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Special Study on Divisor Cordial Labeling of Spider Graph

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Abstract **Let G= (V, E) be a graph with p vertices and q edges. A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,3…,|V|} such that each edge uv is assigned the label 1 if either f(u)|f(v) or f(v)|f(u) and label 0 if f(u) does not divide f(v) or f(v) does note divide f(u) with the condition that the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. The graph that admits a divisor cordial labelling is called divisor cordial graph. In this paper we prove that the spider graph $SP\left(1^m,2^t\right), SP\left(1^m,3^1\right), SP\left(1^m,3^2\right)$ is divisor cordial labelling graph and further study on the generalization of labelling spider graph $SP\left(1^m,3^t\right)$.**

***Keywords* — Divisor cordial graphs, Divisor cordial labelling, Spider Graph**

I. Introduction

A graph G is a finite nonempty set of objects called vertices and edges. We consider all graphs here as finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelling can be attributed to Rosa. Of all the different labelling introduced divisor cordial labelling was one among them which motivates us to study further on different types of graphs. Divisor cordial labelling was introduced by Varatharajan . R and Navaneethakrishnan.S, Nagarajan [2,3,4]. Some more interesting study on divisor cordial labelling of graphs was presented[5,6,7] .In this paper we like to establish some results on divisor cordial labelling of spider graph $SP\left(1^m,2^t\right), SP\left(1^m,3^1\right), SP\left(1^m,3^2\right)$ and identified some mechanism to prove that the spider graph $SP\left(1^m,3^t\right)$ for $t \geq 3$ is divisor cordial labelling. For all the preliminary concepts in graphs we refer to Handbook of graph theory [8]

II. PRELIMINARIES

Definition 2.1: A tree is called a spider if it has a centre vertex C of degree R>1 and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1's of length a_1, X_2's paths of length a_2 etc. We shall denote the spider by $SP\left(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \ldots a_m^{x_m}\right)$ where $a_1 \prec a_2 \prec a_3 \prec \ldots a_m$ and $x_1 + x_2 + \ldots + x_m = R$

Definition.2.2: A graph G is called a divisor cordial labelling if there exists a bijective function from set of all vertices V to {1,2,3,…|V|} such that each edge uv in G assign the label 1 if either f(u)|f(v) or f(v)|f(u) and label 0 if f(u) does not divide f(v) or f(v) does not divide f(u) and further it satisfies the condition $\left|e_f(0) - e_f(1)\right| \leq 1$. The graph is thus called divisor cordial graph.

III. MAIN RESULTS

Theorem .3.1: The Spider graph $SP\left(1^1,2^t\right)$ is a divisor cordial labelling graph

Proof: Let $SP\left(1^1,2^t\right)$ be a Spider graph.

We know that a tree is called a spider if it has a centre vertex C of degree R>1 and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1's of length a_1, X_2's paths of length a_2 etc. We shall denote the spider by $SP\left(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \ldots a_m^{x_m}\right)$ where $a_1 \prec a_2 \prec a_3 \prec \ldots a_m$ and $x_1 + x_2 + \ldots + x_m = R$

Define the Vertex set $V(SP(1^1,2^t)) = \{u,\ v,\ u_j : 1 \leq j \leq 2t\}$ and edge set $E(SP(1^1,2^t)) = \{e = uv\}$ $\cup \{e'_i = u\,u_j\!: 1 \leq j \leq t\} \cup \{e''_i = u_j u_{t+j} : 1 \leq j \leq t\}$

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Now to label the vertices let us consider the bijective function $f: V \to \{1,2,3...|V|\}$ such that such that each edge uv is assigned the label 1 if either f(u)|f(v) or f(v)|f(u) and label 0 if f(u) does not divide f(v) with the condition that the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. We define the labelling of vertices as follows

$f(u) = 1$

$f(v) = 2$

$f(u_i) = i + 2$ for $1 \le i \le 2t$

Then the induced edge labelling for the graph $G = SP(1^1, 2^t)$ are

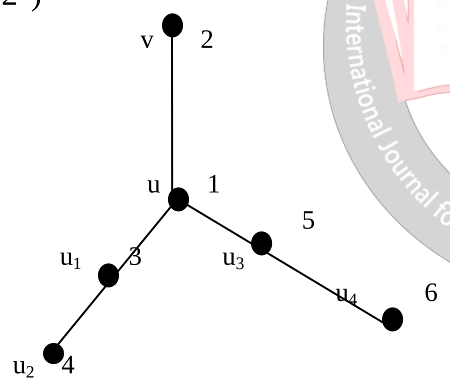
$f^*(uv) = 1$

$f^*(uu_{2j-1}) = 1$ where $1 \le j \le t$

$f^*(u_{2j-1}u_j) = 0$ where $1 \le j \le t$

Noticing the induced edge labelling we find that the number of edges labelled with 1 is t+1 and the number of edges labelled with 0 is t. Hence $|e_f(0) - e_f(1)| \le 1$. Therefore the Spider graph $G = SP(1^1, 2^t)$ is a divisor cordial labelling graph.

Figure.1 : Divisor Cordial labelling of Spider graph $SP(1^1, 2^2)$



We established that the spider graph $G = SP(1^1, 2^t)$ is divisor cordial labelling graph by fixing m=1 and increasing t by 1 from 1.

We understand from the following table that for all values of t and setting m=1 the spider graph $G = SP(1^1, 2^t)$ is divisor cordial labelling graph.

Table.1

Spider Graph	Number of Edges labelled with	
	1	0
$SP(1^1, 2^1)$	2	1
$SP(1^1, 2^2)$	3	2
$SP(1^1, 2^3)$	4	3
$SP(1^1, 2^4)$	5	4
$SP(1^1, 2^5)$	6	5

And so on .

Observation.1: From the table.1 we find that the Spider graph $G = SP(1^1, 2^t)$ is divisor cordial labelling graph and number of edges labelled with 1 is one more than the number of edges labelled with 0 for all values of t.

Theorem .3.2: The Spider graph $SP(1^1, 3^t)$ is a divisor cordial labelling graph when t=1 and t=2.

Proof: Let $SP(1^1, 3^t)$ be a Spider graph.

We know that a tree is called a spider if it has a centre vertex C of degree R>1 and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1's of length a_1, X_2's paths of length a_2 etc. We shall denote the spider by $SP(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} ... a_m^{x_m})$ where $a_1 \prec a_2 \prec a_3 \prec ... a_m$ and $x_1 + x_2 + ... + x_m = R$

Define the Vertex set $V(SP(1^1, 3^t)) = \{u,\ v,\ u_j : 1 \le j \le 3t\}$ and edge set

$E(SP(1^1, 3^t)) = \{e = uv\}$

$\cup \{e'_i = u\,u_{j:} 1 \le j \le 2t\}$

$\cup \{e"_i = u_j u_{t+j} :\ 2t+1 \le j \le 3t\}$

Now to label the vertices let us consider the bijective function $f: V \to \{1,2,3...|V|\}$ such that such that each edge uv is assigned the label 1 if either f(u)|f(v) or f(v)|f(u) and label 0 if f(u) does not divide f(v) with the condition that the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. We define the labelling of vertices as follows

$f(u) = 1$

$f(v) = 2$

$f(u_i) = i + 2$ for $1 \le i \le 3t$

Then the induced edge labelling for the graph $SP(1^1, 3^t)$ are

$f^*(uv) = 1$

$f^*(uu_{3j-2}) = 1$ where $1 \le j \le t$

$f^*(u_{3j-2}u_{3j-1}) = 0$ where $1 \le j \le t$

$f^*(u_{3j-1}u_{3j}) = 0$ where $1 \le j \le t$

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Noticing the induced edge labelling we find that the number of edges labelled with 1 is t+1 and the number of edges labelled with 0 is t. Hence $\left|e_f(0)-e_f(1)\right|\leq 1$. Therefore the Spider graph $G=SP\left(1^1,3^t\right)$ is a divisor cordial labelling graph

From the above theorem.3.2 we find that the spider graph $G=SP\left(1^1,3^t\right)$ by fixing m=1 and increasing t by 1 from the standard spider graph $SP\left(1^m,3^t\right)$ the graph in general ceases to be divisor cordial labelling for $t\geq 3$. The result is only true for t=1 and t=2.

Now we pose the problem in a different approach and try to find out suitable solution In order to prove that the graph is divisor cordial. It is also noted that on increasing the value of m by 1 instead of fixing it as 1 also will not help always. Hence we try to find out the value of m in in terms of t so as to prove it divisor cordial labelling graph.

Table.2

Spider Graph	Number of Edges labelled with	
	1	0
$SP(1^1,3^1)$	2	2
$SP(1^1,3^2)$	3	4
$SP(1^1,3^3)$	4	6
$SP(1^1,3^4)$	5	8
$SP(1^1,3^5)$	6	10

And so on and we find that the number of zero's computed for the induced edge labelling for the spider graph $SP\left(1^1,3^t\right)$ increases and which ceases the graph from being divisor cordial. Hence to balance the number of zeros that are increasing we add on edges to the powers of 1 so as to prove it divisor cordial graph.

As per the condition the edges labelled with 1 and 0 should differ by at most 1 . But from table.2 we find that for the spider graph $SP(1^1,3^3)$ we find that the edges labelled with 1 and 0 differ by 2 . Hence it is not divisor cordial labelling. In order to that let us now add 1 or 2 to the value of m. If we add 1 to m then the resulting spider graph is $SP(1^2,3^3)$ and if we add 2 to m then the resulting spider graph is $SP(1^3,3^3)$ which causes the resulting edges to have label 1 added to it and the total number of edges labelled with 1 is 5 in case of $SP(1^2,3^3)$ and 6 in case of $SP(1^3,3^3)$ and as usual the edges labelled with 0 is 6 in both the cases enabling the graph to be proved as divisor cordial labelling.

Similarly for the spider graph $SP(1^1,3^4)$ has the number of edges labelled with 1 is 5 and the number of edges labelled with 0 is 8 which causes the spider graph $SP(1^1,3^4)$ to cease divisor cordial labelling. Hence to improve on the number of edges labelled with 1 we add 2 or 3 to the value of m. If we add 2 to m then the resulting spider graph is $SP(1^3,3^4)$ and if we add 3 to m then the resulting spider graph is $SP(1^4,3^4)$ and the total number of edges labelled with 1 increases by 2 or 3 as in the case of the spider graph $SP(1^3,3^4)$ and $SP(1^4,3^4)$ respectively so as to prove as divisor cordial labelling . We can continue the procedure similarly for the higher powers of t of the spider graph $SP\left(1^1,3^t\right)$.

Hence we try to add on edge to the powers of m of the basic spider graph $SP(1^1,3^1)$ according to the requirement of the edges labelled with 1 to prove it as divisor cordial labelling.

In this procedure we identify that m can be represented in terms of t

For instance $SP(1^1,3^3)$ we need to add the value of m by 1 or 2 and the resulting graph is $SP(1^2,3^3)$ or $SP(1^3,3^3)$.

For the spider graph $SP(1^1,3^4)$ we need to add the value of m by 2 or 3 and the resulting graph is $SP(1^3,3^4)$ or $SP(1^4,3^4)$ and so on.

Hence by setting m=t-1 or m=t

The general spider graph $SP(1^m,3^t)$ takes the form $SP(1^{t-1},3^t)$ or $SP(1^t,3^t)$ which is divisor cordial labelling.

Observation :2: From table.2 we find that for the spider graph $SP\left(1^1,3^t\right)$ for t >3 the number of edges labelled with 1 increases by 1 and the number of edges labelled with 0 increases by 2 in a steady manner.

Observation.3: From table.3 we find that for the spider graph $SP\left(1^1,3^t\right)$ for $t\geq 3$ the difference between the number of edges labelled with 1 and number of edges labelled with 0 is 2,3,4,5 and so on.

Definition 3.3: For a spider graph $SP\left(1^1,3^t\right)$ let us denote the total number of edges labelled with 1 by $T\left(e_f(1)\right)$ and total number of edges labelled with 0 by $T\left(e_f(0)\right)$

Theorem.3.4: For a Spider graph $SP\left(1^1,3^t\right)$ we have $T\left(e_f(1)\right)=t+1$ and $T\left(e_f(0)\right)=2t$

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Proof: Given a spider graph $SP\left(1^1,3^t\right)$ from the labelling procedure adopted in theorem 3.2 we find that induced edges which results with the edge labelling 1 are from those from computing through

$f^*(uv)=1$, $f^*(uu_{3j-2})=1$ where $1\le j\le t$

Now for t=1. The total edges labelling with 1 is 2

Similarly for t=2 the total edges labelling with 1 is 3 and hence in general for the spider graph $SP\left(1^1,3^t\right)$

$T\left(e_f(1)\right)=t+1.$

Similarly the induced edges which results with the edge labelling 0 are from those from computing through

$f^*(u_{3j-2}u_{3j-1})=0$, $f^*(u_{3j-1}u_{3j})=0$ where $1\le j\le t$.

Now for t=1 the total edges labelled with 0 is 2 ,

For t=2 the total edges labelled with 0 is 4 .

Hence in general for the spider graph $SP\left(1^1,3^t\right)$

$T\left(e_f(0)\right)=2t$. Hence the proof of the theorem.

Definition:3.5: For a spider graph $SP\left(1^1,3^t\right)$ let us denote the t^{th}difference between the total number of edges labelled with 1 and the total number of edges labelled with 0 as $D^t\left(T(e_f(1)),T(e_f(0)\right)$

Theorem.3.6: For a spider graph $SP\left(1^1,3^t\right)$, $t\ge 3$ we have $D^t\left(T(e_f(1),T(e_f(0)\right)=t-1$

Proof: Given for a spider graph $SP\left(1^1,3^t\right),t\ge 3$ we can compute the total number of edges labelled with 1 denoted as $T\left(e_f(1)\right)$and total number of edges labelled with 0 denoted as $T\left(e_f(0)\right)$ from theorem 3.4 and it is evident from the result that

$T\left(e_f(1)\right)=t+1$ and $T\left(e_f(0)\right)=2t$. Hence

$D^t\left(T(e_f(1)),T(e_f(0)\right)=2t-(t+1)$

$D^t\left(T(e_f(1)),T((e_f(0))\right)=t-1.$

Hence the proof.

Definition.3.7 : The value of m to be added to the spider graph $SP\left(1^1,3^t\right)$, $t\ge 3$ to prove it as divisor cordial labelling is denoted by $N(SP(1^1,3^t))$

Theorem.3.8 : For a spider graph $SP\left(1^1,3^t\right),t\ge 3$

$N(SP(1^t,3^t))=D^t\left(T(e_f(1)),T(e_f(0))\right)$

$N(SP(1^t,3^t))=D^t\left(T(e_f(1)),T(e_f(0))\right)-1$

Proof: Given a spider graph $SP\left(1^1,3^t\right)$, $t\ge 3$ We can identify the value of m to be added to the spider graph in order to prove the spider graph $SP\left(1^1,3^t\right)$ divisor cordial labelling graph is through the relation $N(SP(1^1,3^t))=D^t\left(T(e_f(1)),T(e_f(0))\right)$ and $N(SP(1^1,3^t))=D^t\left(T(e_f(1)),T(e_f(0))\right)-1$. For claiming the relation let us compute $D^t\left(T(e_f(1)),T(e_f(0)\right)$ for t=3,4,…

On computing and substituting in the relation we find the value of m to be added as $N(SP(1^1,3^t))$.

i.e for t=3 , $N(SP(1^1,3^3))=1$ or $N(SP(1^1,3^3))=2$ for t=4, $N(SP(1^1,3^4))=2$ or $N(SP(1^1,3^4))=3$ and so on. Hence the proof.

IV. Results

In this paper we have proved that the spider graphs $SP\left(1^m,2^t\right),SP\left(1^m,3^1\right),SP\left(1^m,3^2\right)$and identified some mechanism to prove that the spider graph $SP\left(1^m,3^t\right)$ for $t\ge 3$ is divisor cordial labelling.

V. Conclusion

The different labeling techniques teaches us on the various ways in which the vertices or edges or both can be labelled so as to prove that the graph is of desired type. In our study we have taken the spider graphs of the type $SP\left(1^m,2^t\right),SP\left(1^m,3^1\right),SP\left(1^m,3^2\right)$and have proved that it divisor cordial labeling But on identifying that in general the spider graphs $SP\left(1^1,3^t\right)$ for $t\ge 3$ ceases to be divisor cordial labeling graph we have introduced a way in which the number of edges labelled with 0 and number of edges labelled with 1 follows the required condition namely the difference between them is less than or equal to 1 . We in our future study wish to identify some more graph which can be proved as divisor cordial labeling graph and also wish to establish the generalization of the graph to be divisor cordial..

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