circ f: (X, \tau, \tau_f) \to (Z, \rho, \rho_f)$ is $f\gamma - P_S - g$ −continuous.

Definitio 4.2. A function $f: (X, \tau, \tau_f) \to (Y, \tau', \tau_f')$ is called fine-$\gamma - P_S$ −irresolute ($f\gamma - P_S$ −irresolute) map if $f^{-1}(V)$ is fine-$\gamma - P_S -$open in X for every fine- $\gamma - P_S$ −open set V of Y.

Example 4.2. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & if\ \ A = \{a\} \\ A \cup \{c\} & if\ A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. Set of all $f\gamma -$ open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}\}$ and the class of all $f\gamma - P_S -$ open sets $f\tau_\gamma - P_S O(X) = \{\phi. X, \{a\}, \{b, c\}\}$. Let $Y = \{1,2,3\}$, with topology $\tau' = \{\phi, Y, \{1\}, \{2\}, \{1,2\}\}$ and $\tau_f{}' = \{\phi, Y, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,3\}\}$ define an operation γ on $\tau_f{}'$ such that

$$\gamma(S) = \begin{cases} S & if\ \ S = \{1\} \\ S \cup \{3\} & if\ S \neq \{1\} \end{cases}$$

for every $S \in \tau_f'$. Set of all $f\gamma -$ open sets $f\tau_\gamma{}' = \{\phi, Y, \{1\}, \{2,3\}, \{1,3\}\}$ and the class of all $f\gamma - P_S$ −open sets $f\tau_\gamma' - P_S O(Y) = \{\phi. Y, \{1\}, \{2,3\}\}$. We define a mapping $f: X \to Y$ such that $f(a) = 1, f(b) = 2. f(c) = 3$. It may be checked that the pre-image of $f\gamma - P_S$ −open sets of Y viz. $\{1\}, \{2,3\}$ are $\{a\}, \{b, c\}$ respectively, which are $f\gamma - P_S -$ open in X. Therefore f is $f\gamma - P_S$ −irresolute, but it is not continuous.

Definition 4.3. A function $f: (X, \tau, \tau_f) \to (Y, \tau', \tau_f')$ is called fine- $\gamma - P_S -$ irresolute ($f\gamma - P_S -$ irresolute) homeomorphism if

(1) f is one-one and onto.

(2) Both the function f and inverse function $f^{-1}: (Y, \tau', \tau_f') \to (X, \tau, \tau_f)$ are $f\gamma - P_S$ −irresolute.

Example 4.3. Let $X = \{a, b, c\}$, with topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ define an operation γ on τ_f such that

$$\gamma(A) = \begin{cases} A & if\ \ A = \{a\} \\ A \cup \{c\} & if\ A \neq \{a\} \end{cases}$$

for every $A \in \tau_f$. Set of all $f\gamma -$ open sets $f\tau_\gamma = \{\phi, X, \{a\}, \{b, c\}, \{a, c\}\}$ and the class of all $f\gamma - P_S -$ open sets $f\tau_\gamma - P_S O(X) = \{\phi. X, \{a\}, \{b, c\}\}$. Let $Y = \{1,2,3\}$, with topology $\tau' = \{\phi, Y, \{1\}, \{2\}, \{1,2\}\}$ and $\tau_f{}' = \{\phi, Y, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,3\}\}$ define an operation γ on $\tau_f{}'$ such that

$$\gamma(S) = \begin{cases} S & if\ \ S = \{1\} \\ S \cup \{3\} & if\ S \neq \{1\} \end{cases}$$

for every $S \in \tau_f'$. Set of all $f\gamma -$ open sets $f\tau_\gamma{}' = \{\phi, Y, \{1\}, \{2,3\}, \{1,3\}\}$ and the class of all $f\gamma - P_S$ −open sets $f\tau_\gamma' - P_S O(Y) = \{\phi. Y, \{1\}, \{2,3\}\}$. We define a mapping $f: X \to Y$ such that $f(a) = 1, f(b) = 2. f(c) =$

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3. By construction f is one-one and onto. It may be seen that the pre-image of $f\gamma - P_S -$ open sets of Y viz. $\{1\},\{2,3\}$ are $\{a\},\{b,c\}$ respectively, which are $f\gamma - P_S -$ open in X. Therefore f is $f\gamma - P_S -$ irresolute. Similarly, it may be checked that the inverse function $f^{-1}: Y \rightarrow X$ is also $f\gamma - P_S -$ irresolute. Thus f is $f\gamma - P_S -$irresolute homeomorphism.

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