# Vertex Coloring in Complement Fuzzy Graph 

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#### Abstract

Let $G=\left(V_{F}, E_{F}\right)$ be the basic undirected connected graph with fuzzy numbers and where $V_{F}$ be a set of all fuzzy nodes and each node has a membership value of $\mu$ and also $E_{F}$ be a set of all fuzzy arcs and each arc has a membership value of $\sigma$. A function of node coloring which has to assign colors to the nodes so as to that an adjacent nodes receives different colors. In this paper, We have to introduce the node coloring of complement of fuzzy graph and also coloring this complement fuzzy graph using $\alpha$ - cuts. Also we have to find that the chromatic number of a fuzzy graph.


Keywords Complement fuzzy graph, node coloring and chromatic number.

## I. INTRODUCTION

The concept of a graphs are simply models of relation. A graph is a convenient way of represents an information involving relationship between objects. In 1736 Euler and Leohard came out with the solution is terms of graph theory[2]. A coloring function of a graph is a mapping $C: V \rightarrow N$ in a graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ and it is denoted by $C(i)$ as the color of node $i \in V$ such that two adjacent nodes cannot be shared the same color i.e., $C(i) \neq C(j)$ if $i \neq j$. A graph H is called the complement of a graph G if two distinct nodes of H are adjacent if and only if they are not adjacent in G. In this paper, we discussed the concept of fuzzy graph by using the fuzzy relations with membership function $[0,1]$. And also, we introduce the concept of a complement, fuzzy coloring and chromatic number of fuzzy graph. Here, we proposed the node coloring of $\alpha$-complement of a fuzzy graph.

In 1975 Rosenfield[9] discussed the concept of fuzzy graph by using fuzzy relations whose basic idea was introduced by kauflmann in 1973[3]. Coloring of fuzzy play a vital role in solving complication in network. A large number of variation in coloring of fuzzy graphs are available in literature coloring of fuzzy graphs, were introduce by monoz et. al[7]. Anjali and sunitha introduced chromatic number of fuzzy graph and developed aldoeithms to the same[4]. Samanta and Pal introduced fuzzy coloring of fuzzy graph[10]. Moderson[6] introduced the concept of complement of fuzzy graph. Later Suita and Vijaykumar re-defined the complement of fuzzy graph[11]. In this paper we studied about the node coloring of $\alpha$ complement fuzzy graph for
given fuzzy graphs. We given to color the node independent for each $\alpha$ belongs to [0,1].
V.K. Balakrishnan Graph theory McGraw-Hill in 1997 premilinary concepts and definitions are studied in [12]. In the year 2001 L.S. Bershtein and A.V. Bozhenuk describes and some results on fuzzy coloring for fuzzy graph [5]. Arindam Dey and et-al[1]. Investigated and given some results on Edge coloring of a complement Fuzzy graph in the year 2012. In this year 2018, Parimaleswari. R and et-al[8]. Discussed about vertex coloring of complement Fuzzy graph based on $\alpha-$ cut .

This paper consist of four sections. Introductory concepts are introduced in the first section. Second section gives the preliminaries, which are very support to consecutive sections. Section three, we find the complement of a fuzzy graph and define a coloring function which is based on $\alpha$ cut to color the complement fuzzy graph and finding the chromatic number for this fuzzy graph. Conclusion are given in section four.

## II. PRELIMINARIES

## Definition 2.1

The node coloring of a graph is an assignment of labels or colors to each node of a graph such that no arc connects two identically colored nodes.

## Definition 2.2

A proper node coloring of a simple graph $G=(V, E)$ is defined as a node coloring from set of colors such that no two adjacent nodes share a same color.

## Definition 2.3

The $\alpha$ cut of fuzzy graph defined as $G_{\alpha}=\left(V_{\alpha}, E_{\alpha}\right)$
where

$$
V_{\alpha}=\{v \in V \mid \sigma \geq \alpha\}
$$

and
$E_{\alpha}=\{e \in E \mid \mu \geq \alpha\}$.

## Definition 2.4

The complement of a fuzzy graph $G:(\sigma, \mu)$ is $G^{C}:\left(\sigma^{C}, \mu^{C}\right)$, the advantage of this defined was that, for every fuzzy graph G. $\left(G^{C}\right)^{C}=G$, where $\sigma^{C}=\sigma$ and $\mu^{C}(x, y)=(\sigma(x) \wedge \sigma(y))-\mu(x, y)$.

## Definition 2.5

The minimum number if colors required to color the nodes of the given fuzzy graph is known as chromatic number . The chromatic number is denoted by, $\chi(G)$. $\chi(G)=\left\{\left(x_{\alpha}, \alpha\right)\right\}$ where $x_{\alpha}$ is the chromatic number of $G_{\alpha}$ and $\alpha$ values are the same different membership value of nodes and arcs of graph $G=\left(V_{F}, E_{F}\right)$.

## III. VERTEX COLORING IN <br> COMPLEMENT FUZZY GRAPH

We have calculated the complement of the fuzzy graph. Consider $\alpha$ value from different membership


## Fig-1 G (Fuzzy graph)

Let $G=\left(\mathrm{V}_{\mathrm{F}}, \mathrm{E}_{\mathrm{F}}\right)$ be a fuzzy graph where $\mathrm{V}_{\mathrm{F}}=$ $\left\{\left(v_{1}, 0.45\right), \quad\left(v_{2}, 0.55\right), \quad\left(v_{3}, 0.65\right), \quad\left(v_{4}, 0.75\right), \quad\left(v_{5}, 0.85\right)\right.$, $\left.\left(\mathrm{v}_{6}, 0.95\right)\right\}$ and $\mathrm{E}_{\mathrm{F}}=\left\{\left(\mathrm{e}_{1}, 0.1\right),\left(\mathrm{e}_{2}, 0.2\right),\left(\mathrm{e}_{3}, 0.3\right),\left(\mathrm{e}_{4}, 0.4\right)\right.$, $\left.\left(e_{5}, 0.5\right),\left(\mathrm{e}_{6}, 0.6\right)\right\}$.

## Stage 2:

To find the complement graph of the fuzzy graph $\left(\mathrm{G}^{\mathrm{C}}\right)$


Fig-2 G ${ }^{\text {c }}$ Complement of fuzzy graph

## Stage 3:

Graph $\mathrm{G}^{\mathrm{C}}$ we have $\alpha$ values $\{0.1,0.2,0.3,0.4,0.5$, 0.6,0.7, $0.80 .9,0.95\}$. By the Definition, we have colored the nodes of $G_{\alpha}^{C}$ for each $\alpha$ belongs to the above set and also find its chromatic number by the definition.

For $\alpha=0.1$, the fuzzy graph $\quad G_{\alpha}^{C}$ where $\sigma=\{0.45,0.55,0.65,0.75,0.85,0.95\}$ and the adjacent matrix is


Fig-3 $\chi_{(0.1)}=3$
For $\alpha=0.2$, the fuzzy graph $G_{0.2}^{C}$ where $\sigma=\{0.45,0.55,0.65,0.75,0.85,0.95\} \quad$ and the adjacent matrix is

$$
\mu_{4}=\begin{array}{rcccccc} 
& V_{1} & V_{2} & V_{3} & V_{4} & V_{5} & V_{6} \\
V_{1} & 0.0 & 0.0 & 0.3 & 0.2 & 0.0 & 0.0 \\
V_{2} & \begin{array}{l}
0.0 \\
V_{3}
\end{array} & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\
V_{4} & 0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\
V_{5} & 0.2 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\
V_{6} & 0.0 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0
\end{array}
$$

For $\alpha=0.2$, we find the graph $G_{0.2}^{C}$ (Fig-4). Then we

$$
\mu_{3}=\begin{array}{r}
V_{1} \\
V_{2} \\
V_{3} \\
V_{4} \\
V_{5} \\
V_{6}
\end{array}\left[\begin{array}{llllll}
0.0 & 0.0 & 0.3 & 0.2 & 0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\
0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\
0.2 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\
0.1 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0
\end{array}\right]
$$

For $\alpha=0.1$, we found the fuzzy graph $G_{0.1}^{C}$ (Fig-3). Then we use the proper coloring to color the nodes of the fuzzy graph $G_{\alpha}^{C}$ and the chromatic number of this graph 3.
proper color all the node of this graph and the chromatic number of this graph is 3 .


Fig-4 $\chi_{(0.2)}=3$

For $\alpha=0.3$, the fuzzy graph $G_{0.3}^{C}$ where $\sigma=\{0.45,0.55,0.65,0.75,0.85,0.95\}$ and the adjacent matrix is

$$
\mu_{5}=\begin{array}{ccccccc} 
& V_{1} & V_{2} & V_{3} & V_{4} & V_{5} & V_{6} \\
V_{1} & 0.0 & 0.0 & 0.3 & 0.0 & 0.0 & 0.0 \\
V_{2} \\
V_{3} \\
V_{4} \\
V_{5} \\
V_{6}
\end{array}\left[\begin{array}{lllllll}
0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\
0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\
0.0 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\
0.0 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0
\end{array}\right]
$$

For $\alpha=0.3$ we find the graph $G_{0.3}^{C}($ Fig-5). Then we proper color all the node of this graph and the chromatic number of this graph is 3 .


Fig-5 $\chi_{(0.3)}=3$

For $\quad \alpha=0.4$, the fuzzy graph $G_{0.4}^{C}$ where $\sigma=\{0.45,0.55,0.65,0.75,0.85,0.95\} \quad$ and the adjacent matrix is
$\mu_{7}=\begin{gathered} \\ V_{2} \\ V_{3} \\ V_{4} \\ V_{5} \\ V_{5}\end{gathered}\left[\begin{array}{ccccc}V_{2} & V_{3} & V_{4} & V_{5} & V_{6} \\ 0.0 & 0.0 & 0.6 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.5 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.9 & 0.0 & 0.0\end{array}\right]$
For $\alpha=0.5$, we find the graph $G_{0.5}^{C}$ (Fig-7). Then we proper color all the node of this graph and the chromatic number of this graph is 3 .
$\mathrm{V}_{6}(0.95)$


Fig-7 $\chi_{(0.5)}=3$

For $\alpha=0.6$, the fuzzy graph $G_{0.6}^{C}$, where $\sigma=\{0.65,0.75,0.85,0.95\}$

$\mu_{8}=$|  |
| :---: |
| $V_{2}$ |
| $V_{3}$ |
| $V_{4}$ |
| $V_{5}$ |
| $V_{5}$ |\(\left[\begin{array}{ccccc}V_{2} \& V_{3} \& V_{4} \& V_{5} \& V_{6} <br>

0.0 \& 0.0 \& 0.6 \& 0.0 \& 0.0 <br>
0.0 \& 0.0 \& 0.0 \& 0.7 \& 0.8 <br>
0.6 \& 0.0 \& 0.0 \& 0.0 \& 0.9 <br>
0.0 \& 0.7 \& 0.0 \& 0.0 \& 0.0 <br>
0.0 \& 0.8 \& 0.9 \& 0.0 \& 0.0 <br>
\end{array}\right.\)
[7]

For $\alpha=0.7$, the fuzzy graph $G_{0.7}^{C}$, where

$$
\sigma=\{0.75,0.85,0.95\}
$$

$$
\mu_{9}=\begin{gathered}
\\
V_{3} \\
V_{4} \\
V_{5} \\
V_{6}
\end{gathered}\left[\begin{array}{llll}
0.0 & V_{4} & V_{5} & V_{6} \\
0.0 & 0.0 & 0.7 & 0.8 \\
0.7 & 0.0 & 0.0 & 0.0 \\
0.8 & 0.9 & 0.0 & 0.0
\end{array}\right]
$$

For $\alpha=0.7$, we find the graph $G_{0.7}^{C}$ (Fig-9). Then we use the proper coloring to color the node of graph and chromatic number of this graph is 2 .

$$
\mathrm{V}_{6}(0.95) \quad \mathrm{V}_{2}(0.55)
$$

For $\alpha=0.6$, we find the graph $G_{0.6}^{C}$ (Fig-8). Then we use the proper coloring to color the node of graph $G$ and the chromatic number of this graph is 2 .
$\mathrm{V}_{6}(0.95)$


For $\alpha=0.8$, the fuzzy graph $G_{0.8}^{C}($ Fig-10). Where $\sigma=\{0.85,0.95\}$ and use the proper coloring the node and chromatic number of graph is 2 .

Fig-8 $\chi_{(0.6)}=2$


Fig-10 $\chi_{(0.8)}=2$

For $\alpha=0.9$, the fuzzy graph $G_{0.9}^{C}($ Fig-11). Where $\sigma=\{0.95\}$ and use the proper coloring to color the node and chromatic number is 2 .


Fig-11 $\chi_{(0.9)}=2$
$\alpha=0.95$, the fuzzy graph $G_{0.95}^{C}$ (Fig-12). Then use the in Engineeri Let
$\left.\mu_{12}=\begin{array}{c} \\ V_{4}\end{array} \begin{array}{cc}V_{4} & V_{6} \\ V_{6} & 0.0 \\ 0.0 \\ 0.0 & 0.0 \\ & \end{array}\right]$

○ $\mathrm{V}_{6}(0.95)$

Fig-12 $\chi_{(0.95)}=1$

Let $\xi=(V, \sigma, \mu)$ be a fuzzy graph. $\xi_{\alpha}=\left(V_{\alpha}, E_{\alpha}\right)$ such that $V_{\alpha}=\{u \in V \mid \sigma(u) \geq \alpha\}$ and $E_{\alpha}=\{(u, v), u, v, \in V \mid \mu(u, v) \geq \alpha\}$.

## Theorem:

Let $\xi$ be a fuzzy graph. If $0 \leq \alpha \leq \beta \leq 1$, then $\xi^{\beta} \subseteq \xi^{\alpha}$.

## Proof:

Let $\xi=(V, \sigma, \mu)$ be a fuzzy graph and $0 \leq \alpha \leq \beta \leq 1 . \quad$ Now $\quad \xi^{\alpha}=\left(V^{\alpha}, E^{\alpha}\right) \quad$ where $V^{\alpha}=\left\{u \in V \mid I_{v} \geq \alpha\right\}$ and
$E^{\alpha}=\left\{(u, v), u, v, \in V \mid I_{(u, v)} \geq \alpha\right\} . \quad$ Also, $\xi^{\beta}=\left(V^{\beta}, E^{\beta}\right)$ where $V^{\beta}=\left\{u \in V \mid I_{v} \geq \beta\right\}$ and $E^{\beta}=\left\{(u, v), u, v, \in V \mid I_{(u, v)} \geq \beta\right\}$. Let x be any element of $V^{\beta}$. Then, $I_{x} \geq \beta \geq \alpha$.

Therefore, $x \in V^{\alpha}$. Similarly, for any element $(x, y) \in E^{\beta}$.

Therefore, $\xi^{\beta} \subseteq \xi^{\alpha}$.
Hence proved.

## Theorem:

Let $\xi$ be a fuzzy graph. If $0 \leq \alpha \leq 1$, then $\xi^{\beta} \subseteq \xi^{\alpha}$.

## Proof:

Again $\xi^{\alpha}=\left(V^{\alpha}, E^{\alpha}\right)$ such that $V^{\alpha}=\left\{u \in V \mid I_{v} \geq \alpha\right\}$ and
$E^{\alpha}=\left\{(u, v), u, v, \in V \mid I_{(u, v)} \geq \alpha\right\}$.
Let $\quad x, y \in V_{\alpha} \quad$ and $\quad(x, y) \in E_{\alpha}$. Therefore $\sigma(x) \geq \alpha, \sigma(y) \geq \alpha \quad \& \quad \mu(x, y) \geq \alpha$. This results along with $\alpha \leq 1$ implies that $\frac{\mu(x, y)}{\sigma(x) \wedge \sigma(y)} \geq \alpha$.
Hence $I_{(x, y)} \geq \alpha$. So , $(x, y) \in E^{\alpha}$. Thus for every edges of $\xi_{\alpha}$, their exist an edge in $\xi^{\alpha}$. Now, clearly
from the definitions of strength of nodes, $V_{\alpha} \subseteq V^{\alpha}$. Hence the results $\xi_{\alpha} \subseteq \xi^{\alpha}$ is true.

Hence proved.

## IV.CONCLUSION

In this paper, we computed chromatic number for the node coloring of a complement fuzzy graph using $\alpha$-cut. We took $\alpha$ value from the different membership values of nodes and arcs in G. we found the chromatic number (node coloring) for the different $\alpha$ value of complement fuzzy graph G as follows

$$
\begin{aligned}
\chi(G)= & (3,0.1),(3,0.2),(3,0.3), \\
& (3,0.4),(3,0.5),(2,0.6), \\
& (2,0.7),(2,0.8),(2,0.9),(1,0.95)
\end{aligned}
$$

Thus this paper mainly focused, that the chromatic number for the different $\alpha$ values of complement fuzzy graph $G$. If the $\alpha$ value approach to 1 then the chromatic number is 1 . This is the main conclusion of the paper.

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