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Applications of Game Problems in Fuzzy Enviornment

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Abstract **In this paper a new approach is proposed to deal with fuzzy game problems with imprecise payoffs. The study deals with computational procedure to rank octagonal fuzzy numbers by using incentre of centroids. In this paper, fuzzy game problem in which payoffs are octagonal fuzzy numbers are converted into crisp problem and then solve it by using any traditional game theory method. A numerical example is given to illustrate the proposed ranking method.**

Keywords: Octagonal fuzzy numbers, Centroid of centroids, Incentre of centroids, Ranking function, Fuzzy game theory.

I. INTRODUCTION

Game theory is the study of mathematical tools that deals with decision making in conflict and competitive situations between two or more players. Game theory is originated by Mathematician John von Neumann and Economist Oskar Morgenstren (1947). In this game theory each competitor choose his strategy from a set of available strategies. John Nash (1949) proved that there is an equilibrium point in a game theory in which the players select their best actions, when the opponent's choices are given.

However in the real life exact Information to solve competitive situations is not available. This lack of information may be modeled by using fuzzy set theory.

Fuzzy set theory was firstly proposed by Zadeh (1965). This concept of decision making is elaborated by Bellman and Zadeh (1970) in the fuzzy environment. Jain (1976) described the method of ordering of fuzzy numbers and gives optimal alternatives. Yager (1980) used the concept of connectives, union and intersection of fuzzy numbers. Chu and Tsao (2002) presented a method to rank fuzzy number by finding the area between the centroid and original points. Cevikel and Ahlatcioglu (2010) considered two person zero sum game which is based on solution of games with payoffs and goals are fuzzy. Thorani et al. (2012) used orthocenter of centroids for ordering of generalized trapezoidal fuzzy numbers. A new method to rank fuzzy numbers by using incentre of centroids is proposed by Rajarajeswari and Sudha (2014). Kumar et al. proposed a new approach for the ranking of generalized trapezoidal fuzzy numbers. Dhanalaxmi and Kennedy (2014) described some ranking methods for octagonal fuzzy numbers. Jatinder and Neha (2016) presented an ordering of dodecagonal fuzzy numbers with incentre of centroids. Namarta and Neha (2016) proposed a method to rank hendecagonal fuzzy numbers by using centroid of centroids. Kumar et al.3 (2013) proposed an interactive method that integrates the concept of fuzzy ranking and minimax principle to get an imprecise game value. Selvakumari and Lavanya (2014) used octagonal fuzzy numbers to solve fuzzy game problem. Selvakumari and Lavanya (2015) considered a solution of game theory based on ranking of triangular and trapezoidal fuzzy numbers. Thirucheran et al. (2017) considered a two person zero sum game and used ranking criteria to solve it without converting into crisp problem.

The present paper describes the method of ranking octagonal fuzzy numbers using centroid of centroids and incentre of centroids. In octagonal fuzzy number, firstly the octagon is split into two trapezoidal and one hexagon and then computes the centroid of these plane figures. Secondly, it computes the centroid of these centroids and the centroid is followed by calculation of incentre. In this paper, the method of ranking fuzzy numbers with an area between the incentre and the original point is also proposed.

The paper is organized into four sections. Section 2 presents octagonal fuzzy numbers and ranking method in which procedure to find incentre of centroids is described. In Section 3, applications of ranking of octagonal fuzzy numbers to fuzzy game problems are described. Finally this paper concludes in Section 4.

II. PROPOSSED RANKING METHOD

Octagonal Fuzzy Numbers: A generalised fuzzy number $\widetilde{A_o} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\,; w)$ is said to be octagonal fuzzy number if its membership function $\mu_{\widetilde{A_o}}(x)$ is given below:

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$$\mu_{\widetilde{A_0}}(x) = \begin{Bmatrix} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \left(\frac{w}{2}\right) & a_2 \le x \le a_3 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ w & a_4 \le x \le a_5 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{a_5-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ \left(\frac{w}{2}\right) & a_6 \le x \le a_7 \\ \frac{w}{2}\left(\frac{a_7-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 0 & x \ge a_8 \end{Bmatrix}$$

To find the balancing point of the octagon, firstly, divide the octagon into two trapezoidal AIJD, EMNH and one hexagon DJKLME (Fig 1.) and then find the centroid of these plane figures. Let the centroid of these plane figures be G_1, G_2 and G_3 respectively. The Centroid of centroids, that is, point G, is taken as the point of reference to define the ranking of generalized octagonal fuzzy numbers. Further the incentre of centroids G_1, G_2 and G_3 is also calculated. Then ranking of octagonal fuzzy numbers is defined by using incentre of centroids. Consider the generalized octagonal fuzzy number $\widehat{A}_o = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\ ;\ w)$

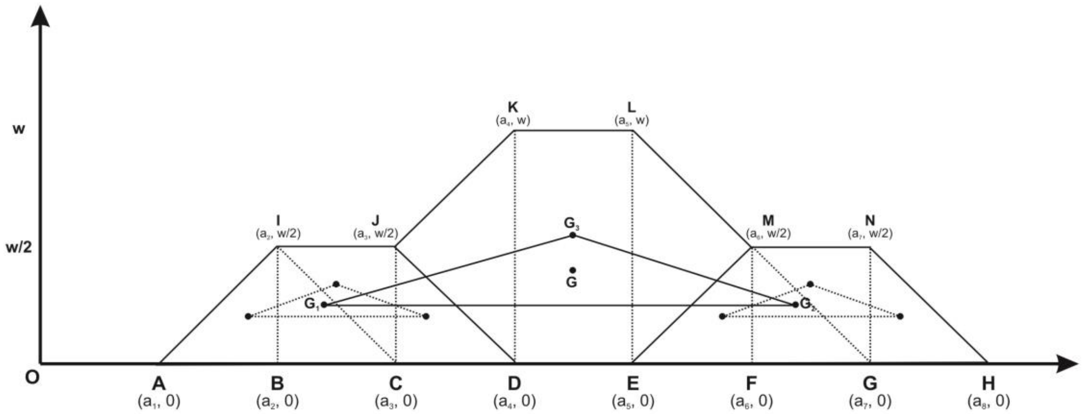

Fig.1

The centroid of these figures are

$$G_1 = \left(\frac{2a_1+7a_2+7a_3+2a_4}{18}, \frac{7w}{36}\right);$$

$$G_2 = \left(\frac{2a_5+7a_6+7a_7+2a_8}{18}, \frac{7w}{36}\right);$$

$$G_3 = \left(\frac{2a_4+a_3+2a_5+a_6}{6}, \frac{w}{2}\right)$$

As G_1, G_2 and G_3 are non collinear and they form a triangle. Therefore Centroid $G_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0})$ of a triangle is

$$G_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a_1+7a_2+10a_3+8a_4+8a_5+10a_6+7a_7+2a_8}{54}, \frac{11w}{54}\right)$$

As a special case, for heptagon fuzzy number $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_6, a_7, a_8; w)$ i.e., $a_4 = a_5$, the centroid of centroids of a heptagon is given by

$$G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a_1+7a_2+10a_3+16a_4+10a_6+7a_7+2a_8}{54}, \frac{11w}{54}\right)$$

Now the Incentre $I_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0})$ of a triangle whose vertices are G_1, G_2 and G_3 is given by

$$I_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0}) = \left(a_{\widetilde{A_o}}\left(\frac{2a_1+7a_2+7a_3+2a_4}{18}\right) + b_{\widetilde{A_o}}\left(\frac{2a_5+7a_6+7a_7+2a_8}{18}\right) + c_{\widetilde{A_o}}\left(\frac{2a_4+a_3+2a_5+a_6}{6}\right),\ a_{\widetilde{A_o}}\left(\frac{7w}{36}\right) + b_{\widetilde{A_o}}\left(\frac{7w}{36}\right) + c_{\widetilde{A_o}}\left(\frac{w}{2}\right)\right)$$

Where

$$a_{\widetilde{A_o}} = \frac{\sqrt{(6a_3+12a_4+8a_5-8a_6+14a_7-4a_8)^2+(11w)^2}}{36}$$

$$b_{\widetilde{A_o}} = \frac{\sqrt{(-4a_1-14a_2-8a_3+8a_4+12a_5+6a_6)^2+(11w)^2}}{36}$$

$$c_{\widetilde{A_o}} = \frac{2a_1+7a_2+7a_3+2a_4-2a_5-7a_6-7a_7-2a_8}{18}$$

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The Ranking function of the generalized octagonal fuzzy number is defined as:

$$R\,(\widetilde{A_H}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$$

III. APPLICATION OF RANKING OF FUZZY NUMBERS TO GAME THEORY

1. Consider A and B are two players with payoffs octagonal fuzzy numbers:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} (8,9,10,11,12,13,14,15) & (2,3,4,5,6,7,8,9) \\ (0,1,2,3,4,5,6,7) & (-1,0,1,2,3,4,5,6) \end{bmatrix} \end{array}$$

By using the ranking method defined in section 2, the fuzzy game problem is converted into crisp problem, then solved by maximin-minimax principle and find the value of the game.

$\tilde{a}_{11} = (8,9,10,11,12,13,14,15)$	$M_0^{oct}(\tilde{a}_{11}) = 11.5$
$\tilde{a}_{12} = (2,3,4,5,6,7,8,9)$	$M_0^{oct}(\tilde{a}_{12}) = 5.5$
$\tilde{a}_{21} = (0,1,2,3,4,5,6,7)$	$M_0^{cot}(\tilde{a}_{21}) = 3.5$
$\tilde{a}_{22} = (-1,0,1,2,3,4,5,6)$	$M_0^{oct}(\tilde{a}_{22}) = 2.5$

The reduced payoff matrix is

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 11.5 & 5.5 \\ 3.5 & 2.5 \end{bmatrix} \end{array}$$

Minimum of 1st row = 5.5
Minimum of 2nd row = 2.5
Maximum of 1st column = 11.5
Maximum of 2nd column = 5.5
Max (min) = 5.5 and Min (max) =5.5
Here Max (min) = Min (max)

It has saddle point. Value of Game = 5.5

2. Consider A and B are two players with payoffs octagonal fuzzy numbers:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} (0,1,2,3,4,5,6,7) & (2,4,5,6,7,8,9,11) & (-1,0,1,2,3,4,5,6) \\ (-3,-1,0,1,2,4,6,7) & (-4,-3,-2,-1,0,1,2,3) & (-3,-2,-1,0,1,2,3,4) \\ (8,9,10,11,12,13,14,15) & (2,3,4,5,6,7,8,9) & (4,5,6,7,8,9,10,11) \end{bmatrix} \end{array}$$

In section 2, the defined ranking method is used convert fuzzy game problem into crisp problem, then solve it by using arithmetic method.

$\tilde{a}_{11} = (0,1,2,3,4,5,6,7)$	$M_0^{oct}(\tilde{a}_{11}) = 3.5$
$\tilde{a}_{12} = (2,4,5,6,7,8,9,11)$	$M_0^{oct}(\tilde{a}_{12}) = 6.5$
$\tilde{a}_{13} = (-1,0,1,2,3,4,5,6)$	$M_0^{oct}(\tilde{a}_{13}) = 2.5$
$\tilde{a}_{21} = (-3,-1,0,1,2,4,6,7)$	$M_0^{oct}(\tilde{a}_{21}) = 2.2$
$\tilde{a}_{22} = (-4,-3,-2,-1,0,1,2,3)$	$M_0^{oct}(\tilde{a}_{22}) = 0.5$
$\tilde{a}_{23} = (-3,-2,-1,0,1,2,3,4)$	$M_0^{oct}(\tilde{a}_{23}) = 0.5$
$\tilde{a}_{31} = (8,9,10,11,12,13,14,15)$	$M_0^{oct}(\tilde{a}_{31}) = 11.5$
$\tilde{a}_{32} = (2,3,4,5,6,7,8,9)$	$M_0^{oct}(\tilde{a}_{32}) = 5.5$
$\tilde{a}_{32} = (4,5,6,7,8,9,10,11)$	$M_0^{oct}(\tilde{a}_{33}) = 7.5$

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The payoffs of fuzzy game problem is converted into crisp payoff matrix as:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3.5 & 6.5 & 2.5 \\ 2.2 & 0.5 & 0.5 \\ 11.5 & 5.5 & 7.5 \end{bmatrix} \end{array}$$

Minimum of 1st row = 2.5
Minimum of 2nd row = 0.5
Minimum of 3rd row = 5.5
Maximum of 1st column = 11.5
Maximum of 2nd column = 6.5
Maximum of 3rd column = 7.5
Max (min) = 5.5 and Min (max) =6.5
Here Max (min) $\neq$ Min (max)

It has no saddle point. In order to solve the crisp game problem, we use principle of dominance. As all the elements of first column are greater than third column that means first column is dominated by third column. Hence first column is eliminated,

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6.5 & 2.5 \\ 0.5 & 0.5 \\ 5.5 & 7.5 \end{bmatrix} \end{array}$$

Again, all the elements of second row are less than third row, it means third row dominates second row, hence second row is eliminated, we get

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6.5 & 2.5 \\ 5.5 & 7.5 \end{bmatrix} \end{array}$$

Now 2×2 payoff matrix is obtained. In the reduced matrix, equilibrium point is not obtained, so oddment method is applied to get the value of the game.

			Row oddments
	6.5	2.5	2
	5.5	7.5	4
Column oddments	5	1	6

Since the sum of column oddments and row oddments is 6. The strategies for both maximization and minimization players are $\left(\frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{5}{6}, \frac{1}{6}\right)$ respectively and the value of the game is 5.83.

IV. CONCLUSION

The paper describes ranking method for octagonal fuzzy numbers based on area. The process of ranking involves computation of centroids of two trapezoidal figures and one heptagon formed by octagon numbers, then obtaining centroid of these centroids. Finally, the centroid of centroids and incenter of centroids is used to rank octagonal fuzzy numbers for solving fuzzy game problem and illustrated by an example.

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