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Fixed Point Theorems For Compatible type α ,β and K in Fuzzy 2 - Metric Space

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Abstract - In this paper, we give some common fixed point results related to compatible type α ,β and K and prove fixed point theorems for compatible mapping types in fuzzy 2-metric space.

Keywords - Fuzzy Set, Fuzzy Metric Space, Compatible α , Compatible β, Compatible K, Fuzzy 2-Metric Space.

I. Introduction

In 1965, the notion of fuzzy set was introduced by Zadeh [14] in fuzzy mathematics. A . George, P.veeramani [3] give another path, the concept of fuzzy metric space introduced by Kramosil and Michalek [6].The concept of common fixed point theorem for commuting mapping was given by jungck and more generalized commutability is called compatiblility.G. Jungck [7]discussed the compatible. Jain Singh [5], Junck et.al [8] proved a fixed point theorem for compatible mapping type in fuzzy metric space. Sharma and Iseki[12] gave the contraction type mapping in 2-metric space. The concept of 2-metric space has investigate by S.Gahler[1,2].The study of fuzzy-2 metric space and fuzzy -3 metric space have been active and interesting field of research world.

In this paper, we show the fixed point theorems in compatible type with self maps in fuzzy 2- metric space with illustrative examples.

II. Preliminaries

Definition:2.1[14]

A fuzzy set A in X is a function with domain X and values in [0,1]

Definition:2.2[11]

A binary operation $*$: [0,1] × [0,1] × [0,1]→ [0,1] is a continuous t-norm if {[0,1],$*$} is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 \leq a_2 * b_2 * c_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$, for all $a_1, b_1, c_1, a_2, b_2, c_2 \in$ [0,1].

Definition2.3 [13]

The triple (X,M, $*$) is a fuzzy 2-metric space if X is an arbitrary set, $*$ is a continuous t-norm, M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following condition for all x,y,z,,u ∈ X and t_1, t_2, t_3>0.

(i) M(x,y,z,0) = 0

(ii) M(x,y,z,t) = 1, ∀ t >0 if and only if x=y

(iii) M(x,y,z,t) = M(x,z,y,t) = M(y,z,x,t) ,(Symmetric about three variables)

(iv) M(x,y,u,t_1) $*$ M(x,u,z,t_2) $*$M(u,y,z,t_3) ≤ M(x,y,z, $t_1 + t_2 + t_3$)

(This corresponds to tetrahedron inequality in 2-metric space)

(v)M(x,y,z,.) : [0,1) → [0,1] is left continuous.

(vi)$\lim_{t\to\infty} M(x,y,t)$ = 1, ∀ x,y ∈ X.

Definition2.4 [5]

Two mappings A and B on a fuzzy 2-metric space X are said to be compatible type (α) if

$\lim_{n\to\infty} M(ABx_n, BBx_n$, z , t) = 1 , $\lim_{n\to\infty} M(BAx_n, AAx_n$, z , t) = 1

for all t>0, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x for some x∈X.

Definition2.5[10]

Two mappings A and B on a fuzzy 2-metric space X are said to be compatible type (β) if

$\lim_{n\to\infty} M(A^2x_n, B^2x_n$, z , t) = 1

for all t>0, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ = x for some x∈X.

Definition2.6 [6]

Two mappings A and B on a fuzzy 2-metric space X are said to be compatible type (K) if

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$\lim_{n\to\infty} M(AAx_n, Bx, z, t) = 1$, $\lim_{n\to\infty} M(BBx_n, Ax, z, t) = 1$

for all t>0, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some x ∈X.

Definition2.7[13]

A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M, ∗) is called Cauchy if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$ for all a in X and t>0, p > 0.

Definition2.8 [13]

A sequence $\{x_n\}$ in a fuzzy 2- metric space (X,M, ∗) is said to be Convergent to x ∈ X $\lim_{n\to\infty} M(x_n, x, a, t) = 1$ for all a in X and t>0 .

Definition:2.9 [13]

A fuzzy 2- metric space (X,M, ∗) is said to be complete if every Cauchy sequence in X converges in X.

Definition:2.10 [13]

A function M is continuous in fuzzy 2- metric space if and only if whenever $x_n \to x$, $y_n \to y$ then $\lim_{n\to\infty} M(x_n, y_n, a, t)$ = M(x,y,a,t) for all a in X and t>0.

Lemma:2.11 [4]

Let M(x,y,z,.) is non-decreasing for all x,y,z ∈ X

Lemma:2.12[4]

Let M(X,M,*) be a fuzzy 2- metric space. If there exist k ∈ (0,1) such that M(x,y,z,kt) ≥ M(x,y,z,t) for all x,y,z ∈ X with z ≠ x, z≠ y and t>0 then x=y.

Example:2.13[13]

Let (X,d) be a 2- metric space and a*b =ab, for every a,b ∈ [0,1], Let M_d be a fuzzy set in X^3 x [0,∞), given by $M_d(x,y,z,t) = \frac{t}{t+d(x,y,z)}$ if t>0 and $M_d(x,y,z,0) = 0$.Then (X, M_d,*) is a fuzzy 2-metric space and M_d is called the standard fuzzy 2- metric space induced by 2-metric d. Thus every 2-metric d induces a fuzzy 2-metric M_d on X.

Example:2.14

Let (X,M,*) be a fuzzy metric space, where X=[0,2], t-norm is defined by a*b = min {a,b} for all a,b ∈ [0,1] and $M(x,y,z,t) = \frac{t}{t+d(x,y,z)}$ for all x,y ∈ X and all t>0.Define self-maps A ,B .and Z on X as follows:

$$Ax = \begin{cases} 3-2x & if \ x \in [0,1) \\ 2 & if \ x \in [1,2] \end{cases}$$

$$Bx = \begin{cases} x & if \ x \in [0,1) \\ 2 & if \ x \in [1,2] \end{cases}$$

z = 1 z ∈ X

Take $x_n = 1 - \frac{1}{n}$, then $x_n \to 1$, $x_n < 1$, $3-2x_n > 1$ for all n.

Also $Ax_n, Bx_n, Zx_n \to 1$ and n→∞.

$AAx_n = 2$, $BBx_n = x_n$, $ABx_n = 3-2x_n$, $BAx_n = 2$

Now, we show A & B are compatible type (α)

$$M(ABx_n, BBx_n, z, t) = \frac{t}{t+d(ABx_n, BBx_n, z)} = 1$$

$$M(BAx_n, AAx_n, z, t) = \frac{t}{t+d(BAx_n, AAx_n, z)} = 1$$

as n→∞. So A ,B are compatible type (α)

Next, we show that A & B are compatible type (β)

$$M(AAx_n, BBx_n, z, t) = \frac{t}{t+d(ABx_n, BBx_n, z)} = 1$$

as n→∞. So A ,B are compatible type (β).

III. MAIN RESULTS

Theorem:3.1

Let (X,M,*) be a complete fuzzy 2-metric space. Let S and T be continuous mappings of X in X, then S and T have common fixed point in X if there exist a continuous mapping A of X into S(X) and B of X into T(X) satisfying

(A,S) and (B,T) are compatible type α,

M(Ax,By,a,qt) ≥ φ [min {M(Ax,Sx,a,t) , M(Ax,Ty,a,t) , M(Ty,By,a,t)}]

For all x,y,z ∈ X, t>0 and 0<q<1 where φ: [0,1] → [0,1] is a continuous function such that φ(t) >t and φ(t) = 1 Then S,T,A and B have a unique fixed point.

Proof:

Take a point x_0 ∈ X ,there is a point x_1 ∈ X in A(X) ⊆ T(X) such that A(x_0) =T(x_1) .

For this point x_1 and there exist a point x_2 ∈ X in B(X) ⊆ S(X) such that B(x_1) =S(x_2) and so on continuing this step we build a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}$$
$$y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$$

Using the inequality

$M(Ax_{2n}, Bx_{2n+1}, a, qt) \geq \varphi[\min\{M(Ax_{2n}, Sx_{2n}, a, t), M(Ax_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n+1}, Bx_{2n+1}, a, t)\}]$

$M(y_{2n}, y_{2n+1}, a, qt) \geq \varphi[\min\{M(y_{2n}, y_{2n-1}, a, t), M(y_{2n}, y_{2n}, a, t), M(y_{2n}, y_{2n+1}, a, t)\}]$

$M(y_{2n}, y_{2n+1}, a, qt) \geq \varphi[M(y_{2n}, y_{2n-1}, a, t)] > M(y_{2n}, y_{2n-1}, a, t)$

Thus $\{M(y_{2n}, y_{2n+1}, t), n \geq 0\}$ is an increasing sequences of real number in [0,1] and therefore tends to a limit L ≤ 1. Claim that L = 1 if not L < 1 which on letting n→ ∞ L>L which is contradiction. Finally thereby L = 1 therefore for every n ∈ N with the above assertion one can show that$\{M(y_{2n+1}, y_{2n+2}, t), n \geq 0\}$ is a sequence of real number

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in [0,1] which tends to limit L =1 . $M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t)$ and $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$

Now,

$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * \ldots\ldots * M(y_{n+p-1}, y_{n+p}, t/p)$

where p is a positive integer

Since $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$ for t > 0 it follows that

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * \ldots\ldots * 1 = 1$$

which shows that $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete , there is a point $u \in X$ such that $y_n \to z$, implying the sequence $\{Ax_{2n}\}$ and $\{Bx_{2n+1}\}$ converges to z, as such the subsequences $\{Sx_{2n+1}\}$ and $\{Tx_{2n+2}\}$ also converges to z.

We show that u is common fixed point of A,B,S and T.

Case :1

Since (A,S) is compatible type (α)

$ASx_n = SSx_n$ and $SAx_n = AAx_n$

whenever x_n is a sequence such that $Ax_n = Sx_n = z$

ie) Az = Sz

Now,

$M(AAx_{2n}, Bx_{2n+1}, a, qt) \geq \varphi\,[\min \{M(AAx_{2n}, SAx_{2n}, a, t), M(AAx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n+1}, Bx_{2n+1}, a, t)\}]$

Taking limit as $n \to \infty$, we get

$M(Az,z,a,qt) \geq \varphi[\min \{M(Az,Sz,a,t), M(Az,z,a,t), M(z,z,a,t)\}]$

$M(Az,z,a,qt) \geq \varphi\,[\min \{M(Az,Az,a,t), M(Az,z,a,t), M(z,z,a,t)\}]$

$$\geq \varphi\,[M(Az,z,a,t)] > M(Az,z,a,t)$$

which is contracdiction .

Hence z is a fixed point of A

Az = z

ie) Az = z = Sz

Case :2

Since (B,T) is compatible type (α)

$BTx_n = TTx_n$ and $TBx_n = BBx_n$

whenever x_n is a sequence such that $Bx_n = Tx_n = z$

ie) Bz = Tz

$M(Az, BBx_{2n+1}, a, qt) \geq \varphi\,[\min \{M(Az,Sz,a,t), M(Az, TBx_{2n+1}, a, t), M(TBx_{2n+1}, BBx_{2n+1}, a, t)\}]$

Taking limit as $n \to \infty$, we get

$M(Az,Bz,a,qt) \geq \varphi[\min \{M(Az,Sz,a,t), M(Az,Tz,a,t), M(Tz,Bz,a,t)\}]$

$M(z,Bz,a,qt) \geq \varphi[\min\{M(z,z,a,t), M(z,Bz,a,t), M(Bz,Bz,a,t)\}]$

$$\geq \varphi\,[M(z,Bz,a,t)] > M(z,Bz,a,t)$$

which is contracdiction .

Hence z is a fixed point of B

z = Bz

ie) Bz=z = Tz

hence z is a common fixed point of A,B,S and T.

Corollary:3.2

Let (X,M,*) be a complete fuzzy 2-metric space.Let A,B,S and T be continuous self mappings of X, satisfying

1.. $A(X) \subset T(X)$, $B(X) \subset S(X)$,

2. (A,S) is compatible type α and (B,T) is weak- compatible.

$M(Ax,By,a,qt) \geq \varphi\,[\min \{M(Ax,Sx,a,t), M(Ax,Ty,a,t), M(Ty,By,a,t), M(Sx,By,a,t), M(Sx,Ty,a,t)\}]$

For all $x,y,z \in X$, t>0 and 0<q<1.where $\varphi: [0,1] \to [0,1]$ is a continuous function such that $\varphi(t) > t$ and $\varphi(t) = 1$ Then S,T,A and B have a unique fixed point.

Theorem:3.3

Let (X,M,*) be a complete fuzzy 2-metric space.Let S and T be continuous mappings of X in X, then S and T have common fixed point in X if there exist a continuous mapping A of X into $S(X) \cap T(X)$ which commutes with S & T and the pair (A,S) and (A,T) satisfying the compatible type β

$M(Ax,Ay,a,qt) \geq \min \{M(Ax,Ty,a,t), M(Ax,Sx,a,t), M(Ay,Ty,a,t), M(Ay,Sx,a,t)\}$

For all $x,y,z \in X$, t>0 and 0<q<1. Then S,T and A have a unique common fixed point.

Proof:

Take a point $x_0 \in X$,there is a point $x_1 \in X$ in $A(X) \subseteq S(X)$ such that $A(x_0) = T(x_1)$ .For this point x_1 and there exist a point $x_2 \in X$ in $A(X) \subseteq T(X)$ such that $A(x_1) = S(x_2)$ and so on continuing build this step we get a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}$$
$$y_{2n+1} = Ax_{2n+1} = Sx_{2n+2}$$

Using the inequality

$M(Ax_{2n}, Ax_{2n+1}, a, qt) \geq \min\{M(Ax_{2n}, Tx_{2n+1}, a, t), M(Ax_{2n}, Sx_{2n}, a, t), M(Ax_{2n+1}, Tx_{2n+1}, a, t), M(Ax_{2n+1}, Sx_{2n}, a, t)\}$

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$M(y_{2n}, y_{2n+1}, a, qt) \geq \min \{M(y_{2n}, y_{2n}, a, t), M(y_{2n}, y_{2n-1}, a, t), M(y_{2n+1}, y_{2n}, a, t), M(y_{2n+1}, y_{2n-1}, a, t)\}$

$M(y_{2n}, y_{2n+1}, a, qt) \geq M(y_{2n-1}, y_{2n}, a, t)$

Thus $\{M(y_{2n}, y_{2n+1}, t), n \geq 0\}$ is an increasing sequences of real number in [0,1] and therefore tends to a limit $L \leq 1$. Now, clear that L = 1. if not L < 1 which on letting $n \to \infty$ L > L which is contradiction. Finally thereby L = 1 therefore for every $n \in N$ .For the above assert one can show that$\{M(y_{2n+1}, y_{2n+2}, t), n \geq 0\}$ is a sequence of real number in [0,1] which tends to limit L =1 .

$M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t)$

and $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$

Now, $M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * \ldots\ldots\ldots\ldots * M(y_{n+p-1}, y_{n+p}, t/p)$

where p is a positive integer

Since $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$ for t > 0 it follows that

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * \ldots\ldots\ldots\ldots * 1 = 1$$

which shows that $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete , there is a point $u \in X$ such that

$y_n \to z$, implying the sequence $\{Ax_{2n}\}$ and $\{Ax_{2n+1}\}$ converges to z, as such the subsequences $\{Sx_{2n+1}\}$ and $\{Tx_{2n+2}\}$ also converges to z.

Show that u is common fixed point of A,S and T.

Case :1

Since (A,S) is compatible type (β)

$A^2x_n = S^2x_n$

whenever x_n is a sequence such that $Ax_n = Sx_n = z$

ie) Az = Sz

Now,

$M(A^2x_{2n}, Ax_{2n+1}, a, qt) \geq \varphi [\min \{M(A^2x_{2n}, Tx_{2n+1}, a, t), M(A^2x_{2n}, SAx_{2n+1}, a, t), M(Ax_{2n+1}Tx_{2n+1}, a, t), M(Ax_{2n+1}, SAx_{2n} a, t)\}]$

Taking limit as $n \to \infty$, we get

$M(Az, z, a, qt) \geq \min \{M(Az, z, a, t), M(Az, Sz, a, t), M(z, z, a, t), M(z, Sz, a, t)\}]$

$M(Az, z, a, qt) \geq \min \{M(Az, z, a, t), M(Az, Az, a, t), M(z, z, a, t), M(z, Az, a, t)\}$

$M(Az, z, a, qt) \geq M(Az, z, a, t)$

which is contradiction Az =z

Therefore Az = z = Sz

Case :2

Since (A,T) is compatible type (β)

$A^2x_n = T^2 x_n$

whenever x_n is a sequence such that $Ax_n = Tx_n = z$

ie) Az = Tz

$M(Az, A^2x_{2n+1}, a, qt) \geq \min \{M(Az, TAx_{2n+1}, a, t), M(Az, Sz, a, t), M(A^2x_{2n+1}, TAx_{2n+1}, a, t), M(A^2x_{2n+1}, Sz, a, t)\}$

$M(Az, T^2x_{2n+1}, a, qt) \geq \min \{M(Az, TAx_{2n+1}, a, t), M(Az, Sz, a, t), M(T^2x_{2n+1}, TAx_{2n+1}, a, t), M(T^2x_{2n+1}, Sz, a, t)\}$

Taking limit as $n \to \infty$, we get

$M(Az, Tz, a, qt) \geq \min \{M(Az, Tz, a, t), M(Az, Az, a, t), M(Tz, Tz, a, t), M(Tz, Sz, a, t)\}$

$M(z, Tz, a, qt) \geq \min\{M(z, Tz, a, t), M(z, z, a, t), M(Tz, Tz, a, t), M(Tz, z, a, t)\}$

$\geq [M(z, Tz, a, t)]$

which is contracdiction .

Hence z is a fixed point of T

z = Tz

ie) Az=z = Tz

Hence z is a common fixed point of A,S and T.

Uniqueness:

Suppose there is another fixed point w ≠ z , then

$M(Ax, Ay, a, qt) \geq \min \{M(Ax, Ty, a, t), M(Ax, Sx, a, t), M(Ay, Ty, a, t), M(Ay, Sx, a, t)\}$

$M(Az, Aw, a, qt) \geq \min \{M(Az, Tw, a, t), M(Az, Sz, a, t), M(Aw, Tw, a, t), M(Aw, Sz, a, t)\}$

$M(z, w, a, qt) \geq \min \{M(z, w, a, t), M(z, z, a, t), M(w, w, a, t), M(w, z, a, t)\}$

which is a contradiction , which implies w = z.

Hence A,S and T have unique common fixed point.

Corollary:3.4

Let (X,M,*) be a complete fuzzy 2-metric space.Let A,B,S and T be continuous self mappings of X, satisfying

1. $A(X) \subset PQ(X)$, $B(X) \subset ST(X)$
2. (A,S) is compatible type β and (B,T) is compatible.

$M(Ax, By, a, qt) \geq \min \{M(Ax, STy, a, t), M(Ax, PQx, a, t), M(By, STy, a, t), M(By, PQx, a, t)\}$

For all $x, y, z \in X$, t>0 and 0<q<1. Then S,T,P,Q, A and B have a unique common fixed point.

Theorem:3.5

Let (X,M,*) be a complete fuzzy 2-metric space.Let S and T be continuous mappings of X if there exist a continuous mapping A of X into S(X) and B of X into T(X) satisfying

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(A,S) and (B,T) are compatible type (k),

M(Ax,By,a,qt) ≥ M(Sx,Ty,a,t) ∗ M(Ax,Sx,a,t) ∗ M(By,Ty,a,t) ∗ M(Ax,Ty,a,t)

For all x,y,z ∈ X, t>0 and 0<q<1. Then S,T,A and B have a unique common fixed point.

Proof:

Take a point x_0 ∈ X ,there is a point x_1 ∈ X in A(X) ⊆ T(X) such that A(x_0) =T(x_1) .For this point x_1 and there exist a point x_2 ∈ X in B(X)) ⊆ S(X) such that B(x_1) =S(x_2) and so on continuous this step we build a sequence $\{y_n\}$ in X such that

$y_{2n} = Ax_{2n} = Tx_{2n+1}$

$y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$

Using the inequality

M(Ax_{2n} ,Bx_{2n+1},a,qt) ≥ M(Sx_{2n},Tx_{2n+1},a,t) ∗ M(Ax_{2n},Sx_{2n},a,t) ∗ M(Bx_{2n+1},Tx_{2n+1},a,t) ∗ M(Ax_{2n},Tx_{2n+1},a,t)

M(y_{2n} , y_{2n+1},a,qt) ≥ M(y_{2n-1}, y_{2n} ,a,t) ∗M(y_{2n} , y_{2n-1},a,t) ∗ M(y_{2n+1}, y_{2n} ,a,t) ∗ M(y_{2n} , y_{2n} ,a,t)

M(y_{2n} , y_{2n+1},a,qt) ≥ M(y_{2n} , y_{2n-1},a,t) ∗ M(y_{2n+1}, y_{2n} ,a,t) ∗ M(y_{2n} , y_{2n} ,a,t)

≥ M(y_{2n} , y_{2n-1},a,t)

Thus {M(y_{2n} , y_{2n+1}, t), n≥ 0} is an increasing sequences of real number in [0,1] and therefore tends to a limit L ≤ 1. Claim that L = 1.If not L < 1 which on letting n→ ∞ L> L which is contradiction. Finally thereby L = 1 therefore for every n ∈ N with the above assert one can show that{M(y_{2n+1}, y_{2n+2}, t),n≥0} is a sequence of real number in [0,1] which tends to limit L =1 . M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n,t) and $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$

Now, M(y_n, y_{n+p}, t) ≥ M(y_n, y_{n+1}, t/p) ∗…………..∗M(y_{n+p-1}, y_{n+p}, t/p)

where p is a positive integer

Since $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$ for t > 0 it follows that

$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1$∗…………..∗ 1= 1

which shows that $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete , there is a point u ∈ X such that $y_n \to$ z, implying the sequence {Ax_{2n}} and {Ax_{2n+1}} converges to z, as such the subsequences {Sx_{2n+1}} and {Tx_{2n+2}} also converges to z.

We show that u is common fixed point of A,S and T.

Case :1

Since (A,S) is compatible type (k)

A^2x_n = Sz and S^2x_n = Az

whenever x_n is a sequence such that Ax_n = Sx_n = z

Now,

M(A^2x_{2n},Bx_{2n+1},a,qt) ≥ M(SAx_{2n},Tx_{2n+1},a,t) ∗ M(A^2x_{2n},SAx_{2n},a,t) ∗ M(Bx_{2n+1},Tx_{2n+1},a,t) ∗ M(A^2x_{2n},Tx_{2n+1},a,t)

Taking lim n tends to infinty

M(Sz,z,a,qt) ≥ M(Sz,z,a,t) ∗ M(Sz,Sz,a,t) ∗ M(z,z,a,t) ∗ M(Sz,z,a,t)

M(Sz,z,a,qt) ≥ M(Sz,z,a,t)

Which is contradiction Sz = z

Similarly Az = z

Therefore Az = z = Sz

Case :2

Since (B,T) is compatible type (k)

B^2x_n = Tz and T^2x_n = Bz

whenever x_n is a sequence such that Bx_n = Tx_n = z

M(Ax_{2n}, B^2x_{2n+1},a,qt) ≥ M(Sx_{2n},TBx_{2n+1},a,t) ∗ M(Ax_{2n},Sx_{2n},a,t) ∗ M(B^2x_{2n+1}, TBx_{2n+1},a,t) ∗ M(Ax_{2n},TBx_{2n+1},a,t)

Taking lim n tends to infinty

M(z,Tz,a,qt) ≥ M(z,Tz,a,t) ∗ M(z,z,a,t) ∗ M(Tz,Tz,a,t) ∗ M(z,Tz,a,t)

M(z,Tz,a,qt) ≥ M(z,Tz,a,t)

which is contradiction. Hence Tz = z

Similarly Bz =z

Therefore Bz = z = Tz

Hence Az = Sz = Bz = Tz, z is a common fixed point.

IV. Conclusion

In this paper, discussion is three type of compatible in fuzzy metric space then prove fixed point.

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