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A Statistical Analysis on fuzzy Project Network using Fuzzy Data

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Abstract - In this present paper a new approach is made on fuzzy project network with time as fuzzy numbers such as pentagonal numbers .These numbers are converted into normal time by statistical parameter Mean and also by defuzzification process ie. Ranking of pentagonal numbers .Then statistical concept such as Correlation and Regression is used to analyse relationship between two such values ie. mean and ranking value , results are discussed and conclusions are listed

***Keywords**: Fuzzy sets, Pentagonal fuzzy numbers, statistical mean, fuzzy network ,Correlation and Regression analysis.*

I. INTRODUCTION

One possible choice of a proper model architecture can be characterized through correlation and regression models, which, in comparison of fuzzy inference systems or networks, have the advantage that they describe dependencies between process variables in a closed analytical formula and can be computed in a fast and efficient way

Fuzzy logic has become the most explosive new concept in science with pioneering work of L.A Zadeh [3] .It can be applied to the solution of realistic problems particularly in the realm of decision analysis, artificial intelligence, management science and Operation research[8].

The regression equations can be used to evaluate the functional relationship between the two variables in a fuzzy environment. Fuzzy regression analysis can be categorized into eight classes based on the fuzzy (or crisp)characteristic of the input, fuzzy (or crisp) parameters and fuzzy (or crisp) output data .In general, there are two approaches in the analysis of fuzzy regression models: minimum fuzziness methods and the fuzzy least-squares methods. Those approaches are used to model fuzzy regression equations for a variety of cases.

The science of statistics[7] is essentially a branch of applied mathematics and may be regarded as mathematics applied to the given observation characteristics data .The statistical techniques [1],[8] are frequently used in solving the problems of various sciences , such as network analysis in Operation research based on different assumptions . Lee and Li[1] proposed a comparison of fuzzy numbers by considering the mean and SD based on probability distribution.

Lee and Li [1] proposed a comparison of fuzzy numbers by considering the mean and dispersion (variance)based on the concept of statistics.Chang proposed the Coefficient of variance (CV index) ie.CV= $\sigma / |\mu|$ where $\sigma>0$,$\mu \neq 0$

In a classical network problem[10] weights of the edges are supposed to be real numbers, however , in most practical applications the parameters are not naturally precise in general, therefore , in real world situation they may be considered to be a fuzzy.Lin depicted a new line of method to a fuzzy critical path method for activity network created on statistical buoyancy interval estimates and a ranking method for level (1-α) fuzzy numbers. His focus was to introduce an approach that combined fuzzy set theory with statistics that incorporates the signed distance ranking of level of (1-α) fuzzy number.

Measures of Central Tendency shows the tendency to some central value around which data tends to cluster. It is one of the most powerful tools for analysis is to calculate a single average value that represents the entire mass of data. Such a value is neither the smallest nor the largest value , but is a number whose value is somewhere in the middle of the group. It describes the characteristics of entire data and to facilitate comparison. This paper aims to perform correlation and regression analysis on pentagonal fuzzy numbers in comparison with statistical mean

This paper is organized as follows. In section 2 some basic concepts on fuzzy sets, fuzzy numbers, are defined .In section 3 Construction of Mean for PFN is discussed. In section 4,Ranking methods, ranking of some fuzzy numbers, Correlation and Regression[8] methods are discussed. In section 5 description of the model with algorithm are given. In section 6 a numerical example given and discussions are made. finally conclusions are listed.

II. PRELIMINARIES

In this section some basic definitions related to fuzzy set, fuzzy numbers are reviewed.

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2.1 DEFINITIONS: [FUZZYSET]

A Fuzzy set is characterized by a membership function mapping element of a domains, space, or the universe of discourse X to the unit interval [0,1].

2.2 DEFINITION [FUZZY NUMBERS][6]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1.

A Fuzzy number is a convex normalized fuzzy set on the real line R such that ,there exist at least on

i)$x \epsilon$ **X** with $\mu_A(x) = 1$,ii) $\mu_A(x)$ is piece wise continuous.

2.3 DEFINITION(TRIANGULAR FUZZY NUMBER)

A fuzzy number ***A(x),***it can be represented by ***A*(a,b,c;1**) with membership function μ(x) is given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & x=b \\ \frac{(c-x)}{(c-b)} & c \le x \le b \end{cases}$$

2.4.DEFINITION(TRAPEZOIDAL FUZZY NUMBER)

A fuzzy number A defined on the universal set of real numbers R denoted by $\tilde{A}$ **(a,b,c,d;1)** is said to be a Trapezoidal fuzzy number if its membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & x=b \\ \frac{(d-x)}{(d-c)} & c \le x \le d \end{cases}$$

2.5.DEFINITION (PENTAGONAL FUZZY NUMBER)

A fuzzy number **Ā** is said to be a Pentagonal fuzzy number $\bar{A}_{PFN}$(a,b,c,d,e) where a,b,c,d,e are real numbers and its membership is given by

$$\mu_{\bar{A}PFN}(x) = \begin{cases} \frac{1}{2}\frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ (1/2) + (1/2)\frac{(x-b)}{(c-b)} & \text{for } b \le x \le c \\ 1 - (1/2)\frac{(x-c)}{(d-c)} & \text{for } c \le x \le d \\ (1/2)\frac{(e-x)}{(e-d)} & \text{for } d \le x \le e \end{cases}$$

III. CONSTRUCTION OF EXPECTED TIME (NORMAL) USING STATISTICAL DATA

A Statistical data has minor variations inbuilt into it due to bias. To minimize this bias we require an unbiased estimator. From statistical estimation theory, we know that sample mean is an unbiased estimate of the population mean.

We use this concept to compress the five pentagonal numbers into a single number by taking their average. that is if x_1, x_2, x_3, x_4, x_5, are pentagonal numbers then

$$D_{ij} = \dot{P}FN(\dot{X}) = \frac{x1+x2+x3+x4+x5}{5} \quad \text{--- (I)}$$

gives the unbiased estimate of activity. Then we compare this with number obtained by ranking method.

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IV. RANKING METHODS

Ranking fuzzy numbers plays an important role in practical use such as in approprorimate reasoning ,decisions –making optimization,[15] forecasting analysis. Fuzzy numbers are frequently employed to describe the performance of alternatives in modeling a real world problem.The method of ranking fuzzy numbers has been proposed by Jain[2] Since then, a large variety of methods above been developed ranging from the trivial to the complex , including one fuzz number attribute to many fuzzy numbers attribute[11-13].Some researchers have made the ranking methods as classified into several concepts such as mean etc.. The ranking formula of some fuzzy numbers are given

4.1. triangular fuzzy number (a,b,c) $\underline{R}(A_T)$ = (1/4) [a+2b+c] with weights (1,2,1)

trapezoidal fuzzy number (a,b,c,d) $\underline{R}(A_{TRP})$ = (1/4) [a+b+c+d] with weights (1,1,1,1)

Pentagonal fuzzy number (a,b,c,d,e) $\underline{R}(A_P)$**= (1/4) [2a+3b+2c+3d+2e]with weights (2,3,2,3,2) -- (II)**

4.2. FUZZY CORRELATION COEFFICIENT / REGRESSION LINES

The Pearson product correlation coefficient is a measure of the degree of linear relationship between two variables. It takes on any value between plus and minus one ie.$-1\leq\rho_{\dot{X}\underline{R}(AP)}\leq 1$ since the sign of correlation coefficient (+,-) defines the direction of the relationship. It can be defined by the formula

$$\rho_{\dot{X}[\underline{R}(AP)]} = \frac{n\,\Sigma(\dot{X})\,\underline{R}(A_P) - \Sigma\,\underline{R}(A_P)\,\Sigma\,\dot{X}}{\sqrt{n\Sigma\dot{X}^2 - (\Sigma\dot{X})^2}\;\sqrt{n\Sigma\underline{R}(A_P)^2 - [\Sigma\underline{R}(A_p)]^2}} \quad - \quad - \quad - \quad - \quad \text{III}$$

FUZZY REGRESSION LINES ARE

$$\left.\begin{aligned}[\underline{R} - E(\underline{R})] &= b_{R\dot{X}}[(\dot{X} - E(X)] \text{ and} \\ [\dot{X} - E(\dot{X})] &= b_{\dot{X}\underline{R}}[\underline{R} - E(\underline{R})]\end{aligned}\right\} \quad \text{--- IV}$$

4.3. OBJECTIVE / AIM

(i) In triangular fuzzy number middle number is given more weight and rank $\underline{R}(A_T)$ will be inside the interval [a,c]

(ii) In trapezoidal numbers all numbers are given equal weights and hence rank $\underline{R}(A_{TR})$ = mean of these numbers and lies in the interval [a,d]

(iii) In pentagonal fuzzy numbers the distribution of weights are (2,3,2,3,2,) .ie. second numbers from either ends is given more weight,viz 3 than other numbers. Also division by 4 in the ranking $\underline{R}(A_P)$ nakes it to lie outside the range namely [a,e]

This shift in the pentagonal ranking system necessitates to test statically whether the pentagonal ranking is in conformity with Statistical Mean.

The objective of this study is to verify that statistical methods applied to solve operation research problems yield results contradicting to the method of fuzzy logic when applied to the same problem .

This is in way very much important in the present day scenario as fuzzy logic pushes back all other crisp scientific methods in Electronics, telecommunications, share markets, etc.. where multiple factors are studied all at a time to come to a decision which may beneficiary or at time harmful.

To evaluate the conformity of these two , a simple test is made .ie. we evaluate the correlation coefficient between statistical mean $\dot{X}$ ***and* ranking** $\underline{R}(A_p)$ **to find any non-conformity.**

V. DESCRIPTION OF THIS PAPER

Pentagonal fuzzy numbers are converted into normal value for each activity by using mean and ranking method ,then correlation and regression analysis are carried out [15].

5.1 Algorithm

Step I : Calculate the statistical Mean $\dot{X}$ for each activity with the given PFNby**(I)**

Step II : Determine the Ranking value of PFN by the formula **(II)**

Step III: Determine the coefficient of correlation between statistical Mean (**$\dot{X}$**) and **$\underline{R}(A_P)$**

Step IV: Determine the Two regression lines between (**$\dot{X}$**) and **$\underline{R}(A_P)$**

VI. NUMERICAL EXAMPLES

Consider a project network diagram with ten activities . The distance between them is represented by PFN. Any network attempts to solve one of the following problems.1) shortest route between start node and end node.2) shortest distance between start node to every other nodes.3) shortest time path [11] from one place to another place. 4) critical path [6] (the longest time path) from the start node to the end node.5) Maximum flow of goods from origin to destination.

In all the above problems, the time or distance or arc capacity are single numbers represented on the activities or arcs. When more than one number appears on an activity

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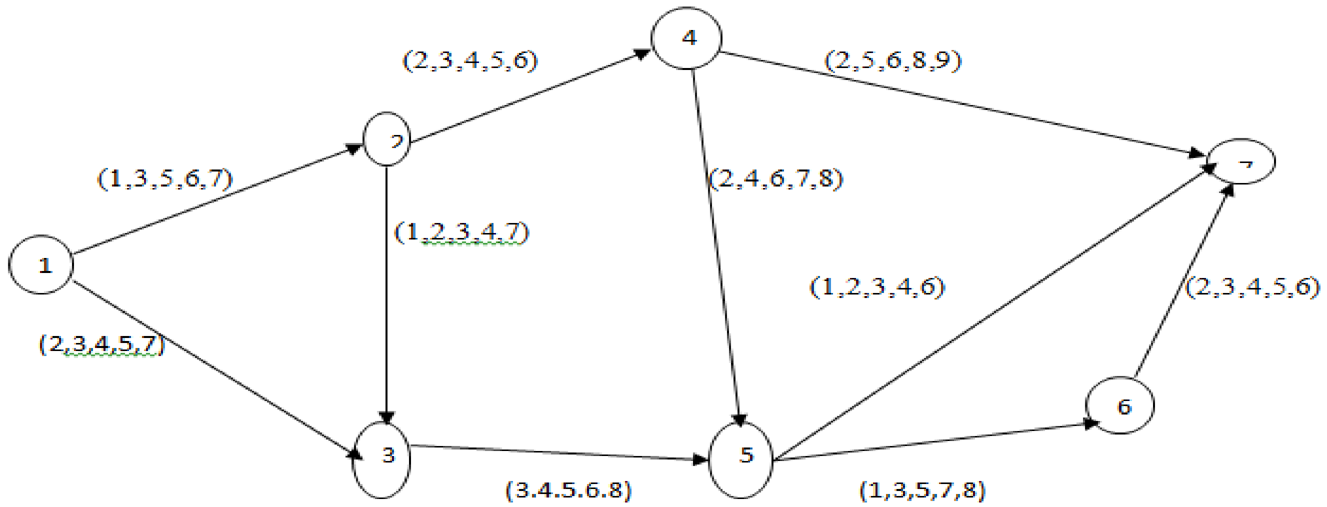
like as in the case of (optimistic , most likely, pessimistic time estimates) we convert them into a single number by formulas(such as $T_o+47T_m+T_p$ / 6) [9]. Such single number conversions can be done in several ways .Our aim is to study whether these different conversion methods are consistent in giving the required results.

A better way to find the compatibility of two different conversion methods is subjecting them to "Correlation and Regression analysis." In this section 6 the five PFN number are converted into single numbers by using 1) statistical method (Mean) 2) Ranking method. In the ensuing paragraphs (Discussions and Comparison) these two method are subjected to correlation and regression analysis and were found to give the same results. Herewith we proceed with network problem.



By algorithms they converted times are

Activity :	1-2	1-3	2-3	2-4	3-5	4-5	4-7	5-6	5-7	6-7
Mean($\dot{X}$):	**4.4**	**4.2**	**3.4**	**4**	**5.2**	**5.4**	**6**	**3.2**	**4.8**	**4**
$\underset{\sim}{R}(A_P)$:	**13.25**	**12.5**	**10**	**12**	**15.5**	**16.25**	**18.25**	**9.5**	**14.5**	**12**

By **(III)**We get the correlation coefficient is**) 0.9996** almost equal to **+1**

By **(IV)**the two regression lines are $\underset{\sim}{R} = 3.1056\dot{X} - 0.476$ **and** ---(V)

$\dot{X} = 0.3217\underset{\sim}{R} + 0.1573$

VII. DISCUSSIONS AND COMPARISON

a) The Correlation coefficient between statistical mean and ranking of pentagonal fuzzy numbers are found to be **+ 1**.

b) This shows that **PERFECT POSITIVE** correlation exists between statistical mean and pentagonal ranks,

c) Hence statistical methods and fuzzy methods do not contradict each other .

d)In fact they go hand in hand and can be invariably used one in place of other.

e) The regression equation **(V)** shows that **$\underset{\sim}{R}$ varies** 3.1 times faster then**$\dot{X}$ and $\dot{X}$ varies 0.32 times (ie reciprocal of 3.1 [1/ 3.1=0.32]) slower than $\underset{\sim}{R}$. (ie)** $b_{\dot{X}\underset{\sim}{R}(AP)} = 1/b_{\underset{\sim}{R}(AP)\dot{X}}$

This shows that a **perfect linear regressional relationship** exists between statistical means and ranking of pentagonal fuzzy numbers.

f) Though they are fuzzy numbers they have the imamate tendency to behave like a central tendency in statistics.

g) The fuzzy pentagonal numbers behave in a controlled fashion and are in tune with statistical measure of location and central tendency like mean. Hence these numbers can be used in practice like any other numbers because of their conformity with statistics.

VIII. ADVANTAGES OF STATISTICAL STUDY

The focus of this paper is to introduce an approach that combined fuzzy set theory with statistics. Whenever uncertainty arises in complex project we use statistical concept to arrive at some better conclusion. since statistics

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parameters are important tool for real life situations for solving many problems. The following significant results are obtained based on characteristics of statistical parameters using fuzzy numbers.

IX. Conclusion

1. As long as there is no significant abnormality in the usage of statistical methods such methods can be used wherever necessary and can be attained without any fear or reservations as they are basic frame work of our society. In this paper a new approach is developed on fuzzy project network based on fuzzy pentagonal numbers.

2. Once drastic abnormalities are observed we must resolve to fuzzy logic which can handle higher level of variability.
3. The given pentagonal fuzzy numbers are converted as statistical mean and also as defussified value

ie .ranking method, and then correlation analysis is made between these two values and results are discussed and also regression analysis is made between statistical mean and ranking , simultantaneously inference is made , .

4. A Combination of fuzzy logic statistics and operations research throw more light in such kind of complex fuzzy network problems which can be analyzed with the help of statistics..

5. Hence the conclusions arrived using fuzzy logic are time bound and valid for at that moment whereas the conclusions reached using statistical methods are stable and trust worthy as far as social sciences are considered whereas in the realm of highly fluctuating areas like electronics, telecommunication, etc.,, fuzzy logic holds upper hand and leads the decision maker faster towards the desired goal.

6. . Hence this method provides new tool and ideas for the project researchers on how to approach fuzzy network using statistical data

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