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Computation of High-speed Inviscid Compressible Flow by Upwind Schemes

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ABSTRACT: Dynamics of inviscid compressible flow is governed by the Euler equations. Unsteady Euler equations are always hyperbolic. Computation of flows governed by compressible Euler equations around such configurations like a wing, aircraft, missile, and launch vehicles are some situations where upwind schemes are extensively used. These schemes are not uniformly good for all situations. The main aim of the present work is an attempt to study the relative merits and demerits of different upwind schemes by applying them for different flow situations. Different aspects of these methods such as accuracy, robustness, etc., are intended to be studied and conclusions are drawn so as to assist making decisions on schemes to be used under different flow situations. Keeping this in view, different upwind schemes such as Van Leer scheme, Steger and Warming scheme, Advection Upstream Splitting Method (AUSM) are used to compute the flow in a shock tube and a quasi-one-dimensional nozzle. Van Leer's scheme is performing well when there is a shock also. However, the Steger-Warming scheme is greatly smeared the contact discontinuity. MacCormack's scheme is responding very well when there is no contact discontinuity and expansion wave. For mixed subsonic-supersonic flow (nozzle flow) Steger and Warming scheme is closer to analytical result as compared to other upwind schemes. It tells us that Steger and Warming scheme works well when there is no shock and contact discontinuity.

Keywords — upwind scheme, shock tube, quasi-one-dimensional nozzle, CFL condition, Mach wave, expansion wave.

I. Introduction

In nature, most of the flows are high speed and are governed by hyperbolic equations. As the analytical solution of these non-linear partial differential equations is not possible in most cases, numerical solution is the only alternative. Numerical method of one-sided differencing is the *upwind scheme*. Upwind methods take into account the wave-like behavior of the hyperbolic systems and they are found to be more successful in numerical computation compared with central difference methods accompanied by artificial viscosity. As upwind schemes are well known for their ability to capture shocks and compute flows over a wide range of speed and geometry, their popularity is on the rise and considerable research is going on to refine technique and extend their range of applicability. They are extensively used in the aerodynamic design of different aerospace configurations.

For numerical analysis of flow at high Mach numbers around objects like missiles, launch vehicles, etc. Euler equations are frequently used. The modern development of numerical schemes for time-dependent Euler equations is found in the pioneering work of Lax and Wendroff [1] and [2]. The first explicit upwind scheme was introduced by Courant [3].

The first explicit upwind scheme was introduced by Courant [3] and several extensions to second-order accuracy and implicit time integrations have been developed. In 1960, Lax and Wendroff [1][2] introduced a method for computing flows with shocks that was second-order accurate and avoided the excessive smearing of the earlier approaches. The MacCormack's [4] version of this technique became one of the most widely used numerical schemes, however, it is not an upwind scheme. Godunov [10] proposed solving multidimensional compressible fluid dynamics problems by using a solution to a Riemann problem for flux calculations at all faces. Van Leer [5] showed how higher-order schemes could be constructed using the same idea. The concept of flux splitting was also introduced as a technique for treating convection-dominated flows. Since the beginning of the 1980's, upwind schemes have become very popular for the sound theoretical basis of characteristic theory for hyperbolic systems and thus for their capability of capturing discontinuities. Steger and Warming [6] introduced splitting where fluxes were determined using an upwind approach. Van Leer [5] also proposed a new flux splitting technique to improve on the existing methods. The application of flux vector splitting scheme with the implicit relaxation algorithms became very attractive for their efficiency, simplicity and ability to capture the sharp shock waves. Van Leer's [5] scheme showed better behavior than Steger and Warming [6] scheme for its smooth transition at sonic and stagnation points.

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In view of foregoing discussions, the importance of studying the relative merits and demerits of the upwind schemes cannot possibly be overemphasized. And this work attempts to do in a small measure. It is intended to apply as many of these schemes to compute flows of varying complexity. Different aspects of these methods are in terms of speed, accuracy and robustness are expected to be brought out by the present study so that the designer can have something to fall back on before deciding upon the scheme to be used for the particular design situation. Keeping this in view, two problems have been taken, one is the computation of fluid flow through shock tube and the other is quasi-one-dimensional-nozzle. Different upwind schemes including Steger-Warming scheme [6], Van Leer scheme [5], AUSM [9] and Zha-Bilgen [7] scheme and two-step predictor-corrector scheme, MacCormack's scheme [4] are used.

II. Numerical Procedure

The governing equations are quasi-one-dimensional Euler equations.

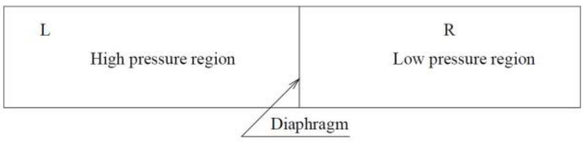
$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}-H=0 \qquad (1)$$

where U, F are vectors containing conservative variables and conservative fluxes respectively and S is the cross-sectional area and are given by,

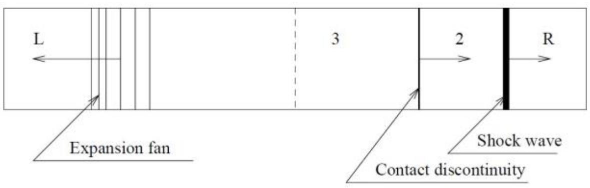
$$U=\begin{bmatrix}\rho\\ \rho u\\ e\end{bmatrix};\quad F=S\begin{bmatrix}\rho u\\ p+\rho u^2\\ (e+p)u\end{bmatrix};\ H=\frac{\partial S}{\partial x}\begin{bmatrix}0\\ p\\ 0\end{bmatrix}$$

Upwind schemes are used to solve two problems one is the shock tube problem. Schematic of the shock tube is shown in Fig. 1 for which cross-sectional area is constant and the equation (1) becomes, $\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=0$.

A. *The Shock Tube Problem*



(a) Shock tube at initial state t = 0



(b) Shock tube at initial state t > 0

Fig. 1. Schematic of shock tube flow.

Shock is a physical disturbance in the flow. For moving wave all properties depend not only on space but also time. This is an unsteady flow and the wave motion is called *unsteady wave motion*.

An important application of unsteady wave motion is a shock tube, where properties change with both space and time, and we find steady wave motion in a quasi-one-dimensional nozzle. The shock tube is a long tube, closed at both ends, with the diaphragm separating a region of high-pressure gas on one side and the low-pressure gas on the other side. Velocity is zero everywhere. The pressure distribution is shown in Fig. 1. When the diaphragm is broken (by electrical current or by mechanical means), a shock wave propagates towards R (low-pressure side) and expansion wave propagates into section L (high-pressure side). The interface between the high-pressure region and the low-pressure region is called contact discontinuity, which also moves towards the low-pressure side. Changes of all thermodynamic properties across all regions are of interest. The shock tube problem constitutes a particularly interesting and difficult test case. Since it requires a solution to the full system of one dimensional Euler equations containing simultaneously a shock wave, a contact discontinuity, and an expansion fan.

The particular problem, also called the Riemann problem, what is altogether of practical and theoretical interest. It can be realized experimentally by the sudden breakdown of a diaphragm in a long one-dimensional tube separating two initial gas states at different. The above governing equation (1) is solved by using different schemes for the Sod [8] initial conditions.

Problem (1): $P_L = 10^5$ Pa; $\rho = 1$ kg/m^3; $u_L = 0$

$P_R = 10^4$ Pa; $\rho = 0.125$ kg/m^3; $u_R = 0$

To solve the shock tube problem, the section is divided into number of grid points in the x- direction. The spacing between the adjacent grid points is Δx. Now assume the flow field variables at all grid points as initial conditions at time t = 0 and apply the time marching procedure.

B. *The Quasi-one-dimensional nozzle*

The second problem solved is a quasi-1D nozzle and the geometry of the nozzle used is shown in Fig. 2. The formulations describing the geometry is given by,

$$A(x)=1+1.5(1-x/5)^2 \quad \textit{for } 0\le x\le 5$$

$$A(x)=1+0.5(1-x/5)^2 \quad \textit{for } 5\le x\le 10$$

The following initial conditions were used for the nozzle problem:

Problem (2): Total temperature = 300 K

Total pressure = 1013 K Pa.

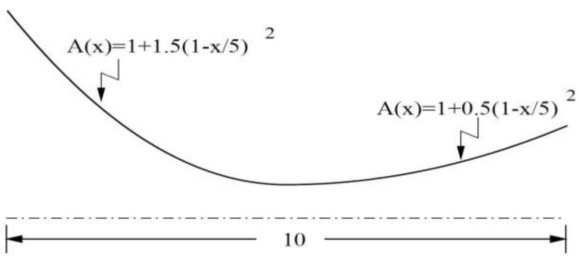


Fig. 2. Area distribution of the nozzle.

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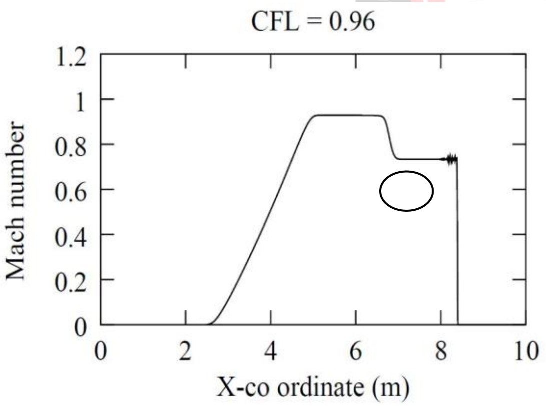
To solve a nozzle problem, the domain is divided into a number of grid points in the x-direction and the spacing between the adjacent grid points is Δx. Now assume the flow field variables at all grid points as initial conditions at time t=0. For faster time marching procedure, one has to choose the initial conditions very carefully. Generally, initial conditions should be closer to final steady-state results for faster convergence. The first step in solving the nozzle problem is to feed the nozzle shape and initial conditions into the program. Calculate all the flow properties for the next time step and compare with the previous time. Repeat this procedure until steady-state is reached. For the subsonic boundary conditions at the entrance, the velocity is extrapolated at the inner domain, and the other variables are calculated from total pressure 1 bar and total temperature 300 K. For supersonic exit boundary conditions, all of the variables are extrapolated from inside of the nozzle.

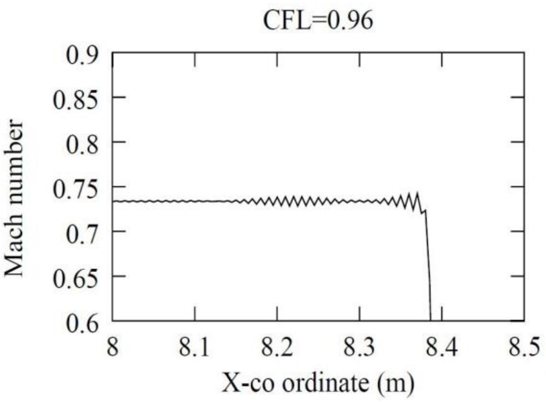
III. RESULTS AND DISCUSSIONS

A. *Numeric result for the Shock tube problem*

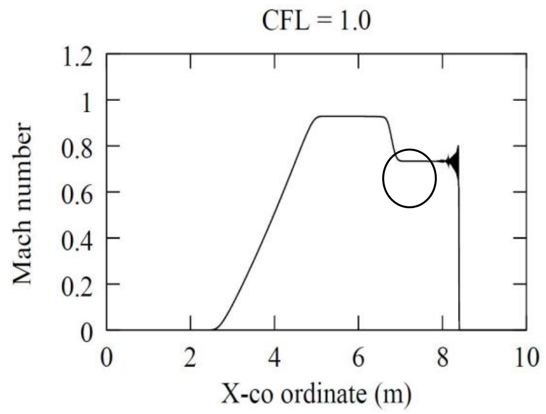
Effect of Courant, Friedrich, and Lewy (CFL) number on the upwind scheme is studied initially. Fig. 3 shows the effect of CFL number on Steger and Warming splitting scheme for the shock tube problem with initial pressure discontinuity of 10. If CFL goes beyond the stability limit (0.95), the scheme becomes unstable and large oscillations occurred in the shock region. It is clearly seen in Fig. 3 for CFL numbers 0.96 and 1.0. At the shock region (circled), oscillations observed for CFL=0.96 and were increased when CFL=1.0.



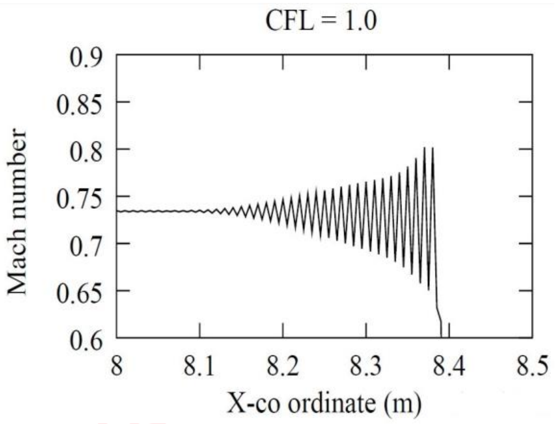
(a) Mach number behavior for CFL = 0.96



(b) Mach number behavior for CFL = 0.96 (enlarged view)



(c) Mach number behavior for CFL = 1.0



(d) Mach number behavior for CFL = 1.0 (enlarged view)

Fig. 3. (a)-(d) Effect of CFL number on the upwind scheme.

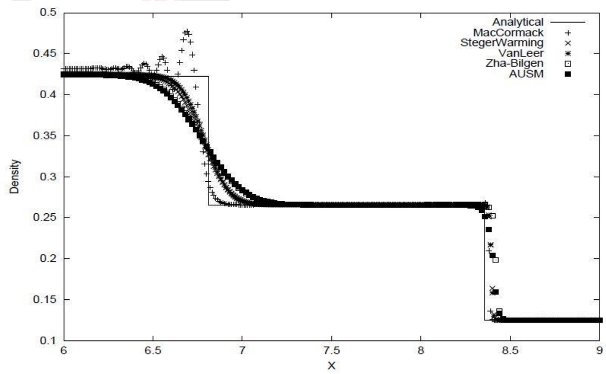


Fig. 4. Comparison of different schemes for the shock tube problem (1) - density variation after time =6.1 msec.

Comparison of different upwind schemes for shock tube problem (1) for density variation is shown in Fig. 4. The plot is shown for the important regions of the shock tube, namely, contact discontinuity and shock region with a grid size of 1000. MacCormack's scheme gives results very close to the exact result in the shock region but will give a number of oscillations in the contact discontinuity. It shows that Van Leer scheme works well in the shock region. Steger and Warming scheme also work reasonably well compared to other schemes. AUSM and Zha-Bilgen are performing equally well at the contact discontinuity, but at the shock region, AUSM is performing well.

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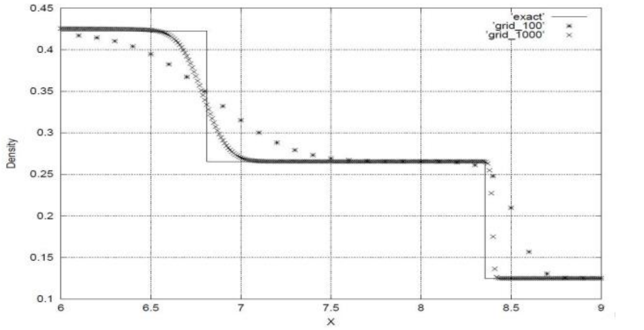

Fig. 5. Effect of the grid on the variation of density for the problem (1) with a numerical scheme.

The effect of grids on the performance of numerical schemes is studied. For the coarse grid of 100 points, it shows that contact discontinuity is greatly smeared. The finer grid (1000) brings great improvement to the results. From Fig. 5, it is found that steepness of the shock wave, contact discontinuity increased as the mesh is refined from 100 to 1000. This is because numerically generated viscosity decreases with mesh refinement.

B. Numerical result for quasi-one-dimensional nozzle

Fig. 6 shows the variation of Mach number as a function of distance for the steady subsonic-supersonic isentropic flow through a nozzle with different schemes for the problem (2) at important throat region. Important conclusions can be drawn by studying the variation of Mach number at the throat region. It is clearly demonstrated, MacCormack's scheme agrees the best with the analytical result at the throat region, but with a little jump. This scheme is performing extremely well when there is no shock and contact discontinuity. Van Leer's scheme also agrees well with analytical result throughout, but at the transition region, there is a large jump. Steger and Warming scheme perform well without any jump at the throat region. But the AUSM scheme is giving a small jump at the throat region.

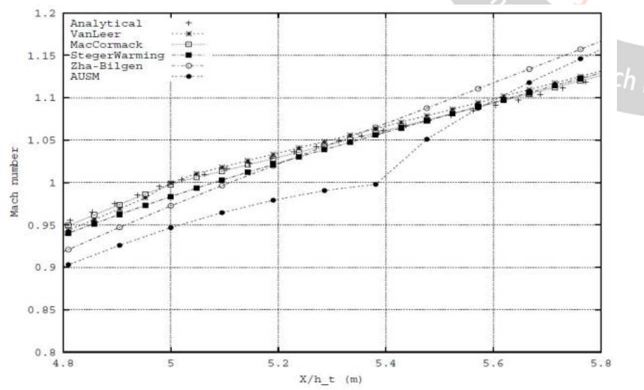

Fig. 6. Comparison of Mach number variation at throat for nozzle problem (2) with different upwind schemes.

IV. Conclusions

Different upwind schemes, including Van Leer scheme [5], Steger and Warming scheme [6], Zha-Bilgen scheme [7], AUSM and a two-step predictor-corrector technique, MacCormack's scheme [4] was used to compute the flow through shock tube and steady subsonic-supersonic flow through a quasi-one-dimensional nozzle. For the shock tube problem, the results obtained are compared with the exact solution given by Charles Hirsch [11], and for the nozzle problem, the results obtained are compared with the analytical solution.

Van Leer's scheme [5] is performing well when there is a shock also. The Steger-warming scheme [6] is greatly smeared the contact discontinuity. MacCormack scheme [4] is responding very well when there is no contact discontinuity and expansion wave. Computational time for AUSM is more and with Zha-Bilgen scheme [7], results are deviating from the analytical results at the outlet.

Stability condition (CFL) plays very important role in the performance of upwind schemes of explicit nature. If it goes beyond the stability limit, the scheme becomes unstable and large oscillations occurred in the shock region.

For mixed subsonic-supersonic flow without shocks (as in the nozzle), the results obtained with Steger and Warming scheme [6] is closer to analytical result as compared to other upwind schemes. It tells us that Steger and Warming scheme [6] works well when there is no shock and contact discontinuity. Van Leer's scheme [5] gives a large jump at the transition region.

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