

# **Increasing Minimal g-open Sets of Type1**

G.Venkateswarlu, Research scholar, Dravidian University, Kuppam, Chittore (District),

A.P., India, gvrjkcmaths @gmail.com

V.Amarendra Babu, Department of Mathematics, Acharya Nagarjuna University, Nagarjuna

Nagar, Guntur(District), Andhra Pradesh, India, amarendravelisela@ymail.com

Abstract The definition of minimal open sets of Type1, minimal g-open sets of Type1 was introduced by [12]. In this article we instigate increasing-minimal open sets Type1, increasing-g-open sets of Type1, decreasing-minimal open sets of Type1, balanced-minimal g-open sets of Type1 and also we have proved that the class of i $\alpha$ -open sets (resp. d $\alpha$ -open sets, b $\alpha$ -open sets,) properly contains in the class of i-minimal g-open sets of Type1 and also we investigate some there fundamental properties with several counter examples.:

 $Keywords - i - m_i O(Z, T1), i - m_i gO(Z, T1), d - m_i O(Z, T1), d - m_i gO(Z, T1), b - m_i O(Z, T1), b - m_i gO(Z, T1)$ 

# **I. INTRODUCTION**

L. Nachbin[6] Topology and order, D.Van Nostrand Inc., Princeton, New Jersy studied increasing [resp. decreasing, balanced] open sets in 1965. K. Bhagya Lakshmi, V. Amarendra Babu, M.K.R.S. Veera kumar [11] studied gaiclosed sets, gαd-closed sets, and gαb-closed sets in topological ordered spaces in 2015.

G.Venkareswarlu, V.Amarendra Babu, and M.K.R.S Veera kumar [12] introduced and studied minimal open sets of Type1 sets, minimal g-open sets of Type1 sets in 2016.

In this paper we introduce increasing-minimal open sets of Type1, Increasing-minimal g-open sets of Type1 (resp. d-minimal g-open sets of Type1) bets and also We have proved that the class of ia-open sets (resp. da-open sets, ba-open sets,) properly contains in the class of i-minimal g-open sets of Type1 and also we investigate some there fundamental properties with several counter examples.

# **II. PRELIMINARIES**

**DEFINITION 2.1**: A subset A of a topological space (X, T) is called

- 1. a generalized closed set (brifly g-closed [3]) if cl(A)
  ⊆ U whenever A⊆ U and U is open in (X, T). The complement of a g-closed is called g-open set.
- 2. a Ψ -close set [13] if scl(A) whenever A⊆ U and U is sg-open in (X, α). The complement of a Ψ-closed is called Ψ-open set.
- 3.  $\alpha$ -open set [7] if A $\subseteq$ int(cl(int(A))) and  $\alpha$ -closed if cl(int(cl(A)))  $\subseteq$  A.
- 4. B-open set [14] if  $A \subseteq cl(int(cl(A)))$  and  $\beta$ -closed if  $int(cl(intA))) \subseteq A$ .

**DEFINITION 2.2:** [12] In a topological space (X, T), an open sub set U of X is called a minimal open sets

of Type! If  $\exists$  at least one non-empty closed set F such that  $F \subseteq U$  or  $U = \Phi$ .

**DEFINITION 2.2:** [12] In a topological space (X, T), an open sub set U of X is called a minimal g- open sets

of Type! If  $\exists$  at least one non-empty g- closed set F such that  $F \subseteq U$  or  $U = \Phi$ .

**DEFINITION 2.3:** [11] A sub set A of a topological space  $(X, T, \leq)$  is called an ia (brifly da- and ba-open

set) if A is both  $\alpha$ -open and increasing (resp. decreasing and balanced) set.

# III. 3. MAIN RESULT

Now we state and prove our first main result.

Before that we first introduce an increasing, decreasing and ballenced open sets in minimal g-open set of Type 1 sets and some notations.

**DEFINITION 3.1**: A subset A of a topological ordered space  $(Z, T, \leq)$  is called increasing minimal open sets

of Type1 ( brifly i-m\_iO(Z, T1) ) if A is both increasing and minimal open sets of Type1.

**DEFINITION 3.2** A subset A of a topological ordered space  $(Z, T, \leq)$  is called increasing minimal g- open sets of **Type1** ( brifly i-m<sub>i</sub>gO(Z, T1) ) if A is both increasing and minimal g- open sets of Type1.

**DEFINITION 3.3**: A subset A of a topological ordered space  $(Z, T, \leq)$  is called decreasing minimal g- open sets of Type1 (brifly d-m<sub>i</sub>gO(Z, T1)) if A is both decreasing and minimal g- open sets of Type1.

**DEFINITION 3.4**: A subset A of a topological ordered space (Z, T,  $\leq$ ) is called balanced minimal g- open



sets of Type1 ( brifly  $b-m_iO(Z, T1)$  ) if A is both balanced and minimal g- open set of Type1.

#### Note.3.5:

The collections of all increasing minimal open sets of Type1, increasing minimal g-open sets

Type1, (resp. decreasing and balanced) set in topological ordered space is denoted by  $i-m_iO(Z, T1)$ , i-

 $\label{eq:migO} \begin{array}{l} m_igO(Z,\,T1) \ (resp. \ d-m_iO(Z,\,T1), \ d-m_igO(Z,\,T1), \ b-m_iO(Z,\,T1), \ b-m_iO(Z,\,T1), \ b-m_igO(Z,\,T1). \end{array}$ 

**THEOREM3.6**: In a topological ordered space (Z, T,  $\leq$ ), every i-m<sub>i</sub>O(Z, T1) is an i-open set.

Proof. As for the fact, every i-  $m_iO(Z\ ,\ T1)$  is  $m_iO(Z,\ T1)$  and that every  $m_iO(Z,\ T1)$  is i-open set.

**REMARK 3.7**: The converse of theorem 3.6 need not be true.

**EXAMPLE 3.8**: Let Z={  $i_1$ ,  $i_2$ ,  $i_3$  } and T= {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }  $\leq_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1)$ }

 $m_iO(Z,\,T1)\;sets = \{\;\Phi,\!Z,\;\{i_1,\;\,i_2\},\!\{i_1,\;\,i_3\}\;\}$ 

 $i-m_iO(Z, T1)$  sets = {  $\Phi,Z$ , { $i_1, i_2$ }, { $i_1, i_3$ } }

open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

i- open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }. Then A = { $i_1$ } is i-open set but not i-m<sub>i</sub>O(Z, T1).

**THEOREM 3.9**: In a topological ordered space  $(Z, T, \leq)$ , every d-m<sub>i</sub>O(Z, T1) is an d-open set.

**Proof**: As for the fact, every  $d-m_iO(Z, T1)$  is  $m_iO(Z, T1)$  and that every  $m_iO(Z, T1)$  is d-open set.

**REMARK 3.10**: The converse of theorem 3.9 need not be true.

**EXAMPLE 3.11**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_2\}, \{i_1, i_2\} \}$ 

 $\leq_1 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3), (i_2, i_3) \}.$ 

 $m_iO(Z,\,T1)=\{\Phi,\,Z\}$ 

 $d-m_iO(Z, T1) = \{\Phi, Z\}$ 

open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ , i<sub>2</sub>} }.

d- open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$ } }. Then B = { $i_1$ } is d-open set but not d-m<sub>i</sub>O(Z, T1).

**THEOREM 3.12**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>O(Z, T1) is an b-open set.

**Proof**: As for the fact, every  $b-m_iO(Z, T1)$  is  $m_iO(Z, T1)$  and that every  $m_iO(Z, T1)$  is b-open set.

**REMARK 3.13**: The converse of theorem 3.12 need not be true.

**EXAMPLE 3.14**: Let Z={  $i_1$ ,  $i_2$ ,  $i_3$  } and T= {  $\Phi$ , Z, { $i_1$ }, {  $i_2$ }, { $i_1$ , {  $i_1$ , {  $i_2$ }, { $i_1$ ,  $i_2$ } }  $\leq_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \}$ .

 $m_iO(Z, T1) = \{\Phi, Z\}$ 

 $b-m_iO(Z, T1) = \{\Phi, Z\}$ 

open set = {  $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$ } }

b-open sets = {  $\Phi,Z,$  {i\_1, i\_2} }. Then C = {i\_1, i\_2} is b-open set but not b-m\_iO(Z, T1)

The theorems of this sketch are given above.

1.i-m<sub>i</sub>gO(Z, T1)  $\Leftrightarrow$  i-m<sub>i</sub>O(Z, T1)  $\Rightarrow$  i-open set. 2.d-m<sub>i</sub>gO(Z, T1)  $\Leftrightarrow$  d-m<sub>i</sub>O(Z, T1)  $\Rightarrow$  d-open set. 3.b-m<sub>i</sub>gO(Z, T1)  $\Leftrightarrow$  b-m<sub>i</sub>O(Z, T1)  $\Rightarrow$  b-open set.

#### Diagram3.1

**THEOREM 3.15:** In a topological ordered space (Z , T,  $\leq$ ), Arbitary union of i-m<sub>i</sub>O(Z, T1) is also i-m<sub>i</sub>O(Z, T1).

**Proof:**  $\{\mathcal{A}\}_{i\in\mathcal{I}}$  be a class of  $i-m_iO(Z, T1) \Longrightarrow \mathcal{A}_i$  is a  $i-m_iO(Z, T1)$  for each  $i\in\mathcal{I}$  clearly  $\bigcup_{i\in I}A_i$  is an open

set. If  $\bigcup_{i \in I} A_i = \Phi$  then there is nothing to prove. Suppose  $\bigcup_{i \in I} A_i \neq \Phi$ . Then there exist some  $j \in \mathcal{I}$  such

that  $\mathcal{A}_j$  is a i- $\frac{m_iO(Z, T1)}{m_iO(Z, T1)}$ 

⇒∃ a non-empty open set  $\mathcal{B}$  such that  $\mathcal{B}\subseteq A_j \Rightarrow \mathcal{B}\subseteq \bigcup_{i\in I}A_i$ . There for  $\bigcup_{i\in I}\mathcal{A}_i$  is also a i-m<sub>i</sub>O(Z, T1).

**THEOREM 3.16**: In a topological ordered space  $(Z, T, \leq)$ , Arbitary union of i-m<sub>i</sub>gO(Z, T1) is also i-m<sub>i</sub>gO(Z, T1).

**Proof**: follows from the theorem 3.15

**THEOREM 3.17**: In a topological ordered space (Z, T,  $\leq$ ), Arbitary union of d-m<sub>i</sub>gO(Z, T1) is also d-m<sub>i</sub>gO(Z, T1).

**Proof**: follows from the theorem3.15

**THEOREM 3.18**: Arbitary union of  $b-m_igO(Z, T1)$  is also  $b-m_igO(Z, T1)$ .

**Proof**: follows from the theorem3.15

**THEOREM3.19**: In a topological ordered space (Z, T,  $\leq$ ), Finite Intersection of i-m<sub>i</sub>O(Z, T1) is also a i-m<sub>i</sub>O(Z, T1).

**Proof:** Let Z={  $i_1$ ,  $i_2$ ,  $i_3$  } and T= {  $\Phi$ , Z, { $i_1$ }, {  $i_2$ ,  $i_3$ }},

 $\leq_1 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3), (i_2, i_3) \}.$ 

 $m_iO(Z, T1)$  sets = {  $\Phi, Z, \{i_1\}, \{i_2, i_3\}$  }

 $i-m_iO(Z, T1)$  sets = {  $\Phi, Z, \{i_2, i_3\}$  }

finite intersection of  $i-m_iO(Z, T1) = \Phi \in i-m_iO(Z, T1)$ .



**THEOREM 3.20**: In a topological ordered space (Z, T,  $\leq$ ), Finite Intersection of d-m<sub>i</sub>gO(Z, T1) is also a d-m<sub>i</sub>gO(Z, T1).

**Proof:** Let Z={  $i_1$ ,  $i_2$ ,  $i_3$  } and T= {  $\Phi$ , Z, { $i_1$ }, {  $i_2$ }, { $i_1$ , {  $i_2$ }, { $i_1$ , {  $i_2$ }, {  $i_2$ ,  $i_3$  },

 $\leq_3 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3) \}.$  $m_i gO(Z, T1) \text{ sets} = \{ \Phi, Z, \{i_1\}, \{i_1, i_2\}, \{i_2, i_3\} \}$ 

 $d\text{-}m_igO(Z,\,T1) \;\; \text{sets} = \; \{\; \Phi,\,Z,\,\{i_1\},\,\{i_1,\;i_2\}\;\}.$ 

finite intersection of d-m<sub>i</sub>gO(Z, T1) =  $\Phi \in d$ -m<sub>i</sub>gO(Z, T1).

**THEOREM 3.21**: In a topological ordered space (Z, T,  $\leq$ ), Finite Intersection of b-m<sub>i</sub>gO(Z, T1) is also a b-m<sub>i</sub>gO(Z, T1).

**Proof:** Let Z={  $i_1$ ,  $i_2$ ,  $i_3$  } and T= {  $\Phi$ , Z, { $i_1$ ,  $i_2$  } },  $\leq_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \}.$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $b\text{-}m_igO(Z,\,T1) \;\; sets = \; \{\; \Phi, Z\; \}$ 

finite intersection of  $b-m_igO(Z, T1) = \Phi \in b-m_igO(Z, T1)$ 

**THEOREM 3.22**: In a topological ordered space  $(Z, T, \leq)$ , every i-m<sub>i</sub>gO(Z, T1) is an i $\alpha$ -open set.

**Proof:** As for the fact, every  $i-m_igO(Z, T1)$  is  $i-m_iO(Z, T1)$  and that every  $i-m_iO(Z, T1)$  is i $\alpha$ -open set.

**REMARK 3.23**: The converse of theorem 3.22 need not be true.

**EXAMPLE 3.24**: Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1\}, \{i_2\}, \{i_1, i_2\}\}$ 

 $\leq_2 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_3, i_2) \}.$ 

 $m_i gO(Z, T1) \ sets = \ \{ \ \Phi, \ Z \ \}$ 

 $i-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }

 $\alpha$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$  }

ia- open serts = { $\Phi$ , Z, {  $i_2$ }, { $i_1$ ,  $i_2$ }. Then D = { $i_1$ ,  $i_2$ } is ia-open set but not i-m<sub>i</sub>gO(Z, T1)

**THEOREM 3.25**: In a topological ordered space (Z, T,  $\leq$ ), every b-m<sub>i</sub>gO(Z, T1) is an b $\alpha$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is ba-open set.

**REMARK 3.26**: The converse of theorem 3.25 need not be true.

**EXAMPLE 3.27**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_3\}, \leq_7 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \}$ .

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $b-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\alpha \text{-open sets} = \ \{ \ \Phi, \ Z, \ \{ \ i_1 \}, \ \{ i_1, \ i_2 \}, \ \{ i_1, \ i_3 \ \} \ \}.$ 

ba-open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$ } }. Then the set E = { $i_1$ ,  $i_2$ } is ba-open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.28**: In a topological ordered space  $(Z, T, \leq)$ , every d-m<sub>i</sub>gO(Z, T1) is an d $\alpha$ -open set.

**Proof:** As for the fact, every  $d-m_igO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is d $\alpha$ -open set.

**REMARK 3.29**: The converse of theorem 3.28 need not be true.

**EXAMPLE 3.30**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_3\}, \}$ 

 $\leq_2 \ = \ \{ \ (i_1, i_1), \ (i_2, \ i_2), \ (\ i_3, \ i_3), \ (i_1, \ i_2), \ (\ i_3, \ i_2) \ \}.$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }.

 $\alpha \text{-open sets} = \ \{ \ \Phi, \ Z, \ \{ \ i_1 \}, \ \{ i_1, \ i_2 \}, \ \{ i_1, \ i_3 \ \} \ \}.$ 

**THEOREM 3.31**: In a topological ordered space  $(Z, T, \leq)$ , every i-m<sub>i</sub>gO(Z, T1) is an d $\alpha$ -open set.

**Proof:** As for the fact, every  $i-m_i gO(Z, T1)$  is  $d-m_iO(Z, T1)$ and that every  $d-m_iO(Z, T1)$  is  $d\alpha$ -open set.

**REMARK 3.32**: The converse of theorem 3.31 need not be true.

**EXAMPLE 3.33**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_3\}, \}$ 

 $\leq_1 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3), (i_2, i_3) \}.$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $i-m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\alpha$ -open sets = {  $\Phi$ , Z, {  $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$  } }.

 $d\alpha$ -open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$  } }. Then the set G = { $i_1$ ,  $i_2$ } is  $d\alpha$ -open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.34**: In a topological ordered space  $(Z, T, \leq)$ , every  $i-m_igO(Z, T1)$  is an b $\alpha$ -open set.

**Proof:** As for the fact, every  $i-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is ba-open set.

**REMARK 3.35**: The converse of theorem 3.34 need not be true.

**EXAMPLE 3.36**: Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1\}, \{i_1, i_3\}, \leq_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1)\}$ .

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $i-m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\alpha$ -open sets = {  $\Phi$ , Z, {  $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$  } }.



ba- open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$ } }. Then the set H = { $i_1$ ,  $i_2$ } is ba-open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.37**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an d $\alpha$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$ and that every  $b-m_iO(Z, T1)$  is d $\alpha$ -open set.

**REMARK 3.38**: The converse of theorem 3.37 need not be true.

**EXAMPLE 3.39**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_2\}, \{i_1, i_3\} \}$ ,

 $\leq_6 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1), (i_1, i_3), (i_2, i_3) \}.$ 

 $m_i gO(Z, T1) \text{ sets} = \{ \Phi, Z, \{i_1, i_2\}, \{i_1, i_3\} \}$ 

 $b\text{-}m_igO(Z, T1) \text{ sets} = \{ \Phi, Z \}.$ 

 $\alpha \text{-open sets} = \{ \ \Phi, \ Z, \ \{ \ i_1 \}, \ \{i_1, \ i_2 \}, \ \{i_1, \ i_3 \ \} \ \}$ 

 $d\alpha$ - open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$ } }. Then the set I= { $i_1$ ,  $i_2$ } is  $d\alpha$ -open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.40**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an i $\alpha$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is i $\alpha$ -open set.

**REMARK 3.41**: The converse of theorem 3.40 need not be true.

**EXAMPLE 3.42**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{ i_2, i_3 \} \}$ ,

 $\leq_2 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_3, i_2) \}.$ 

 $m_i gO(Z, T1) \text{ sets} = \{ \Phi, Z, \{i_1\}, \{ i_2, i_3\} \}$ 

 $b-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\alpha \text{-open sers} = \ \{ \ \Phi, \ Z, \ \{i_1\}, \ \{ \ i_2, \ \ i_3\} \ \}$ 

ia- open sets = {  $\Phi$ , Z, { $i_2$ ,  $i_3$ } }. Then the set J = {  $i_2$ ,  $i_3$ } is ia-open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.43**: In a topological ordered space (Z, T,  $\leq$ ),

every d-m<sub>i</sub>gO(Z, T1) is an i $\alpha$ -open set.

**Proof:** As for the fact, every  $d-m_igO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is ia-open set.

**REMARK 3.44**: The converse of theorem 3.43 need not be true.

Exi<sub>1</sub>mple 3.45 Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_2\}, \{i_1, i_3\} \},\$ 

 $\leq_7 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \} \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z, \{i_1, i_2\}, \{i_1, i_3\}$ 

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$  } }.

open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

 $i\alpha$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }. Then the set K = {  $i_1$ } is i $\alpha$ -open set but not d-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.46**: In a topological ordered space (Z, T,  $\leq$ ), every d-m<sub>i</sub>gO(Z, T1) is an b $\alpha$ -open set.

**Proof:** As for the fact, every  $d-m_igO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is ba-open set.

**REMARK 3.47**: The converse of theorem 3.46 need not be true.

**EXAMPLE 3.48**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1, i_2\} \}, \leq_7 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \} \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }.

open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$  } }

**THEOREM 3.48**: In a topological ordered space  $(Z, T, \leq)$ , every i-m<sub>i</sub>gO(Z, T1) is an i $\beta$ -open set.

**Proof:** As for the fact, every  $i-m_igO(Z, T1)$  is  $i-m_iO(Z, T1)$  and that every  $i-m_iO(Z, T1)$  is  $i\beta$ -open set.

**REMARK 3.49**: The converse of theorem 3.48 need not be true.

**EXAMPLE 3.50**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_2\}, \{i_1, i_2\} \}$ 

$$\leq_{5} = \{ (i_{1}, i_{1}), (i_{2}, i_{2}), (i_{3}, i_{3}), (i_{1}, i_{3}), (i_{2}, i_{3}) \} \}$$
  
m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }

 $i-m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }

 $\begin{array}{l} \text{Parch in Engi $\beta$-open serts} = \{\Phi, Z, \{\{i_1\}, \{i_2\}, \{i_1, i_2\}, \{i_2, i_3\}, \{i_1, i_3\}\} \}. \end{array}$ 

 $i\beta$ -open sets = {  $\Phi$ , Z, { $i_3$ }, {  $i_2$ ,  $i_3$ }, { $i_1$ ,  $i_3$  } Then D = { $i_1$ ,  $i_3$ } is  $i\beta$ -open set but not i-m<sub>i</sub>gO(Z, T1)

**THEOREM 3.51**: In a topological ordered space (Z, T,  $\leq$ ), every i-m<sub>i</sub>gO(Z, T1) is an d $\beta$ -open set.

**Proof:** As for the fact, every  $i-m_i gO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is  $d\beta$ -open set.

**REMARK 3.52**: The converse of theorem 3.51 need not be true.

**EXAMPLE 3.53**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_2\},$ 

 $\leq_2 \ = \ \{ \ (i_1, \, i_1), \, ( \ i_2, \ i_2), \, ( \ i_3, \ i_3), \, (i_1, \ i_2), \, ( \ i_3, \ i_2) \ \}.$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $i-m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\beta\text{-open sets} = \{ \ \Phi, \ Z, \ \{ \ i_1 \}, \ \{ i_1, \ i_2 \}, \ \{ i_1, \ i_3 \ \} \ \}.$ 



 $d\beta$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_3$ } }. Then the set G = { $i_1$ ,  $i_3$ } is d $\beta$ -open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.54**: In a topological ordered space (Z, T,  $\leq$ ), every i-m<sub>i</sub>gO(Z, T1) is an b $\beta$ -open set.

**Proof:** As for the fact, every  $i-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$ and that every  $b-m_iO(Z, T1)$  is  $b\beta$ -open set.

**REMARK 3.55**: The converse of theorem 3.54 need not be true.

**EXAMPLE 3.56**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_3\},$ 

 $\leq_9 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_3)\}$ 

 $m_i gO(Z,\,T1) \;\; \text{sets} = \; \{ \; \Phi, \; Z\{ \;\; i_2\}, \; \{i_1, \;\; i_2\}, \; \{i_1, \;\; i_3 \; \} \; \}.$ 

 $i-m_igO(Z, T1) \text{ sets} = \{ \Phi, Z, \{ i_2 \} \}.$ 

 $\beta\text{-open sets} = \ \{ \ \Phi, \ Z, \ \{ \ i_1\}, \{ \ i_2\}, \ \{i_1, \ i_2\}, \ \{i_1, \ i_3 \ \} \ \}.$ 

b $\beta$ - open sets = {  $\Phi$ , Z, {  $i_2$  }, { $i_1$ ,  $i_3$  }. Then the set H = { $i_1$ ,  $i_3$ } is b $\beta$ -open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.57**: In a topological ordered space  $(Z, T, \leq)$ , every d-m<sub>i</sub>gO(Z, T1) is an d $\beta$ -open set.

**Proof:** As for the fact, every  $d-m_igO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is  $d\beta$ -open set.

**REMARK 3.58**: The converse of theorem 3.57 need not be true.

Example 3.59 Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1\}, \{i_1, i_3\}, \leq_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1)\}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $d-m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\beta$ -open sets = {  $\Phi$ , Z, {  $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$  } }.

dβ- open sets = { Φ, Z, { $i_1$ ,  $i_2$ } }. Then the set F = { $i_1$ ,  $i_1$  Engine is dβ-open set but not d-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.60**: In a topological ordered space (Z , T,  $\leq$ ), every d-m<sub>i</sub>gO(Z, T1) is an i $\beta$ -open set.

**Proof:** As for the fact, every  $d-m_i gO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is  $i\beta$ -open set.

**REMARK 3.61**: The converse of theorem 3.60 need not be true.

**EXAMPLE 3.62:** Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1\}, \{i_2, i_3\}\}, \leq_3 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3)\}$ . m<sub>ig</sub>O(Z, T1) sets =  $\{\Phi, Z, \{i_1\}, \{i_2, i_3\}\}$ .

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z, { $i_1$ }}.

 $\begin{array}{l} \beta\text{-open sets} = \{ \ \Phi, \ Z, \ \{i_1\}, \ \{ \ i_2 \ \}, \ \{i_3\}, \{i_1, \ i_2\}, \{i_2, i_3\}, \ \{i_1, \ i_3\} \}. \ i\beta\text{-open sets} = \{ \ \Phi, \ Z, \ \{ \ i_2 \ \}, \ \{i_3\}, \ \{i_2, i_3\} \ \}. \ Then the set K = \{i_2\} \ is \ i\beta\text{-open set but not } d\text{-}m_igO(Z, \ T1). \end{array}$ 

**THEOREM 3.63**: In a topological ordered space (Z, T,  $\leq$ ), every d-m<sub>i</sub>gO(Z, T1) is an b $\beta$ -open set.

**Proof:** As for the fact, every  $d-m_igO(Z, T1)$  is  $d-m_iO(Z, T1)$  and that every  $d-m_iO(Z, T1)$  is b $\beta$ -open set.

**REMARK 3.64**: The converse of theorem 3.63 need not be true.

**EXAMPLE 3.65**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_2\}, \{i_1, i_2\}, \{i_1, i_3\} \}$ ,

 $\leq_{10} = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_3, i_1), (i_2, i_3), (i_2, i_1) \}$ 

 $m_igO(Z,\,T1) \;\; sets = \; \{ \; \Phi, \; Z, \; \{ \; i_2 \}, \; \{i_1, \; i_2 \}, \; \{i_1, \; \; i_3 \} \}.$ 

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z, {  $i_2$ }}.

B-open sets = {  $\Phi$ , Z, {  $i_2$  }, { $i_3$ }, { $i_1$ ,  $i_2$  }, { $i_1$ ,  $i_3$  }, { $i_2$ ,  $i_3$  }}.

b $\beta$ -open sets = {  $\Phi$ , Z, {  $i_2$  }, { $i_2$ ,  $i_3$  }. Then the set K= { $i_2$ ,  $i_3$ } is b $\beta$ -open set but not d-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.66**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an b $\beta$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is  $b\beta$ -open set.

**REMARK 3.67**: The converse of theorem 3.66 need not be true.

Example 3.68 Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1, i_2\}, \}$ 

 $\leq_9 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), \{i_1, i_3\}\}.$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }.

b-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }.

 $\beta$ -open sets = {  $\Phi$ , Z, {  $i_1$ }, {  $i_2$ }, { $i_1$ ,  $i_2$ }, { $i_2$ ,  $i_3$ }, { $i_1$ ,  $i_3$ }}.

b $\beta$ -open sets = {  $\Phi$ , Z, {  $i_2$  }, { $i_1$ ,  $i_3$  } }. Then the set E = { $i_1$ ,  $i_3$ } is b $\beta$ -open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.69**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an d $\beta$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is d $\beta$ -open set.

**REMARK 3.70**: The converse of theorem 3.69 need not be true.

Example 3.71 Let Z = {  $i_1$ ,  $i_2$ ,  $i_3$  } and T = {  $\Phi$ , Z, { $i_3$ }, { $i_2$ ,  $i_3$  } },

 $\leq_9 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_3)\}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z}

 $b-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\beta\text{-open sets} = \{ \ \Phi, \ Z, \ \{ \ i_3 \}, \ \{i_2, \ i_3 \}, \ \{i_1, \ i_3 \ \} \ \}$ 



 $d\beta$ - open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_3$  }. Then the set I= { $i_1$ ,  $i_3$ } is d $\beta$ -open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.72**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an i $\beta$ -open set.

**Proof:** As for the fact, every  $b-m_i gO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is  $i\beta$ -open set.

**REMARK 3.73**: The converse of theorem 3.72 need not be true.

**EXAMPLE 3.74**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_2\}, \{i_1, i_2\}, \{i_2, i_3\} \}$ .

 $\leq_1 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3), (i_2, i_3) \}.$ 

 $m_i gO(Z,\,T1) \;\; sets = \; \{ \; \Phi, \, Z, \; \{i_1\}, \; \{i_1, \; i_2\}, \; \{ \; i_2, \; i_3\} \}.$ 

 $b-m_i gO(Z, T1) \text{ sets} = \{ \Phi, Z \}.$ 

 $\beta\text{-open sers} = \; \{ \; \Phi, Z, \; \{i_1\}, \; \{i_2\}, \; \{i_1, \; i_2\}, \; \{ \; i_2, \; \; i_3\} \; \}$ 

 $i\beta$ - open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$ }, { $i_2$ ,  $i_3$ } }. Then the set J = { $i_2$ ,  $i_3$ } is  $i\beta$ -open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM** 3.75 In a topological ordered space  $(Z, T, \leq)$ , every i-m<sub>i</sub>gO(Z, T1) is an i $\Psi$ -open set.

**Proof:** As for the fact every  $i-m_igO(Z, T1)$  is  $i-m_iO(Z, T1)$  and that every  $i-m_iO(Z, T1)$  is  $i\Psi$ -open set.

**REMARK 3.76**: The converse of theorem 3.75 need not be true.

**EXAMPLE 3.77**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1, i_2\} \}$ 

 $\leq_4 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_3, i_1), \{i_3, i_2\}\}$ 

 $m_i gO(Z, T1) \text{ sets} = \{ \Phi, Z \}$ 

 $i-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }.

 $\Psi\text{- open sets} = \ \{ \ \Phi, \ Z, \ \{i_1\}, \ \{i_2\}, \ \{i_1, \ i_2\} \}.$ 

 $i\Psi$ -open sets = {  $\Phi$ , Z, { $i_2$ }, { $i_1$ ,  $i_2$ } }. Then the set L = { $i_1$ ,  $i_2$ } is  $i\Psi$ -open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.78**: In a topological ordered space  $(Z,T, \leq)$ , every d-m<sub>i</sub>gO(Z, T1) is an d $\Psi$ -open set.

**Proof:** As for the fact, every d--m<sub>i</sub>gO(Z, T1) is d-m<sub>i</sub>O(Z, T1) and that every d-m<sub>i</sub>O(Z, T1) is d $\Psi$ -open set.

**REMARK 3.79**: The converse of theorem 3.78 need not be true.

**EXAMPLE 3.80**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1, i_2\} \}$ 

 $\leq_1 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_2), (i_1, i_3), (i_2, i_3) \}$ 

 $m_i gO(Z, T1) \ sets = \ \{ \ \Phi, \ Z, \ \{i_1, \ i_2\}, \ \{i_1, \ i_3\} \ \}$ 

 $d\text{-}m_igO(Z,\,T1) \;\; \text{sets} = \; \{ \; \Phi, \; Z, \{i_1, \; i_2\} \; \}.$ 

 $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

 $d\Psi$ - open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ } }. Then the set M = {  $i_1$  } is  $d\Psi$ -open set but not d-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.81**: In a topological ordered space  $(Z, T, \leq)$ , every b-m<sub>i</sub>gO(Z, T1) is an b $\Psi$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is  $b\Psi$ -open set.

**REMARK 3.82**: The converse of theorem 3.81 need not be true.

**EXAMPLE 3.83**: Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1, i_2\}\} \le_7 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1)\}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

b-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }

 $\Psi$ - open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$ }}.

 $b\Psi\text{-open sets} = \{ \ \Phi, \ Z, \ \{i_1, \ i_2\} \ \}. \ \text{Then the set} \ N = \{ \ i_1, \ i_2 \ \} \text{ is } \ b\Psi\text{-open set but not } b\text{-}m_igO(Z, T1).$ 

**THEOREM 3.84**: In a topological ordered space (Z, T,  $\leq$ ), every i-m<sub>i</sub>gO(Z, T1) is an b $\Psi$ -open set.

**Proof:** As for the fact, every i-- $m_i$ gO(Z, T1) is b- $m_i$ O(Z, T1) and that every b- $m_i$ O(Z, T1) is b $\Psi$ -open set.

**REMARK 3.85**: The converse of theorem 3.84 need not be true.

Example 3.86 Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{ i_1 \}, \{ i_1, i_3 \} \} \leq_7 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_2, i_1) \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $i-m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

b $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_2$ } }. Then the set P = {  $i_1$ ,  $i_2$ } is b $\Psi$ -open set but not i-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.87**: In a topological ordered space (Z, T,  $\leq$ ), every i-m<sub>i</sub>gO(Z, T1) is an d $\Psi$ -open set.

**Proof:** As for the fact, every i-- $m_i gO(Z, T1)$  is d- $m_iO(Z, T1)$  and that every d- $m_iO(Z, T1)$  is d $\Psi$ -open set.

**REMARK 3.88**: The converse of theorem 3.87 need not be true.

**EXAMPLE 3.89**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{ i_1, i_2 \} \}$ 

 $\leq_1 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_1, i_3), (i_2, i_3) \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }

 $i-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }

 $\Psi\text{- open sets} = \ \{ \ \Phi, \ Z, \ \{i_1\}, \ \{i_2\}, \ \{i_1, \ i_2\} \}.$ 

 $d\Psi$ - sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ } }. Then the set Q = {  $i_1$  } is  $d\Psi$ -open set but not i-m<sub>i</sub>gO(Z, T1).



**THEOREM 3.90**: In a topological ordered space (Z, T,  $\leq$ ), every b-m<sub>i</sub>gO(Z, T1) is an i $\Psi$ -open set.

**Proof:** As for the fact, every  $b-m_igO(Z, T1)$  is  $b-m_iO(Z, T1)$  and that every  $b-m_iO(Z, T1)$  is  $i\Psi$ -open set.

**REMARK 3.91**: The converse of theorem 3.89 need not be true.

**EXAMPLE 3.92**: Let Z = {  $i_1$  ,  $i_2, i_3$  } and T={  $\Phi, Z, \, \{i_1\}$  }

 $\leq_{10} = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), (i_3, i_1), (i_2, i_3), (i_2, i_1) \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi$ , Z }

 $b-m_igO(Z, T1)$  sets = {  $\Phi$ , Z }

 $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

 $i\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_3$ } }. Then the set R = {  $i_1$ ,  $i_3$ } is  $i\Psi$ -open set but not b-m<sub>i</sub>gO(Z, T1).

**THEOREM 3.93**: In a topological ordered space (Z , T,  $\leq$ ), every b-m<sub>i</sub>gO(Z, T1) is an d $\Psi$ -open set.

**Proof:** As for the fact, every  $b-m_i gO(Z, T1)$  is  $b-m_i O(Z, T1)$  and that every  $b-m_i O(Z, T1)$  is  $d\Psi$ -open set.

**REMARK 3.94**: The converse of theorem 3.92 need not be true.

**EXAMPLE 3.95**: Let  $Z = \{i_1, i_2, i_3\}$  and  $T = \{\Phi, Z, \{i_1\}\}, \leq_8 = \{(i_1, i_1), (i_2, i_2), (i_3, i_3)\}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

 $b-m_igO(Z, T1)$  sets = {  $\Phi, Z$  }

 $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

d $\Psi$ -open sets = {  $\Phi$ , Z,{ $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }.

Then the set  $S = \{i_1, i_2\}$  is d $\Psi$ -open set but not bm<sub>i</sub>gO(Z, T1).

**THEOREM 3.96**: In a topological ordered space  $(Z, T, \leq)$ , every d-m<sub>i</sub>gO(Z, T1) is an i $\Psi$ -open set.

**Proof:** As for the fact, every d--m<sub>i</sub>gO(Z, T1) is d-m<sub>i</sub>O(Z, T1) and that every d-m<sub>i</sub>O(Z, T1) is  $i\Psi$ -open set.

**REMARK 3.97**: The converse of theorem 3.95 need not be true.

**EXAMPLE 3.98**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{ i_2\}, \{i_1, i_2\} \}, \leq_8 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3) \}$ 

 $m_igO(Z,\,T1) \hspace{0.2cm} sets = \hspace{0.2cm} \{ \hspace{0.2cm} \Phi, \hspace{0.2cm} Z \hspace{0.2cm} \}$ 

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }

 $\Psi$ -open sets = { $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$ } { $i_2$ ,  $i_3$ }, { $i_1$ ,  $i_3$ }}

 $i\Psi$ - open sets = { $\Phi$ , Z, { $i_1$ }, { $i_2$ }, { $i_1$ ,  $i_2$ } { $i_2$ ,  $i_3$ }, { $i_1$ ,  $i_3$ }

Then the set  $T=\{\ i_1,\ i_2\ \}$  is  $i\Psi\text{-open set}\,$  but not  $d\text{-}m_igO(Z,\,T1).$ 

**THEOREM 3.99**: In a topological ordered space  $(Z, T, \leq)$ , every d-m<sub>i</sub>gO(Z, T1) is an b $\Psi$ -open set.

**Proof:** As for the fact, every d-- $m_i gO(Z, T1)$  is d- $m_iO(Z, T1)$  and that every d- $m_iO(Z, T1)$  is b $\Psi$ -open set.

**REMARK 3.100**: The converse of theorem 3.98 need not be true.

**EXAMPLE 3.101**: Let  $Z = \{ i_1, i_2, i_3 \}$  and  $T = \{ \Phi, Z, \{i_1\}, \{i_1, i_3\} \}, \leq_9 = \{ (i_1, i_1), (i_2, i_2), (i_3, i_3), \{i_1, i_3\} \}$ 

 $m_i gO(Z, T1)$  sets = {  $\Phi, Z$  }

d-m<sub>i</sub>gO(Z, T1) sets = {  $\Phi$ , Z }

 $\Psi$ -open sets = {  $\Phi$ , Z, { $i_1$ }, { $i_1$ ,  $i_2$ }, { $i_1$ ,  $i_3$ } }

b $\Psi$ - open sets = {  $\Phi$ , Z, { $i_1$ ,  $i_3$  } }. Then the set W={  $i_1$ ,  $i_3$  } is b $\Psi$ -open set but not d-m<sub>i</sub>gO(Z, T1).

#### The above results are shown in following diagram.



# REFERENCES

 K. Krishna Rao: some concepts in topological ordered spaces using semi-open sets, pre-open sets; α-open sets and βopen sets, Ph.D Thesis, Archarya Nagarjuna University, October, 2012.



- K. Krishna Rao and R.Chudamani: α-homomorphisms in topological ordered spaces, International Journal of Mathematics, Tcchnology and Humanities, 52 (2012), 541-560.
- [3]. N. Levine: Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.
- [4]. A.S.Mashhour, I.A.Hasanein and S.N.El.Deeb: αcontinuous and α-open mappings, Acta Math. Hung. 41(3-4) (1983), 213-218.
- [5]. H. Maki, R. Devi and K. Balachandran: Generalized αclosed sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13-21.
- [6]. L Nachbin, Topology and order, D.Van Nostrand Inc., Princeton, New Jersy (1965)
- [7]. O. Njastad: On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.
- [8]. V.V.S. Ramachnadram, B. Sankara Rao and M.K.R.S. Veera kumar: g-closed type, g\*-closed type and sg-closed type sets in topological ordered spaces, Diophantus j. Math., 4(1)(2015), 1-9.
- [9]. M.K.R.S Veera kumar: Homeomorphisms in topological ordered spaces, Acta Ciencia Indica,XXVIII (M)(1)2002, 67-76.
- [10]. M.K.R.S. Veera kumar: Between closed sets and g-closed sets Mem. Fac. Sci.Kochi Univ. Ser.A, Math., 21(2000), 1-19.
- [11]. K. Bhagya Lakshmi, V. Amarendra babu and M.K.R.S. Veera kumar: Generalizwd α-closed type sets in topological ordered spaces, Archimedes J. Maths., 5 stud(1)(2015),1-8.
- [12]. G.venkateswarlu, V. Amarendra babu Veera kumar: International J. Math. And Nature., 2(1)(2016), 1-14.
- [13]. M.K.R.S. Veera kumar: Between semi-closed sets and semipre-closed sets, Rend. Lstit. Mat. Univ. Trieste, XXXII(2000), 25-41.
- [14]. M. E. Abd El-Monsef, S.N El deeb and R.A Mohmoud: βopen sets and β-continous mappings, Bull. Sci. Assiut Univ., 12 (1983), 77-90