# Transformation of Fuzzy Digraphs Viewed as A Fuzzy Intersection Digraphs 

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#### Abstract

In this paper we introduce Fuzzy Line Digraphs,Fuzzy total digraph,Fuzzy middle digraph.And also the conditions for the fuzzy digraphs being a fuzzy line digraphs are discussed.


Keywords - Fuzzy digraph, Fuzzy line Digraph , Fuzzy total digrah, fuzzy middle digraph, Fuzzy intersection graph, Fuzzy out neighbourhood of a set, Fuzzy In neighborhood of a set.

## I. INTRODUCTION

The concept of fuzzy graph was introduced by Rosenfeld [1] in 1975. Fuzzy graph theory has a vast area of applications. It is used in evaluation of human cardiac function, fuzzy neural networks, etc. Fuzzy graphs can be used to solve traffic light problem, time table scheduling, etc. In fuzzy set theory, there are different types of fuzzy graphs which may be a graph with crisp vertex set and fuzzy edge set or fuzzy vertex set and crisp edge set or fuzzy vertex set and fuzzy edge set or crisp vertices and edges with fuzzy connectivity, etc. A lot of works have been done on fuzzy graphs [3], [4], [5]..

The Fuzzy Directed Graph, Fuzzy competition digraphs are well known topic.The fuzzy round digraph, Quasi transitive digraph was defined and discussed.[6],[7]. The Fuzzy in neighbourhood of a set, fuzzy out neighbour hood of a set ,diameter and radius of a fuzzy digraph,,diameter and radius of a Fuzzy Round digraphs are defined and discussed [8].In this article Fuzzy Line Digraphs,Fuzzy total graphs, Fuzzy middle graphs, and Fuzzy intersection graphs are defined, and also we prove the thorem for the fuzzy graph being a fuzzy line digraph.

## II. PRELIMINARIES

## Definition 2.1:

Fuzzy digraph $\stackrel{\rightharpoonup}{\xi}=(\mathrm{V}, \sigma, \vec{\mu})$ is a non-empty set V together with a pair of functions $\sigma: V \rightarrow[0,1]$ and $\vec{\mu}: \mathrm{V} \times \mathrm{V} \rightarrow[0,1]$ such that for all $\mathrm{x}, \mathrm{y} \in \mathrm{V}, \vec{\mu}(\mathrm{x}, \mathrm{y}) \leq \sigma(\mathrm{x}) \wedge \sigma(\mathrm{y})$. Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}(u, v)$ is denoted by the membership value of the edge $\overrightarrow{(u, v)}$. The loop at a vertex x is represented by $\vec{\mu}_{(\mathrm{x}, \mathrm{x})} \neq 0$. Here $\vec{\mu}_{\text {need not be symmetric as }} \vec{\mu}_{(\mathrm{x}, \mathrm{y})}$ and $\vec{\mu}(\mathrm{y}, \mathrm{x})$ may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges

## Definition 2.2:

Fuzzy out-neighbourhood of a vertex v of a directed fuzzy graph $\vec{\xi}=(\mathrm{V}, \sigma, \vec{\mu})$ is the fuzzy set $\quad N^{+}(v)=\left(X_{v}{ }^{+}, m_{v}{ }^{+}\right.$ ) where $X_{v}{ }^{+}=\left\{\mathrm{u} \mid \vec{\mu}_{(\mathrm{v}, \mathrm{u})>0\}}\right.$ and $m_{v}{ }^{+}: X_{v}{ }^{+} \rightarrow[0,1]$ defined by $\quad m_{v}{ }^{+}(\mathrm{u})=\vec{\mu}(\mathrm{v}, \mathrm{u})$. Similarly, fuzzy in-neighbourhood of a vertex v of a directed fuzzy graph $\vec{\xi}=\left(\mathrm{V}, \sigma, \vec{\mu}^{\prime}\right)$ is the fuzzy set $N^{-}(v)=\left(X_{v}{ }^{-}, m_{v}{ }^{-}\right)$where $X_{v}{ }^{-}=\{\mathrm{u} \mid \vec{\mu}(\mathrm{u}, \mathrm{v})>0\}$ and $m_{v}{ }^{-}: X_{v}{ }^{-} \rightarrow[0,1]$ defined by $m_{v}{ }^{-}(\mathrm{u})=\vec{\mu}(\mathrm{u}, \mathrm{v})$.
Definition 2.3:
A Fuzzy Digraph on ' n ' vertices is round if its vertices are labelled as $v_{1}, v_{2}, \ldots \ldots . . v_{n \text { so }}$ that or each ' I'
$N^{+}\left(v_{i}\right)=\left\{\left(X_{v_{i+1}}{ }^{+}, m_{v_{i+1}}{ }^{+}\right) \ldots \ldots . . .\left(X_{\left.v_{i+d^{+}\left(v_{i}\right)}\right)}{ }^{+} m_{v_{i+d^{+}\left(v_{i}\right)}}+\right)\right\}_{\text {and }}$
$N^{-}\left(v_{i}\right)=\left\{\left(X_{v_{i-d^{-}\left(v_{i}\right)}}{ }^{-}, m_{v_{i-d^{-}\left(v_{i}\right)}}{ }^{-}\right) \ldots \ldots\left(X_{v_{i-1}}{ }^{-}, m_{v_{i-1}}{ }^{-}\right)\right\}$
Definition 2.4:

The fuzzy line digraph $\vec{\eta}=\mathrm{L}(\vec{\xi})$ of a fuzzy graph $\vec{\xi}$ is the fuzzy directed graph whose vertex set $\mathrm{V}(\vec{\eta})$ corresponds to the arc set of the $\vec{\xi}$ and having an arc directed from an edge e1 to an edge e2, if in $\vec{\xi}$ the head of e1 meets the tail of e2.

$$
\text { i.e, } \mathrm{V}(\vec{\eta})=\mathrm{A}(\vec{\xi})
$$

$\mathrm{A}(\vec{\eta})=\{\mathrm{e} 1 \mathrm{e} 2:, \mathrm{e} 1, \mathrm{e} 2 \in \mathrm{v}(\mathrm{Q})$,the head of e1 coincide with the tail of e 2$\}$

## Example 2.1:

$\left(\mathrm{V}_{2}\right)$


## Definition.2.5:

TheFuzzy Total digraph, $\vec{\tau}=T(\vec{\xi})$ of the fuzzy digraph $\vec{\xi}$ has a vertex set VUA, and two such elements are connected by an arc in $\vec{\tau}$ iff the corespoinding elements in $\vec{\xi}$ are adjacent in $\vec{\xi}$.


Note: Then the resulting Fuzzy total graphs becomes vertex valued fuzzy graph.

## Definition.2.6:

The Fuzzy Middle digraph denoted $\overrightarrow{\mathcal{M}}(\vec{\xi})$ of the digraph $\vec{\xi}$, has a vertex set VU $\vec{A}$ and two such vertices in $\overrightarrow{\mathcal{M}}(\vec{\xi})$ are connected by arc in $\overrightarrow{\mathcal{M}}(\vec{\xi})$ iff they are not both vertices in $\vec{\xi}$ and the corresponding elements in $\vec{\xi}$ are adjacent in $\vec{\xi}$.

$\mathrm{V}_{1}$ V3

## Definition.2.7:

If $\stackrel{\rightharpoonup}{\xi}=(\mathrm{V}, \sigma, \vec{\mu})$ is a fuzzy digraph and $\overrightarrow{\mathrm{I}}(\mathrm{S}, \lambda, v)$ is the fuzzy intersection digraph defined as

$$
\lambda\left(\mathrm{S}_{\mathrm{i}}\right)=\sigma\left(\mathrm{v}_{\mathrm{i}}\right) \forall S_{i} \in S
$$

$$
\lambda(\mathrm{Si}, \mathrm{Sj})=\mu\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{j}}\right)
$$

where $\mathrm{T}=\left\{\left(S_{i} S_{j} / S_{i} S_{j} \in S, \quad S_{i} \cap S_{j} \neq i \neq j\right\}\right.$
Fuzzy line graph is the fuzzy intersection graph defined on the arc set A.

## III. FUZY LINE DIGRAPHS

Theorem.3.1:
$\forall S_{i} S_{j} \epsilon T$

Let $\vec{\xi}$ be a directed fuzzy pseudograph with vertex set $\{1,2, \ldots \mathrm{n}\}$ and with no parallel arcs and let $\mathrm{M}=\left[\mathrm{m}_{\mathrm{ij}}\right]$ be its adjacency matrix then the following assertions are equivalent.
i) $\quad \vec{\xi}$ is a fuzzy line digraph
ii) If $\overrightarrow{v u}, \overrightarrow{u w}$ and $\overrightarrow{u x}$ are arcs of $\vec{\xi}$ then so is $\overrightarrow{v x}$.

## Proof:

Let $\vec{\xi}=\mathrm{L}(\vec{\eta})$. For each $v_{i} \in \mathrm{~V}(\vec{\eta})$, Let $A_{i}$ and $B_{i}$ be the set of in- coming and outgoing arcs at $v_{i}$, respectively .then the arc set of the subdigraph induced by $\mathrm{A}_{\mathrm{i}} \mathrm{UB}_{\mathrm{i}}$ equals $A_{i} X B_{i}$. Then there exist two partitions $\left\{\mathrm{A}_{\mathrm{i}}\right\}_{\mathrm{i} \in \mathrm{I}}$ $\left\{\mathrm{B}_{\mathrm{i}}\right\}_{\mathrm{i} \in \mathrm{I}}$ of $\mathrm{V}(\vec{\xi})$ such that $\mathrm{A}(\vec{\xi})=\bigcup_{i \in I} A_{i} X B_{i}$. If $\overrightarrow{v u}, \overrightarrow{u w}$ and $\overrightarrow{u x}$ are arcs of $\vec{\xi}$ then there exist I,j such that $\{\mathrm{u}, \mathrm{v}\} \subseteq \mathrm{A}_{\mathrm{i}}$, and $\{\mathrm{w}, \mathrm{x}\} \subseteq \mathrm{B}_{\mathrm{j}}$. Hence $(\mathrm{v}, \mathrm{x}) \in A_{i} X B_{j}$ and $\overrightarrow{v x} \in$ $\vec{\xi}$.

## Theorem.3.2:

Let $\vec{\xi}$ be a directed fuzzy pseudograph with vertex set $\{1,2, \ldots \mathrm{n}\}$ and with no parallel arcs and let $\mathrm{M}=\left[\mathrm{m}_{\mathrm{ij}}\right]$ be its adjacency matrix then the following assertions are equivalent.
i) $\quad \vec{\xi}$ is a fuzzy line digraph
ii) Any two rows of M are either identical or orthogonal.

## Proof:

$\vec{\xi}$ is a fuzzy line digraph then $\overrightarrow{v u}, \vec{u} \vec{w}$ and $\overrightarrow{u x}$ are $\operatorname{arcs}$ of $\vec{\xi}$ then so is $\overrightarrow{v x}$. If it does not hold it means that some rows say $i$ and $j$ are neither identical nor orthogonal.Then there exist $\mathrm{k}, \mathrm{h}$ such that $\mathrm{m}_{\mathrm{ik}}=\mathrm{m}_{\mathrm{jk}}=1$ and $\mathrm{m}_{\mathrm{ih}}=1, \mathrm{~m}_{\mathrm{jh}}=0$. Hence $\mathrm{ik}, \mathrm{jk}$,ih are in $\mathrm{A}(\vec{\xi})$ but jh is not. This contradicts the above theorem. Hence any two rows of M arre either identical or orthogonal.

## Theorem.3.3:

Let $\vec{\xi}$ be a directed fuzzy pseudograph with vertex set $\{1,2, \ldots \mathrm{n}\}$ and with no parallel arcs and let $\mathrm{M}=\left[\mathrm{m}_{\mathrm{ij}}\right]$ be its adjacency matrix then the following assertions are equivalent.
i) $\quad \vec{\xi}$ is a fuzzy line digraph
ii) Any two colums of M are either identical or orthogonal.
Proof: $\quad \vec{\xi}$ is a fuzzy line digraph then $\overrightarrow{v u}, \overrightarrow{u w}$ and $\overrightarrow{u x}$ are arcs of then so is $\overrightarrow{v x}$. If it does not hold it means that some columns say $i$ and $j$ are neither identical nor orthogonal. Then there exist $\mathrm{k}, \mathrm{h}$ such that $\mathrm{m}_{\mathrm{ik}}=\mathrm{m}_{\mathrm{jk}}=1$ and $\mathrm{m}_{\mathrm{ih}}=1, \mathrm{~m}_{\mathrm{jh}}=0$. Hence $\mathrm{ik}, \mathrm{jk}$,ih are in $\mathrm{A}(\vec{\xi})$ but jh is not. This contradicts the above theorem. Hence any two columns of M arre either identical or orthogonal.

## IV. CONCLUSION

Finally we defined the fuzzy line digraphs,Fuzzy middle digraph, Fuzzy Total Digraph and discussed about the conditions for the fuzzy graph being a fuzzy line digraphs.It will use full in the system where the fuzzy line graphs are used.

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