International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-01, April 2019

Determining The Inter-relationship Between Darcy-Weisbach Formula and Hazen-Williams Equation, On The Concern Of Water Supply Distribution System – A Differential Research Initiative

Prasanta Biswas[1]

[1]Assistant Professor, Civil Engineering Department, Global Institute of Management and Technology (Approved By All India Council Of Technical Education, AICTE, India & Affiliated By West Bengal University Of Technology, WBUT, West Bengal, India), NH-34 Bhatjangla, Nadia West Bengal, India. paport2018@gmail.com

ABSTRACT – Differential views and its application to various physical activities are required and to be established in every subject of research study. It is so obvious that without having better inner knowledge, a research could not come out as a 'good' one. Yet, it might although be of little value apparently but the approach and findings give research temptation to a satisfactory level. In this study, this similar intrigued effect could be obtained where various design equations, theoretically, are derived out by applying the methodological philosophy and related approaches. It is in relation to pipe flow wherein the required design formulations have been derived by common form of the Darcy-Weisbach formula and its required essentials, in differential form. These are by the approaches of methodology so found and obtained signifying the possible scopes of research. And, it is using the Hazen-Williams formula which is given the priority in this study in making itself to get compared with the Darcy-Weisbach. By this combination of the two formulas, it has been easily felt with the subject matter to get achieved to its application indeed. Subjectively, this study gives its finding specially to water supply distribution network that exists after the drinking water storage tanks towards its supply to the network.

KEYWORDS – Water Supply, Distribution Network Design, Flow Differentials, Darcy-Weisbach Formula, Hazen-Williams Formula.

I. INTRODUCTION

So far of water supply distribution network, pipes of different sizes and its categorical |and related connections are laid below the ground level in a form of mesh of pipes. This network of pipes is meant for supplying the water from treatment units to the consumer in a township/city (Figure 1). Various distribution layouts are followed in this, by one particular mode or by variety of zones, depending on the importance of locality and related to water pressure requirements, land topography etc. Common conventional practices are usually followed in design, construction and implementation of the pipes in the network. Design flow variables such as head loss, pipe-flow, velocity, pipe length etc. are the design concern of the distribution network design of water supply.

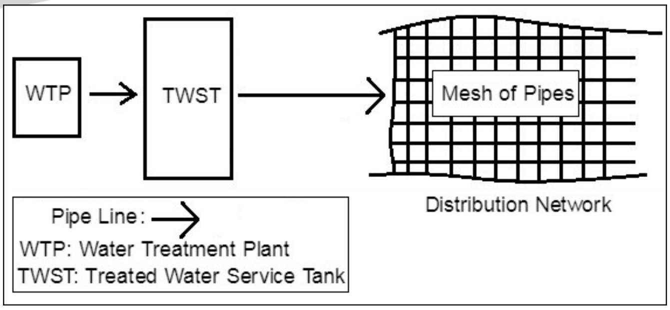

Figure 1: Typical Layout of the Distribution Network

This study may be regarded as a study of the inventions about the formulated equation of the investigation find-out. In this study the finding is determined by observing the Hazen-William (H-W) equation and the Darcy-Weisbach (D-W) equation with the vision of formulating new forms of the network design (Figure 2). Inter-applicability with these famous renowned formulas has built the methodology of this research study to quite a satisfactory level which upon its

 DOI : 10.18231/2454-9150.2019.0368

established forms gives the future possibility of its various research scopes.

In the methodology of this study, the differentials of the D-W formula are given importance in this study. These differentials have the very implication in maintaining the overall flow happenings in the network system, giving due consideration to energy (head-loss) loss during the pipe flow (Figure 3). These derivatives of the flow-variables must have the variable impacts on the head-losses for surface roughness of the pipe flow, from smooth to rough pipes [1]. In finding the flow-derivatives, general principles of pipe flow in the distribution network are applied. Also the momentum balance concept has obviously taken place its position in the derived methodology. The conservation of energy law creates this balance [2]. With the derived derivatives/differentials, the inner features about the applicability and inter-relationship amongst the two formulas have to come to view.

Regarding application of this study, it should be mentioned here that by the outcome of this study the general method of design like Hardy-Cross method and etc. could be developed in estimating and designing the design flow variables particularly for the subjective goals discussed in this paper indeed. This study's derived formulas are having better flexibility to satisfy the design suitability in obtaining and reaching at the better design optimization[3], [4].

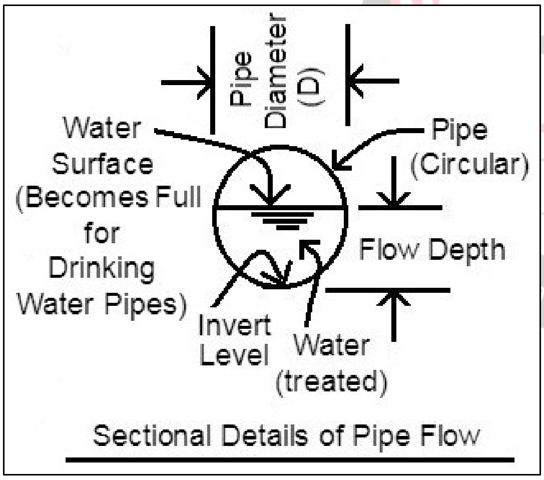

Figure 2: A Water Distribution Pipe

(Length of Pipe is Normal to the Diagram)

Although, it is not the scope to lay down a design approach based on the derived equations of this study, but it could well be comprehended about its usefulness, derived forms and the differential impacts on the design. This study has derived equations are equally to be useful in the pipe section (like section 1, 2, etc. as shown in the Figure 2) and the subjective fields of the pipe flow wherein the similar situation is prevailed of [5], [6]. In fact, the findings are caused by the underlying physicality occurred as described by the Figure 2.

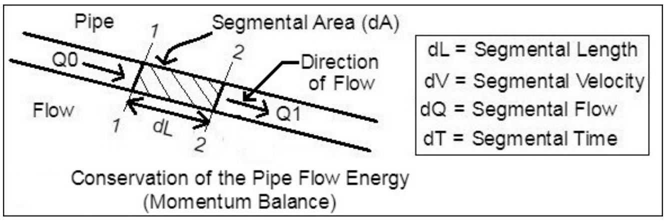

Figure 3: Differential Attributes to Pipe Flow

II. Goals of this Study

The goals to be derived and established by this study are –

a) To search and extract the flow factorials required in deriving the required invention.
b) To give internal perspective views of the pipe flow in the distribution network of water supply.
c) To facilitate idea of design equation required for the system design of the network.
d) To reach at and comprehend the design mobility using the invention in the pipe flow.

III. Assumption

a) The flow of pipe is continuous & follows the continuity principle of flow.
b) The fundamental law of velocity, V=L/T, is applicable for the subject of average flow; where L & T be the length & time of flow respectively.
c) The pipe is circular and having the full flow always.

IV. Methodology

Differentiation gives the inner sight of every physical happening. This study has enunciated one of kind. Here, the formulation used by the conventional formula of the Darcy-Weisbach (D-W) is discussed by using various applications through its required derivatives. This study, although theoretical, has established all the possible effects that could be drawn up by the D-W formula for the goals to be met. In the foregoing discussion it'll be found about this study's limitation, bindings, application zones etc. based on which the related fields as well as the regions of its application could be transparently comprehended. The Eq.(1) is one such that have caused and defined this study to be within the limiting working barriers and it indicates the related field of use in particular. $K = \left(\frac{1}{470}\right)\left(\frac{L}{D^{4.87}}\right)$ ...(1); where, K = coefficient of pipe roughness.

In formulating the derivation, this equation, Eq.(1), has been found to be as useful one to propagate with and establish the subsequent formulation[7]. As it is well known that the Eq.(1) comes from the Hazen-Williams equation, its particular way and field of use should be cognized accordingly in the required places.

Regarding the Figures given in this study, the Figure 1 is a typical layout diagram for a water supply system under the system process wherein the network distribution is shown for having the idea of the direction of flow process from the

 DOI : 10.18231/2454-9150.2019.0368

TWST to the city supply pipes. The subjective specialty and generous applicability of this study is shown by a short description through sectional pipe flow (Figure 2) and its dimensional flow features. And, the Figure 3 is the flow diagram giving the sectional details of the flow variables in order to maintain the particular momentum balance as derived and obtained by the energy law and the continuity principle of the pipe flow – all the differentials which is the primary basement of this study are derived from this. Thereby, all the physical diagrams/Figures given in this study are inter-related to each other, entirely and are, indeed, not in separation. The notations used in this study are –

D = Flow-Depth in Pipe.

L = Pipe Length of Flow.

Q = Flow in pipe.

H_L= Head Loss.

K = Function of L and D = Flow Co-efficient of pipe.

V = Velocity of Pipe Flow.

A = Area of Pipe Flow.

x = constant in the D-W equation.

Now, here, it is started the methodology of this study and the Figures provided must be in relation to the concerned, for consultation, invariably:

For a pipe flow in a distribution network, the head-loss (H_L) in pipe flow is proportional to the flow quantity the pipe discharges or allows to pass through its inner diameter or surfaces. In this regard, the D-W formula [8] is given by, $H_L=KQ^x$; where, x = constant (1.5 for the H-W formula and 2.0 for the D-W formula) ; K = Constant of proportionality = Co-efficient of roughness.

The D-W, in terms of the flow (Q) is, $Q = \left(\frac{H_L}{K}\right)^{\left(\frac{1}{x}\right)}$

Differentiating Q with respect to H_L (considering K as variable),

$$\left(\frac{dQ}{dH_L}\right) = \left(\frac{1}{x}\right)\left(\frac{H_L}{K}\right)\left[\frac{K - H_L\left(\frac{dK}{dH_L}\right)}{K^2}\right]^{\left(\frac{1}{x}-1\right)} \quad \text{... (2)}$$

$$\text{Or,}\left(\frac{dQ}{dH_L}\right) = \left(\frac{1}{x}\right)\left(\frac{H_L}{K}\right)\left[\frac{K-H_L\left(\frac{dQ}{dH_L}\right)\left(\frac{dK}{dQ}\right)}{K^2}\right]^{\left(\frac{1}{x}-1\right)} \quad \text{... (3)}$$

$$\text{Or,}\left(\frac{dQ}{dH_L}\right) = \left(\frac{Q^x}{x}\right)\left[\frac{1}{K} - \left(\frac{H_L}{K^2}\right)\left(\frac{dQ}{dH_L}\right)\left(\frac{dK}{dQ}\right)\right]^{\left(\frac{1-x}{x}\right)}$$

$$\text{Or,}\left(\frac{dQ}{dH_L}\right) = \left(\frac{1}{x}\right)\left[\left(\frac{Q^x}{K}\right) - \left(\frac{Q^{1+x}}{K}\right)\left(\frac{dQ}{dH_L}\right)\left(\frac{dK}{dQ}\right)\right]^{\left(\frac{1-x}{x}\right)}$$

Assuming$\left(\frac{dQ}{dH_L}\right) = J$,

$$J^{\left(\frac{x}{1-x}\right)} + \left(\frac{Q^{1+x}}{K}\right)\left(\frac{dK}{dQ}\right)J - \left(\frac{1}{x}\right)\left(\frac{Q^x}{K}\right) = 0$$

Treating as a quadratic equation (x>0), its roots are

$$J = \left(\frac{dQ}{dH_L}\right)$$

$$= \left(\frac{1}{2}\right)\left[-\left(\frac{Q^{1+x}}{K}\right)\left(\frac{dK}{dQ}\right) \pm \sqrt{\left(\frac{Q^{1+x}}{K}\right)^2\left(\frac{dK}{dQ}\right)^2 + 4\left(\frac{1}{x}\right)\left(\frac{Q^x}{K}\right)}\right] \quad \text{... (4)}$$

By the Hit and Trial Method it is always easily the required variable could be derived out and determined. In the result and discussion segment, given later, the graphical estimations given could also be applied for the Eq.(4) in order to find out the related and required constants of it and its estimation. Regarding the ratio (dK/dQ), it is informed here to see through the elaboration given afterwards.

The ratio (dH_L/dQ) is also found out by its unit concept which by dimensionally, is TL^{-2}, where L = Flow length in metre (m); and T = Time of Flow in sec. in the pipe flow. Dimensionally, the unit of the ratio is,$\left(\frac{dH_L}{dQ}\right) = \frac{L}{L^3/T} = \left(\frac{T}{L^2}\right)$. Applying the suitable flow variables to the ratio in accordance to the functional validity and by units, it, (dH_L/dQ), could be estimated and used through the involved equation to network design. Various other ratios might also be determined by applying this philosophy of unit/dimensional implementation.

Now, again, from the Eq.(3),

$$K^2\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)} = \left(\frac{1}{x}\right)(Q^x)\left[K - (H_L)\left(\frac{dQ}{dH_L}\right)\left(\frac{dK}{dQ}\right)\right]$$

$$\text{Or,}\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)} = \left(\frac{1}{x}\right)\left(\frac{Q^x}{K}\right) - \left(\frac{H_L}{K^2}\right)\left(\frac{Q^x}{x}\right)\left(\frac{dQ}{dH_L}\right)\left(\frac{dK}{dQ}\right)$$

$$\text{Or,}\left(\frac{1}{x}\right)\left(\frac{Q^x}{K}\right) = \left(\frac{dQ}{dH_L}\right)\left(\frac{1}{x}\right)\left(\frac{1}{K^2}\right)\left(\frac{dK}{dQ}\right)(H_L)^2 + \left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)}$$

$$\text{Or,}Q^x = \left(\frac{dQ}{dH_L}\right)\left(\frac{1}{K^2}\right)\left(\frac{dK}{dQ}\right)(H_L)^2 + (xK)\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)} \quad \text{... (5)}$$

This is another form of the Eq.(2). It is thereby clear that the various equations are possible, within the domain of the functional variables (of the dK/dH_L or else as necessary as suitable) under the subjections, using the Eq.(2) by the kinds of formation of the variables by variety of the ratios. This explanation along with the trial and error method and the graphical way of application has demanded this study to explore it to be more with lots of pertinent research scopes. Eq.(5) is the equation in terms of the flow discharge (Q) and others and is thereby having the implication of the functional flow variables (Table 1). The term (dK/dH_L) is thereby the responsible one to have the equations of variety as like the Eq.(5). In the discussion of these differentials, the Figure 3 is to be its related Figure of the consultation, although its details are given in foregoing discussion.

DOI : 10.18231/2454-9150.2019.0368

ISSN : 2454-9150 Vol-05, Issue-01, April 2019

To so far this study has discussed about the development of the various useful equations of the design, now a similar but different by the kind is explained here with its related functional form that could be applied in conventional design method of the network design.

In the network system of distribution in a water supply scheme, the term 'pipe-loop' is defined as a framework [5] consisting of no. of pipes carrying flow variables (Figure 1). In this regard, the Darcy-Weisbach equation ($H_L = KQ^x$) after derivating the H_L with respect to Q is found to be as,

$$\left(\frac{dH_L}{dQ}\right) = \frac{d}{dQ}(KQ^x) = K\left(\frac{dQ^x}{dQ}\right) + Q^x\left(\frac{dK}{dQ}\right)$$

$$\text{Or,}\left(\frac{dH_L}{dQ}\right) = K(x)Q^{x-1} + Q^x\left(\frac{dK}{dQ}\right) = (x)\left(\frac{KQ^x}{Q}\right) + Q^x\left(\frac{dK}{dQ}\right)$$

$$\text{Or,}\left(\frac{dH_L}{dQ}\right) = (x)\left(\frac{H_L}{Q}\right) + (Q^x)\left(\frac{dK}{dQ}\right)$$

$$\text{Or,} H_L = \left(\frac{dH_L}{dQ}\right)\left(\frac{Q}{x}\right) - \left(\frac{Q^{x+1}}{x}\right)\left(\frac{dK}{dQ}\right) \qquad \dots(6)$$

It is thereby found that the general expression of $H_L = KQ^x$ could be presented by the Eq.(5) and Eq.(6) and theses are tabulated in the Table 1.

Determination Of The Ratio (dK/dQ): It is a different segment wherein the evaluation of the derivative of has been described, because of its usefulness on all along this study. The ratio (dK/dQ), once possible to be determined, should be used in the Eq. (6), for the design of the flow-variable as this study's further scope.

The equation of the K by the Hazen-William equation is

$$K = \left(\frac{1}{470}\right)\left(\frac{L}{D^{4.87}}\right)$$

where, L and D be the flow variables.

Differentiating K with respect to Q,

$$\frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{dL}{dQ}\right) - L\frac{d}{dQ}(D^{4.87})\right]$$

Or,

$$\frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{dL}{dQ}\right) - 4.87L(D^{3.87})\frac{d(D)}{dQ}\right] \qquad \dots(7)$$

Now, the components of the (dK/dQ) in Eq.(7) are estimated and determined as follows –

Estimation Of $\left\{\frac{d(D)}{dQ}\right\}$:

As considered the flow area of the pipe (Figure 2 and assumption), A = Pipe-flow area = $\Pi D^2/4$; D = pipe diameter and Value of $\Pi = 3.14$.

Derivating 'A' with respect to Q,

$$\frac{dA}{dQ} = \left(\frac{\Pi}{4}\right)\frac{d(D^2)}{dQ} = \left(\frac{\Pi}{4}\right)(2D)\frac{d(D)}{dQ} = \left(\frac{\Pi D}{2}\right)\frac{d(D)}{dQ} = \left(\frac{p}{2}\right)\left\{\frac{d(D)}{dQ}\right\}$$

$$\text{Or,} \frac{d(D)}{dQ} = \left(\frac{2}{p}\right)\left\{\frac{dA}{dQ}\right\}$$

By ignoring the negligible quantity,

$$\frac{d(D)}{dQ} = \left(\frac{2}{p}\right) = \frac{0.636}{D} = 0.563(A)^{-(1/2)} = \left(\frac{D}{2A}\right) \qquad \dots(8)$$

where, p = perimeter of the pipe-flow = $\Pi(D)$.

$$\text{and, } A = \frac{\Pi D^2}{4} = \frac{(\Pi D)(D)}{4} = \frac{(p)(D)}{4}; \left(\frac{p}{A}\right) = \left(\frac{4}{D}\right)$$

Table 1: Methodological Formulation$

Pipe Flow (Q and/or H_L) Design Equation	
Applying (dQ/dH_L)	**Applying (dH_L/dQ)**
$(Q^x) = \left(\frac{dQ}{dH_L}\right)\left(\frac{1}{K^2}\right)\left(\frac{dK}{dQ}\right)(H_L)^2 + (xK)\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)}$	$H_L = \left(\frac{dH_L}{dQ}\right)\left(\frac{Q}{X}\right) - \left(\frac{Q^{x+1}}{X}\right)\left(\frac{dK}{dQ}\right)$

$subject to applicability of the general velocity propagation and continuity in flow distribution.

Estimation Of $\left\{\frac{Dl}{Dq}\right\}$:

From the definition of velocity of flow in the pipe of length L, velocity V = (L / T); T = unit time

Also by differential magnitude (Figure 3), dL = (dV)(dT)

Equating the value of V of the continuity equation $\left(V = \frac{Q}{A}\right)$ by the conventional equation $\left(V = \frac{L}{T}\right)$, $T = \frac{AL}{Q}$

Now, by derivating the L with respect to Q from the equation L = VT,

$$\frac{dL}{dQ} = \frac{d(VT)}{dQ} = V\frac{dT}{dQ} + T\frac{dV}{dQ} = V\left[\frac{dT}{(dV)(dA)}\right] + T\left[\frac{dV}{(dV)(dA)}\right]$$

$$\text{Or,} \frac{dL}{dQ} = V\left[\frac{dL}{(dV)^2(dA)}\right] + T\left[\frac{1}{(dA)}\right]$$

$$\text{Or,} \frac{dL}{dQ} = \left(\frac{Q}{A}\right)\left[\frac{dL}{(dV)^2(dA)}\right] + \left(\frac{AL}{Q}\right)\left[\frac{1}{(dA)}\right] \text{ where, } T = \left(\frac{AL}{Q}\right)$$

Ignoring the negligible values,

$$\frac{dL}{dQ} = \left(\frac{Q}{A}\right) + \left(\frac{AL}{Q}\right)\left(\frac{dL}{dQ}\right) = \left(\frac{Q^2 + A^2L}{AQ}\right)$$

 DOI : 10.18231/2454-9150.2019.0368

$$\text{Or, } \left(\frac{dL}{dQ}\right) = \left[\left(\frac{Q}{A}\right) + \frac{L}{\left(\frac{Q}{A}\right)}\right] \quad \text{... (9)}$$

The Eq.(9) is thereby the equation in which the effect of the term $\left[\frac{dL}{(dV)^2(dA)}\right]$ shall bring amount of the flow or discharge (Q), to some lower magnitudes by values. Because the term, on further application by its inter-related variables, leads to$\left[\frac{dL}{(dV)(dQ)} = \frac{dT}{(dQ)}\right]$ which is the differential magnitude of the flow (Q) & the flow-time (T). Henceforth the formulation of Q that is going to be furthered should be reckoned with by due consideration of this neglect as formed in forming the Eq.(9), especially to take order of its inclusion on needs line on quantification, precision, further research etc. etc.

Applying the Eq. (8) and Eq. (9), the Eq. (7) is finally here as

$$\frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{dL}{dQ}\right) - 4.87L(D^{3.87})\frac{d(D)}{dQ}\right]$$

$$\text{Or, } \frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{Q^2+A^2L}{AQ}\right) - 4.87L(D^{3.87})\left(\frac{0.636}{D}\right)\right]$$

$$\text{Or, } \frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{Q^2+A^2L}{AQ}\right) - 3L(D^{2.87})\right]$$

Or,

$$\left(\frac{dK}{dQ}\right) = \left(\frac{1}{470D^{6.87}}\right)\left[D^2\left(\frac{Q}{A} + \frac{L}{\left(\frac{Q}{A}\right)}\right) - 3L\right] \quad \text{... (10)}$$

The Eq. (10) is thereby the defining equation of ‘approximate’ kind, by nature of the formation. The actual value estimated from the Eq.(10) may however be less than its ‘absolute’ actual one as it is quite recognizable about the way of its finding in here from the Eq.(9).

Alternatively, by derivating and applying the flow-continuity principle into the equation $L = VT$,

$$\frac{dL}{dQ} = V\left(\frac{dT}{dQ}\right) + T\left(\frac{dV}{dQ}\right) = \left(\frac{Q}{A}\right)\left[\frac{dL}{(dV)^2(dA)}\right] + \left(\frac{AL}{Q}\right)\left[\frac{1}{(dA)}\right]$$

Now, as shown in the Figure 3, applying the momentum balance on the flow variables for a segmental length dL, where $dL = L1 - L0$, over the section 1-1 and section 2-2 could determine the expression of the (dL/dQ). In the Figure 3, a differential length along the pipe flow is considered to find out the differential attributions. This differential concept of the momentum balance is required to be applied to other variables like the dL and these are likedV $= V1 - V0$; $dA = A1 - A0$; $dQ = (Q1 - Q0)$ and so on. The balance is required for the required stability in the flow mechanism so far as the law of conservation of flow as well as the assumption of flow-continuity is so concerned.

It is mentioned that here the values Q1, A1, V1, L1 etc. are the final values(at the section 2-2 in the Figure 3) along the length of the segmental flow and Q0, A0, V0, L0 etc. are the respective initial values (at the section 1-1 in the Figure 3) of the flow-length in the distribution network pipes.

The final values of sectional flow should ‘ultimately’ take into the form of L, V, Q, A etc. which were in use of the application as derived in the equations obtained up to the Eq.(10). Thereby the final values should be regarded in terms of L, V, Q, A etc. of the consideration of the differential flow attributes and vice-versa.

Applying the flow attributions differentially (Figure 3),

$$\frac{dL}{dQ} = \left(\frac{Q1}{A1}\right)\left[\frac{(L1 - L0)}{(V1 - V0)^2(A1 - A0)}\right] + \left(\frac{A1}{Q1}\right)(L1)\left[\frac{1}{(A1 - A0)}\right]$$

Ignoring the initial sectional attributions with regards of the attainments of the final forms as it (flow) goes towards the flow direction,

$$\frac{dL}{dQ} = \left(\frac{Q1}{A1}\right)\left[\frac{(L1)}{(V1)^2(A1)}\right] + \left(\frac{L1}{Q1}\right)\left[\frac{A1}{(A1)}\right]$$

$$\text{Or, } \frac{dL}{dQ} = \frac{(L1)}{(Q1)} + \left(\frac{L1}{Q1}\right)$$

$$\text{Or, } \frac{dL}{dQ} = 2\left(\frac{L1}{Q1}\right) = 2\left(\frac{L}{Q}\right) = \left(\frac{2}{q1}\right) \quad \text{... (11)}$$

where, L0, A0, V0, Q0 be the flow variables whose corresponding dimensions after the differential changes are L, A, V and D (i.e., L1, A1, V1 and D1) and this is occurred due to the momentum changes in the pipe flow over the differential change (dT) in time T; the differential time change from its initial time (T0) of the reference is = dT = (T1 – T0).

and, q1 = the discharge per unit length (final) = (Q/L)

The Eq.(11) is thereby to be made into the application of the Eq.(7), as done by the Eq.(9).

Again by another alternative way of the formulation, the Eq.(9) is written as,

$$\frac{dL}{dQ} = \frac{d(VT)}{dQ} = V\left[\frac{d\left(\frac{AL}{Q}\right)}{dQ}\right] + T\left[\frac{d\left(\frac{Q}{A}\right)}{dQ}\right]$$

$$\text{Or, } \frac{dL}{dQ} = \frac{d(VT)}{dQ} = V\left[\frac{d\left(\frac{AL}{Q}\right)}{dQ}\right] + T\left[\frac{d\left(\frac{Q}{A}\right)}{dQ}\right] \quad \text{... (12)}$$

The terms of the Eq.(12) are obtained as,

$$\frac{d}{dQ}\left(\frac{Q}{A}\right) = \left[\frac{A - Q\frac{d}{dQ}(A)}{A^2}\right]$$

And,

 DOI : 10.18231/2454-9150.2019.0368

$$\frac{d}{dQ}\left(\frac{AL}{Q}\right)=\left[\frac{(Q)\frac{d}{dQ}(AL)-(AL)\frac{d}{dQ}(Q)}{Q^2}\right]$$

Or, $\frac{d}{dQ}\left(\frac{AL}{Q}\right)=\left[\frac{(Q)\left\{L\frac{d}{dQ}(A)+(A)\frac{d}{dQ}(L)\right\}-(AL)\frac{d}{dQ}(Q)}{Q^2}\right]$

Or, $\frac{d}{dQ}\left(\frac{AL}{Q}\right)=\left[\left(\frac{L}{Q}\right)\frac{d}{dQ}(A)+\left(\frac{A}{Q}\right)\frac{dL}{dQ}\right]-(AL)$

Now, from the Eq.(12),

$$\frac{dL}{dQ}=\left(\frac{V}{dQ}\right)\left[\left(\frac{L}{Q}\right)\frac{d}{dQ}(A)+\left(\frac{A}{Q}\right)\left(\frac{dL}{dQ}\right)-(AL)\right]+\left(\frac{T}{dQ}\right)\left[\frac{A-Q\frac{d}{dQ}(A)}{A^2}\right]$$

Or, $\left(\frac{dL}{dQ}\right)\left[1-\frac{VA}{QdQ}\right]$

$$=\left(\frac{V}{dQ}\right)\left[\left(\frac{L}{Q}\right)\frac{d}{dQ}(A)-(AL)\right]+\left(\frac{T}{dQ}\right)\left[\frac{A-Q\frac{d}{dQ}(A)}{A^2}\right]$$

Or, $\left(\frac{dL}{dQ}\right)\left[1-\frac{1}{dQ}\right]$

$$=\left(\frac{V}{dQ}\right)\left[\left(\frac{L}{Q}\right)\frac{d}{dQ}(A)-(AL)\right]+\left(\frac{T}{dQ}\right)\left[\frac{A-Q\frac{d}{dQ}(A)}{A^2}\right]$$

Or, $\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{LV}{dQ}\right)\left[\frac{1}{\left(1-\frac{1}{dQ}\right)}\right]\left[\left(\frac{1}{Q}\right)\frac{d}{dQ}(A)-(A)\right]+\left(\frac{T}{dQ}\right)\left[\frac{A-Q\frac{d}{dQ}(A)}{A^2}\right]$$

Multiplying $D^{4.87}$,

$D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4A}{\Pi}\right)^{2.435}\left[\left(\frac{LV}{dQ}\right)\left\{\frac{1}{\left(1-\frac{1}{dQ}\right)}\right\}\left\{\left(\frac{1}{Q}\right)\frac{d}{dQ}(A)-(A)\right\}+\left(\frac{T}{dQ}\right)\left\{\frac{A-Q\frac{d}{dQ}(A)}{A^2}\right\}\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4A}{\Pi}\right)^{2.435}\left[(LV)\left\{\frac{1}{(dQ-1)}\right\}\left\{\left(\frac{1}{Q}\right)\frac{d}{dQ}(A)-(A)\right\}+\left(\frac{T}{dQ}\right)\left(\frac{1}{A}\right)-\left(\frac{T}{A^2}\right)\frac{d}{dQ}(A)\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4}{\Pi}\right)^{2.435}\left[(LV)\left\{\frac{1}{(dQ-1)}\right\}\left\{\left(\frac{1}{Q}\right)\frac{d}{dQ}(A^{3.435})-(A^{3.435})\right\}+\left(\frac{T}{dQ}\right)(A^{1.435})-\left(\frac{T}{A^2}\right)\frac{d}{dQ}(A^{3.435})\right]$$

For a particular velocity V in the pipe-flow,

$D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4}{\Pi}\right)^{2.435}\left[(LV)\left\{\frac{1}{(dA-1)}\right\}\left\{\left(\frac{1}{Q}\right)\frac{d}{dA}(A^{3.435})-(A^{3.435})\right\}+\left(\frac{T}{dA}\right)(A^{1.435})-\left(\frac{T}{A^2}\right)\frac{d}{dA}(A^{3.435})\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4}{\Pi}\right)^{2.435}\left[(LV)\left\{\frac{1}{(dA-1)}\right\}\left\{\left(\frac{1}{Q}\right)(3.435)(A^{2.435})-(A^{3.435})\right\}+\left(\frac{T}{dA}\right)A^{1.435}-3.435\left(\frac{T}{A^2}\right)(A^{2.435})\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)$

$$=\left(\frac{4}{\Pi}\right)^{2.435}\left[LV\left\{3.435\left(\frac{1}{Q}\right)A^{2.435}-A^{3.435}\right\}(dA-1)+\left(\frac{T}{dA}\right)A^{1.435}(dA-1)-(3.435)TA^{0.435}(dA-1)\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)=$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left[(LV)\left\{\left(\frac{1}{Q}\right)3.435(A^{2.435})dA-A^{3.435}dA-\left(\frac{1}{Q}\right)(3.435)(A^{2.435})dA+A^{3.435}\right\}+T(A^{1.435})-\left(\frac{T}{dA}\right)A^{1.435}-(3.435)(T)A^{0.435}(dA)+(3.435)TA^{0.435}\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)=$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left[L(3.435)(A^{1.435})dA-(LQ)(A^{2.435})dA-(L)(3.435)(A^{1.435})dA+(LQ)(A^{2.435})+(T)(A^{1.435})-\left(\frac{T}{dA}\right)(A^{1.435})-(3.435)(T)(A^{0.435})(dA)+(3.435)(T)(A^{0.435})\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right)=$

DOI : 10.18231/2454-9150.2019.0368

$$\left(\frac{4}{\Pi}\right)^{2.435}[L(3.435)(A^{2.435}) - (LQ)(A^{3.435}) - (L)(3.435)(A^{2.435}) + (LQ)(A^{2.435}) + (T)(A^{1.435}) - (T)(A^{0.435}) - (3.435)(T)(A^{1.435}) + (3.435)(T)(A^{0.435})]$$

where, the quantity (dA/A) is considered negligible.

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left[L(3.435)(A^{2.435}) - (LQ)(A^{3.435}) - (L)(3.435)(A^{2.435}) + (LQ)(A^{2.435}) + \left(\frac{L}{Q}\right)(A^{2.435}) - \left(\frac{L}{Q}\right)(A^{1.435}) - (3.435)\left(\frac{L}{Q}\right)(A^{2.435}) + (3.435)\left(\frac{L}{Q}\right)(A^{1.435})\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left[(LQ)\{(A^{2.435}) - (A^{3.435})\} + \left(\frac{L}{Q}\right)\{(A^{2.435}) - (A^{1.435}) - (3.435)(A^{2.435}) + (3.435)(A^{1.435})\}\right]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left(\frac{L}{Q}\right)(A^{1.435})[(Q^2)(A - A^2) + \{A - 1 - (3.435)A + (3.435)\}]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left(\frac{L}{Q}\right)A^{1.435}[(Q^2)(A - A^2) - (2.435)(A - 1)]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}\left(\frac{L}{Q}\right)(A^{1.435})[(Q^2)A(1 - A) + (2.435)(1 - A)]$$

Or, $D^{4.87}\left(\frac{dL}{dQ}\right) =$

$$\left(\frac{4}{\Pi}\right)^{2.435}(A^{1.435})(1 - A)(Q^2A + 2.435)\left(\frac{L}{Q}\right) \quad \text{... (13)}$$

Applying the Eq.(13) and the Eq.(7),

$$\frac{dK}{dQ} = \frac{1}{470D^{9.74}}\left[D^{4.87}\left(\frac{dL}{dQ}\right) - 4.87L(D^{3.87})\frac{d(D)}{dQ}\right]$$

Or, $\frac{dK}{dQ} =$

$$\frac{1}{470D^{9.74}}\left[\left(\frac{4}{\Pi}\right)^{2.435}(A^{1.435})(1 - A)(Q^2A + 2.435)\left(\frac{L}{Q}\right) - (3.101)LD^{2.87}\right]$$

where,

$$4.87L(D^{3.87})\frac{d(D)}{dQ} = 4.87L(D^{3.87})\left(\frac{2}{\Pi D}\right) = \left(\frac{9.74}{\Pi}\right)(L)(D^{2.87})$$

Or, $4.87L(D^{3.87})\frac{d(D)}{dQ} = (3.101)LD^{2.87}$

Or, $\frac{dK}{dQ} =$

$$\frac{L}{470D^{9.74}}\left[(1.80)(A^{1.435})(1 - A)(Q^2A + 2.435)\left(\frac{1}{Q}\right) - (3.101)D^{2.87}\right]$$

Or, $\frac{dK}{dQ} =$

$$\frac{L}{1528\ A^{4.87}}\left[1.80(A^{1.435})(1 - A)(Q^2A + 2.435)\left(\frac{1}{Q}\right) - (4.38)A^{1.435}\right]$$

where, for the circular pipe of full flow (Figure 2), Flow Area $= A = 0.785D^2$; And, $470D^{9.74} = (1527.5)A^{4.87} \approx (1528)A^{4.87}$

Or, $\frac{dK}{dQ} =$

$$\left[(1.80)(A^{2.435})(1 - A)Q + 4.383\left(\frac{1 - A}{Q}\right)(A^{1.435}) - (4.38)A^{1.435}\right]\left[\frac{L}{1528\ A^{4.87}}\right]$$

Or, $\frac{dK}{dQ} =$

$$\frac{L}{1528\ A^{4.87}}\left[(1.80)(A^{2.435})(1 - A)Q - 4.383\left(\frac{A^{2.435}}{Q}\right) + (4.38)A^{1.435}\left(\frac{1}{Q} - 1\right)\right]$$

Or, $\frac{dK}{dQ} =$

$$\frac{L}{1528\ A^{4.87}}(1.80)\left[\left(\frac{1}{Q} - \frac{A}{Q}\right)Q^2 - (2.435)\left(\frac{1}{Q}\right)A^{2.435} + (2.435)\left(\frac{1}{Q} - 1\right)A^{1.435}\right]$$

Or, $\frac{dK}{dQ} =$

$$\frac{L}{(848.89)\ A^{4.87}}\left[A^{2.435}\left\{\left(\frac{1}{Q} - \frac{A}{Q}\right)Q^2 - (2.435)\left(\frac{1}{Q}\right)\right\} + 2.435(A^{1.435})\left(\frac{1}{Q} - 1\right)\right] \quad \text{... (14)}$$

Thereby, the design formulations of the flow variables, for the Eq.(6) could be determined by equations, as derived by the equations up to the Eq.(14), through the application of the differentials by the related variables and its functional applicability and suitably. The required variables in their differential forms are given in the Table 2.

It is now the stage of accomplishment of finding of all the required presentation. With these, not only the usefulness of the Eq.(5) and Eq.(6) is done, but also the design along with various future scopes of it could be done through several applications. And, this research paper has brought that knowledge into the limelight such that the distribution network design could be viewed so closely.

 DOI : 10.18231/2454-9150.2019.0368

ISSN : 2454-9150 Vol-05, Issue-01, April 2019

In the entire formulation of the inter-related relations, all the associated parameters have been treated as variables & otherwise.

This study has, in this way become, a resourceful article in having the objectives well with its various possible scopes of research application. Thereby, it provides the various insights as well as the applicability of them.

Table 2: Essentials Of The (dK/dQ) Determination

Functional Variables of (dK/dQ)^		
$\frac{d(D)}{dQ}$	$\left(\frac{dL}{dQ}\right)$	
	The General Form	Further Derived Form
$\left(\frac{D}{2A}\right) = \frac{0.636}{D}$ (for circular pipe)	$\left[\left(\frac{Q}{A}\right) + \frac{L}{\left(\frac{Q}{A}\right)}\right]$	$\left(\frac{1}{D^{4.87}}\right)\left[\left(\frac{4}{\Pi}\right)^{2.435}(A^{1.435})(1 - A)(Q^2A + 2.435)\left(\frac{L}{Q}\right)\right]$

^subject to the formulation, T = (AL/Q); also by differential implication.

V. Results and Discussion

Following is the output of this research paper, which, as formulated and discussed, has been aimed to derive out the differential implication on the subjective integrity:

a) Results derived out from each of the derived equations should be compared amongst and it also could be interesting.

b) Design equations should be tested and verified by various Model tests in laboratory. Experiments could be hoped to be giving much insights and transparent ideas about the derived equations about their nature, behavior and theirs patterns on extremities etc.

c) The magnitudes of the values, determined by each of the derived equations, are intended to be giving kinds of values as it should provide indeed and with this intension and others the model tests and etc. of the experimentations are required whose results might change or correct the derived 'correctly' determinations of this study.

d) Graphical description is given as obtained and derived in this study. In this explanation, the straight-line method of the co-ordinate geometry is applied. It is here analyzed, of the Eq.(5) with its (dQ/dH_L) effect, by the co-ordinate theory of the straight line equation, Y = MX + C where M = gradient and C = intercept. A graph of $(H_L)^2$ versus (Q^x) could be plot along the X and Y axis respectively, as shown in the Figure 4 to derive out the constants such as Gradient= $M = \left(\frac{dQ}{dH_L}\right)\left(\frac{1}{K^2}\right)\left(\frac{dK}{dQ}\right)$; Intercept $= C = (xK)\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)}$. From the Eq.(6), the head-loss with its (dH_L/dQ) effect is, $(H_L) = -\left(\frac{dK}{dQ}\right)\left(\frac{Q^{x+1}}{x}\right) + \left(\frac{dH_L}{dQ}\right)\left(\frac{Q}{x}\right)$.

In this case, the straight-line theory of the geometry gives the gradient M as [(-)(dK/dQ)] and the intercept C value as (dH_L/dQ)(Q/x) while the graph is plotted as (H_L) versus $[Q^{(x+1)}]$(1/x) along the X and Y direction respectively (Figure 5).

All these geometrical information is tabulated in a tabulation in the Table 3. The way as the derived equations are explained here (by the Table 3) could also be described by various other ways by choosing the axial values suitably and in that cases, the related corresponding constants could be found out – it is desirably and suitably on the needs and application approaches.

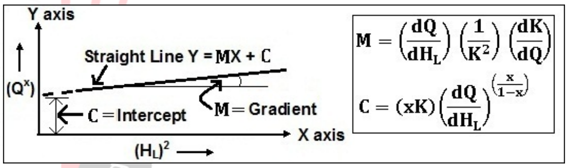

Figure 4: The Graph of $(H_L)^2$ versus (Q^x)

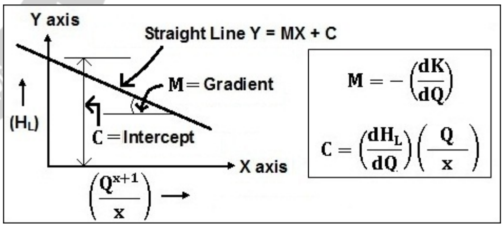

Figure 5: The Graph of (H_L) versus $\left(\frac{Q^{x+1}}{x}\right)$

e) The values derived out from the given Figure (Figure 4, Figure 5) could be compared amongst themselves – each of the values could also be estimated or designed by the statistical equilibrium of equations, known and unknowns.

VI. Conclusion

Hereby, the following should be the concluding points, in addition to the possible others given and discussed in this research paper at the subjective places:

a) Factorials and its determinations are the attraction of this study. As the magnitudes differ by the factorial's presentative formation, at whatever it'd likely or be meant to be, the consequent results shall

DOI : 10.18231/2454-9150.2019.0368

ISSN : 2454-9150 Vol-05, Issue-01, April 2019

be accordingly. There may also be several scopes of this flexibility.

b) The design equation as derived in this study could be used to prepare small to large scale models in laboratory – this making & testing should give the confidence of this study, although this presentation could be presented in the later researches.

c) The way the Eq.(5) is developed in terms of (dK/dQ) from the general consequent formation could also be applied suitably to the formulation process in order to generate the formulation of those kinds which should be equally applicable. In that, the various other formations using dK, dQ, dV, dD, dH_L, dA etc. are possible to be formed, by using alternatingly in the ratio form in the denominator and numerator, and applied in its subsequent subjective cases of importance.

Table 3. Graphical Determination Of Constants@

Subjecting Condition	Straight-line Co-ordinate Method				The Figure
	Axial values		The Constants		
	Along X	Along Y	M value – the Gradient	C value – the Intercept	
Applying (dQ/dH_L)	$(H_L)^2$	(Q^x)	$\frac{\left(\frac{dQ}{dH_L}\right)}{\left(\frac{1}{K^2}\right)\left(\frac{dK}{dQ}\right)}$	$\frac{(xK)}{\left[\left(\frac{dQ}{dH_L}\right)^{\left(\frac{x}{1-x}\right)}\right]}$	Figure 4
Applying (dH_L/dQ)	(H_L)	$\left(\frac{Q^{x+1}}{x}\right)$	$(-)\left(\frac{dK}{dQ}\right)$	$\left(\frac{dH_L}{dQ}\right)\left(\frac{Q}{x}\right)$	Figure 5

@subject to the applicability of the methodological equation as given in the Table 1.

d) All the findings are given in their related tables & graphs & these are at their own extremes so far as discussed in this study.

VII. FUTURE SCOPE

Following future scopes could be its purpose of use:

a) After having all the equations as established by this study, it is now to be going to prepare the equations on desired basis suitably which could develop various interesting pipe flow mechanism.

b) Inter-relation and inter-mixing of the design equations (Table 1) might be done on desirably suitable basis.

c) The ratio (dK/dQ) is described and developed by this study which could be applied to desired level (by its formulation approach as well as by the derived formation) of where a particular factorial is required to be brought into.

d) The experimentation on the famous popular fundamental formulas has never been done elsewhere. So, this study is hereby regarded as the research breakthrough which could enlighten the future levels of the stage of development not only in the water supply sector, but also to the other fields where this study's methodological applications and approaches could be a light in the dark.

REFERENCE

[1] Connell. David, August 2001., Hazen-Williams C-factor Assessment in an Operational Irrigation Pipeline., Department of Agricultural and Bio-systems Engineering McGill University, Montreal.

[2] Yasmina. LAHIOUEL, Rachid. LAHIOUEL, EVALUATION OF ENERGY LOSSES IN PIPES, URL: http://documents.irevues.inist.fr/bitstream/handle/2042/57485/64815.pdf?sequence=1.

[3] Prasanta Biswas, " Iterative Methodology In the Proportionate 'Design' Flow-Kinetics of Water Supply Network of Distribution Zones", *International Journal of Scientific Research in Science, Engineering and Technology*(IJSRSET), Print ISSN : 2395-1990, Online ISSN : 2394-4099, SJIF – 4.916, UGC approved: Journal No. 47147, Vol. 4, Issue 4, Edition: March-April 2018, Page No.:1294-1300.URL: http://ijsrset.com/IJSRSET1844446.php

[4] Prasanta Biswas, "Innovative method of Finding Discharge, Velocity and Diameter Simultaneously in the Drinking Water Supply Distribution Network", *International Journal of Research* (IJR) – EduPedia Publications Pvt. Ltd., ISSN(print): 2348-6848, SJIF – 5.60, Index Copernicus Value (ICV) – 100, UGC approved: Journal No. 44396, Vol. 5, Issue 12, Edition: April-2018, Page No.: 4177-4184. URL:https://edupediapublications.org/journals/index.php/IJR/.

[5] Brkić. Dejan, Similarities and Differences in Optimization of Water- and Gas Distribution Pipeline Networks, Ministry of Science and Technological Development of Serbia., DOI:10.20944/preprints201808.0278.v1.

[6] Yasmina. LAHIOUEL, Rachid. LAHIOUEL, Zitounic AZARI, 2015, Simultaneous layout and pipe size optimization of fluid distribution networks, 22 ème Congrès Français de Mécanique , AFM, Association Française de Mécanique, URL: http://hdl.handle.net/2042/57131.

[7] Garg, S. K., 2014.,Water Supply Engineering.(Vol.I), Khanna Publishers., New Delhi Pin 110006., India.

[8] Allen. R.G, 1996, Relating the Hazen-Willaims and Darcy-Weisbach Friction Loss Equations for Pressurized Irrigation, *American Society of Agricultural Engineers* (ISSN: 0883-8542), Vol.12(6):685-693.

 DOI : 10.18231/2454-9150.2019.0368