Some Conditions on J_S Normal Matrices

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Abstract A matrix $A \in M_n(C)$ is called J_S - normal if $A^{\#}A = AA^{\#}$, where $A^{\#} = JA^{\theta}J$, and $J = I_r \oplus -I_{n-r}$ [1]. Here we derived some properties of J_S -normal matrices.

Keywords —Secondary transpose, Conjugate Secondary transpose, J_S - normal matrix, J_S – Symmetric matrix.

I. Introduction And Preliminaries

Throught this paper we use this following notations.

Notation 1.1 [2,3]. The secondary transpose (Conjugate secondary transpose) of A is defined by $A^S = VA^TV[A^{\theta} = VA^*V]$, where V is the fixed disjoint permutation matrix within its secondary diagonal.

Notation 1.2. Let □ be the set of all real number, □ be the set of all complex be the set of all complex numbers, and $M_n(F)$ be the $n \times n$ matrices over field. Let J_S^n be the set of all $n \times n$, J_S -normal matrices.

II. Main Results

Theorem 2.1. *Let $A \in J_s^n$, then*

1. $S^{-1}K = KS^{-1}$ is J_S - normal.
2. $K^{-1}S = SK^{-1}$ is J_S - normal.
3. $A\overline{S} + SA^{\#} = 2S\overline{S}$ is J_S - normal.
4. $A\overline{K} - KA^{\#} = 2K\overline{K}$ is J_S - normal.
5. $S^{-1}A + A^{\#}S^{-1} = 2I$ is J_S - normal.
6. $K^{-1}A + A^{\#}\overline{K}^{-1} = 2I$ is J_S - normal.
7. $HK = KH$ is J_S - normal.
8. $AH = HA$ is J_S - normal.
9. $AH + HA^{\#} = 2H^2$ is J_S - normal.
10. $AK - KA^{\#} = 2K^2$ is J_S - normal.
11. $H^{-1}K = KH^{-1}$ is J_S - normal.
12. $HK^{-1} = K^{-1}H$ is J_S - normal.
13. $H^{-1}A + A^{\#}H^{-1} = 2I$ is J_S - normal.
14. $K^{-1}A - A^{\#}K^{-1} = 2I$ is J_S - normal.
15. $S\overline{S} - K\overline{K} = AA^{\#}$ is J_S - normal.

Proof of Condition 1.

Let $A = S^{-1} + K$ and $A^{\#} = S^{-1} - K$, so $A^{\#}A = AA^{\#}$

$\Leftrightarrow (S^{-1} - K)(S^{-1} + K) = (S^{-1} + K)(S^{-1} - K)$

$\Leftrightarrow S^{-1}S^{-1} + S^{-1}K - KS^{-1} - KK = S^{-1}S^{-1} - S^{-1}K + KS^{-1} - KK$

$\Leftrightarrow S^{-1}K - KS^{-1} = -S^{-1}K + KS^{-1}$

$\Leftrightarrow S^{-1}K + S^{-1}K = KS^{-1} + KS^{-1}$

$\Leftrightarrow 2S^{-1}K = 2KS^{-1}$

$\Leftrightarrow S^{-1}K = KS^{-1}$

Proof of Condition 2.

Let $A = S^{-1} + K$ and $A^{\#} = S^{-1} - K$, so $A^{\#}A = AA^{\#}$

$\Leftrightarrow (S - K^{-1})(S + K^{-1}) = (S + K^{-1})(S - K^{-1})$

$\Leftrightarrow SS + SK^{-1} - K^{-1}S - K^{-1}K^{-1} = SS - SK^{-1} + K^{-1}S - K^{-1}K^{-1}$

$\Leftrightarrow SK^{-1} - K^{-1}S = -SK^{-1} + K^{-1}S$

$\Leftrightarrow SK^{-1} + SK^{-1} = K^{-1}S + K^{-1}S$

$\Leftrightarrow 2SK^{-1} = 2K^{-1}S$

$\Leftrightarrow SK^{-1} = K^{-1}S$

Proof of Condition 3.

Let $A = S + K$ and $A^{\#} = S - K$, so $A\overline{S} + SA^{\#} = 2S\overline{S}$

$\Leftrightarrow (S + K)\overline{S} + S(S - K) = 2S\overline{S}$

$\Leftrightarrow S\overline{S} + K\overline{S} + SS - SK = 2S\overline{S}$

$\Leftrightarrow S\overline{S} + K\overline{S} + S\overline{S} - \overline{S}K = 2S\overline{S}$

$\Leftrightarrow S\overline{S} + K\overline{S} + S\overline{S} - K\overline{S} = 2S\overline{S}$

$\Leftrightarrow 2S\overline{S} = 2S\overline{S}$

Proof of Condition 4.

Let $A = S + K$ and $A^{\#} = S - K$, so $A\overline{K} - KA^{\#} = 2K\overline{K}$

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$\Leftrightarrow (S+K)\overline{K} - K(S-K) = 2K\overline{K}$
$\Leftrightarrow S\overline{K} + K\overline{K} - KS + KK = 2K\overline{K}$
$\Leftrightarrow SK + K\overline{K} - KS + KK = 2K\overline{K}$
$\Leftrightarrow SK + K\overline{K} - SK + K\overline{K} = 2K\overline{K}$
$\Leftrightarrow 2K\overline{K} = 2K\overline{K}$

Proof of Condition 5.

Let $A = S + K$ and $A^{\#} = S - K$, so $S^{-1}A + A^{\#}S^{-1} = 2I$
$\Leftrightarrow S^{-1}(S+K) + (S-K)S^{-1} = 2I$
$\Leftrightarrow S^{-1}S + S^{-1}K + SS^{-1} - KS^{-1} = 2I$
$\Leftrightarrow S^{-1}S + S^{-1}K + SS^{-1} - S^{-1}K = 2I$
$\Leftrightarrow 2S^{-1}S = 2I$
$\Leftrightarrow 2I = 2I$

Proof of Condition 6.

Let $A = S + K$ and $A^{\#} = S - K$, so $K^{-1}A - A^{\#}\overline{K}^{-1} = 2I$
$\Leftrightarrow K^{-1}(S+K) - (S-K)\overline{K}^{-1} = 2I$
$\Leftrightarrow K^{-1}S + K^{-1}K - S\overline{K}^{-1} + K\overline{K}^{-1} = 2I$
$\Leftrightarrow SK^{-1} + K^{-1}K - SK^{-1} + KK^{-1} = 2I$
$\Leftrightarrow SK^{-1} + KK^{-1} - SK^{-1} + KK^{-1} = 2I$
$\Leftrightarrow 2KK^{-1} = 2I$
$\Leftrightarrow 2I = 2I$

Proof of Condition 7.

Let $A = H + K$ and $A^{\#} = H - K$, so $A^{\#}A = AA^{\#}$
$\Leftrightarrow (H-K)(H+K) = (H+K)(H-K)$
$\Leftrightarrow HH + HK - KH - KK = HH - HK + KH - KK$
$\Leftrightarrow HK - KH = -HK + KH$
$\Leftrightarrow HK + HK = KH + KH$
$\Leftrightarrow 2HK = 2KH$
$\Leftrightarrow HK = KH$

Proof of Condition 8. Let $A = H + K$ and $A^{\#} = H - K$, so $A^{\#}A = AA^{\#}$. Necessary Condition. By Condition 7:
$\Leftrightarrow AH = (H+K)H$
$\Leftrightarrow AH = HH + KH$
$\Leftrightarrow AH = H(H+K)$
$\Leftrightarrow AH = HK.$

Sufficient Condition.

$\Leftrightarrow AH = HA$
$\Leftrightarrow (H+K)H = H(H+K)$
$\Leftrightarrow HH + KH = HH + HK$
$\Leftrightarrow KH = HK$
$\Leftrightarrow HK = KH.$

Proof of Condition 9. Let $A = H + K$ and $A^{\#} = H - K$, so $AH + HA^{\#} = 2H^2$
$\Leftrightarrow (H+K)H + H(H-K) = 2H^2$
$\Leftrightarrow HH + KH + HH - HK = 2H^2$
$\Leftrightarrow HH + HK + HH - HK = 2H^2$
$\Leftrightarrow 2HH = 2H^2$
$\Leftrightarrow 2H^2 = 2H^2$

Proof of Condition 10. Let $A = H + K$ and $A^{\#} = H - K$, so $AK - KA^{\#} = 2K^2$
$\Leftrightarrow (H+K)K - K(H-K) = 2K^2$
$\Leftrightarrow HK + KK - KH + KK = 2K^2$
$\Leftrightarrow KH + KK - KH + KK = 2K^2$
$\Leftrightarrow 2KK = 2K^2$
$\Leftrightarrow 2K^2 = 2K^2$

Proof of Condition 11. Let $A = H^{-1} + K$ and $A^{\#} = H^{-1} - K$, so $A^{\#}A = AA^{\#}$
$\Leftrightarrow (H^{-1}-K)(H^{-1}+K) = (H^{-1}+K)(H^{-1}-K)$
$\Leftrightarrow H^{-1}H^{-1} + H^{-1}K - KH^{-1} - KK = H^{-1}H^{-1} - H^{-1}K + KH^{-1} - KK$
$\Leftrightarrow H^{-1}K - KH^{-1} = -H^{-1}K + KH^{-1}$
$\Leftrightarrow H^{-1}K + H^{-1}K = KH^{-1} + KH^{-1}$
$\Leftrightarrow 2H^{-1}K = 2KH^{-1}$
$\Leftrightarrow H^{-1}K = KH^{-1}$

Proof of Condition 12. Let $A = H^{-1} + K$ and $A^{\#} = H^{-1} - K$, so $A^{\#}A = AA^{\#}$
$\Leftrightarrow (H-K^{-1})(H+K^{-1}) = (H+K^{-1})(H-K^{-1})$
$\Leftrightarrow HH + HK^{-1} - K^{-1}H - K^{-1}K^{-1} = HH - HK^{-1} + K^{-1}H - K^{-1}K^{-1}$
$\Leftrightarrow HK^{-1} - K^{-1}H = -HK^{-1} + K^{-1}H$
$\Leftrightarrow HK^{-1} + HK^{-1} = K^{-1}H + K^{-1}H$
$\Leftrightarrow 2HK^{-1} = 2K^{-1}H$
$\Leftrightarrow HK^{-1} = K^{-1}H$

Proof of Condition 13. Let $A = H + K$ and $A^{\#} = H - K$, so $H^{-1}A + A^{\#}H^{-1} = 2I$

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$\Leftrightarrow H^{-1}(H+K)+(H-K)H^{-1}=2I$

$\Leftrightarrow H^{-1}H+H^{-1}K+HH^{-1}-KH^{-1}=2I$

$\Leftrightarrow H^{-1}H+H^{-1}K+HH^{-1}-H^{-1}K=2I$

$\Leftrightarrow 2H^{-1}H=2I$

$\Leftrightarrow 2I=2I$

Proof of Condition 14. Let $A=H+K$ and $A^{\#}=H-K$, so $K^{-1}A-A^{\#}K^{-1}=2I$

$\Leftrightarrow K^{-1}(H+K)-(H-K)K^{-1}=2I$

$\Leftrightarrow K^{-1}H+K^{-1}K-HK^{-1}+KK^{-1}=2I$

$\Leftrightarrow HK^{-1}+K^{-1}K-HK^{-1}+KK^{-1}=2I$

$\Leftrightarrow HK^{-1}+KK^{-1}-HK^{-1}+KK^{-1}=2I$

$\Leftrightarrow 2KK^{-1}=2I$

$\Leftrightarrow 2I=2I$

Proof of Condition 15. Let $A=\overline{S}+K$ and $A^{\#}=\overline{S}-K$, so $S\overline{S}-K\overline{K}=AA^{\#}$. Take R.H.S,

$\Leftrightarrow AA^{\#}=(\overline{S}+K)(\overline{S}-K)$

$\Leftrightarrow AA^{\#}=\overline{S}\,\overline{S}-\overline{S}K+K\overline{S}-KK$

$\Leftrightarrow AA^{\#}=\overline{S}\,\overline{S}-\overline{S}K+\overline{S}K-KK$

$\Leftrightarrow AA^{\#}=\overline{S}\,\overline{S}-KK$

$\Leftrightarrow AA^{\#}=S\overline{S}-K\overline{K}$

III. Conclusion

In this paper, we derived some properties of J_s -Normal matrices.

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