International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-02, May 2019

Two Sided Complete Bayesian Chain Sampling Plan With Beta Geometric As Prior Distribution

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Abstract: In this paper, the average probability of acceptance of Bayesian Two Sided Complete Chain Sampling Plan is derived considering the Beta-Geometric as the prior distribution. The values for the extract of Two Sided Complete Chain sampling design on the basis of different combinations of argument are tabulated. The performance/discriminating power of the beta-geometric sampling plans is also discussed by determining the operating characteristic curve.

Keywords: Two-Sided Complete Bayesian Chain Sampling Plan, Beta- Geometric prior distribution, Operating Characteristic curve, Probability of acceptance.

I. INTRODUCTION

In a market, it is too tough to maintain the quality of a product at its essential standards .If the product meets the required quality at a reasonable price, the customers buy that product and it will result in goodwill for the product. In the other case, if the manufacturer didn't make attention to the less quality of his product, the customers will not be satisfied with that product and ultimately the manufacturer has to quit the market. Quality means the prescribed standard of the product but it doesn't mean the highest standard of the product. Statistical quality control methods are the methods for finding the variation in the quality of the product. Attributes are the quality characteristics of a product that are not answerable for measurement, but are recognized by their absence or presence. Example, the number of defective items in a sample.

II. BAYESIAN CHAIN SAMPLING PLAN

To symbolize the indiscriminate fluctuations fascinated within acceptance sampling, the Bayesian acceptance sampling technique is related by using all of prior behaviour records for the process of Gamma-Poisson, Beta-Binomial, and Beta-Geometric distributions. Bayesian sampling plans require the purchaser from chance to lot to offer explicitly the choice of defectives. The considered distribution of a chance quality at which the sampling information is mended to function is the prior distribution. The distribution is far formulated previously to the capturing of samples is called the prior distribution.

Dodge [4] developed the concept of chain sampling plan of type ChSp-1. Chiu [3] analyzed that balanced prior distribution of the lot fraction defectives would be finer than the beta prior distribution when sampling is done from the binomial model. The procedures and tables for the construction and selection of ChSP-1 plans developed by Soundararajan [12]. Hald [5] presented the optimum resources of the sampling plans presuming that each lot is formed by a process under binomial model but the process average deviates from lot to lot according to a beta distribution. Case and Keats [2] obtained choice of prior distribution in Bayesian Acceptance Sampling Plans. Calvin [1] uncovered that, when the number of non-conforming units in a sample of n units results in a binomial distribution and the process fraction defective is distributed through a beta distribution, the succeeding distribution of the process fraction non-conforming is furthermore a beta distribution. Latha and Jeyabharati [10] gave the average probability of acceptance by means of attribute under Beta-Geometric distribution of chain sampling plans. Latha and Rajeswari [8] pointed asymptotic extensions that how the sampling plan commits on the précised, such as quality levels, cost parameters and the parameters of the prior distribution. Latha and Rajeswari derived average cost and average regret function for BChSP-1 and are derived regret values for BChSP-1 for different conjunctions of sample sizes and interpreted graphically [6]. Latha and Jeyabharathi added the performance or discriminating power is through considering its operating characteristic curve of Beta-Binomial sampling plans [7]. Latha performed the Bayesian one plan suspension system mutually beta distribution as prior distribution [9]. Rebecca Jebaseeli Edna et al. [11] introduced the algorithm and the selection of Two-sided complete ChSP (0, 1) is indexed on quality standards.

 DOI : 10.35291/2454-9150.2019.0158

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III. Two Sided Complete Chain Sampling Plan

In the current competitive scheme, large amount quality concern practitioners say that if any failure occurs in a sample by the time mentioned not only the preceding ' i' samples anyhow the succeeding 'j' samples should furthermore be proposed, for the order of the advanced lot. Hence an effort has been taking to construct Complete Chain Sampling Plans by the authors. The two sided complete chain sampling plans are made by presuming that there are possibilities in many production industries for past, present and future samples.

The algorithm for framing a lot or bunch was extended by Devaarul and Edna (2011) is as follows:

(i) For each lot, pick a sample of n units and verify each unit for conformance to the specified attribute standard.

(ii) Accept the current lot if d (the observed number of defectives) is zero in the sample of n units and reject the lot if d > 1. If d = 1, go to after step.

(iii) Now execute the current lot if d = 1 and if no defectives are found in the immediately preceding 'i' samples and succeeding 'j' samples from the same consistent state process.

Operating characteristic function for Two Sided Complete Chain Sampling Plan-1 (TSCChSP-1) is given by Rebecca Jebaseeli Edna K (2012).

The OC function for TSCChSP-1 is

$$p_{0,n} + (p_{0,n})^i p_{1,n} (p_{0,n})^j$$

The average probability of acceptance for Two Sided Complete Bayesian Chain Sampling Plan is given as

$$\bar{p} = \int_0^\infty p(n, {}^i/_p) w(p) dp$$

Where, $w(p) = \frac{1}{\beta(s,t)} p^{s-1}(1-p)^{t-1}\ ; s, t > 0$

It is noted that p follows Beta distribution with probability density function.

Therefore, the Average probability of acceptance for Bayesian Two Sided chain sampling plan using Beta-Geometric Distribution in given by

$$\bar{p} = \frac{t}{s+t} + \frac{s(t+i+j)(t+i+j-1)(t+i+j-2)\ldots\ldots t}{(s+t+i+j+1)(s+t+i+j)(s+t+i+j-1)\ldots(s+t)}; i \neq j$$

$$= \frac{t}{s+t} + \frac{s(t+2i)(t+2i-1)(t+2i-2)\ldots\ldots t}{(s+t+2i+1)(s+t+2i)(s+t+2i-1)\ldots(s+t)} \qquad ; i = j$$

When $\mu = {}^s/_{s+t}$, the Average probability of acceptance for Bayesian Two Sided chain sampling plan using Beta-Geometric Distribution in given by the equation

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu i+\mu j)((s-s\mu+\mu i+\mu j-\mu)\ldots\ldots}{(s+\mu i+\mu j+\mu)(s+\mu i+\mu j)\ldots\ldots}\right]; i = j$$

$$= (1-\mu)\left[1 + \frac{s\mu(s-s\mu+2\mu i)((s-s\mu+2\mu i-\mu)\ldots\ldots}{(s+2\mu i+\mu)(s+2\mu i)\ldots\ldots}\right] \qquad ; i = j$$

(A)

IV. Construction of Tables

If i=j=1, $\bar{p}$ is reduced and μ_0 is the point of control

The above equation (A) can be reduced to

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu)(s-s\mu+2\mu)}{(s+\mu)(s+2\mu)(s+3\mu)}\right]$$

If i=j=2, $\bar{p}$ is reduced to,

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu)(s-s\mu+2\mu)(s-s\mu+3\mu)(s-s\mu+4\mu)}{(s+\mu)(s+2\mu)(s+3\mu)(s+4\mu)(s+5\mu)}\right]$$

If i=j=3, $\bar{p}$ is reduced to,

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu)(s-s\mu+2\mu)(s-s\mu+3\mu)(s-s\mu+4\mu)(s-s\mu+5\mu)(s-s\mu+6\mu)}{(s+\mu)(s+2\mu)(s+3\mu)(s+4\mu)(s+5\mu)(s+6\mu)(s+7\mu)}\right]$$

If i=j=4, $\bar{p}$ reduces to,

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu)(s-s\mu+2\mu)(s-s\mu+3\mu)(s-s\mu+4\mu)(s-s\mu+5\mu)(s-s\mu+6\mu)(s-s\mu+7\mu)(s-s\mu+8\mu)}{(s+\mu)(s+2\mu)(s+3\mu)(s+4\mu)(s+5\mu)(s+6\mu)(s+7\mu)(s+8\mu)(s+9\mu)}\right]$$

If i=j=5, $\bar{p}$ reduces to,

$$\bar{p} = (1-\mu)\left[1 + \frac{s\mu(s-s\mu+\mu)(s-s\mu+2\mu)(s-s\mu+3\mu)(s-s\mu+4\mu)(s-s\mu+5\mu)\ldots\ldots(s-s\mu+10\mu)}{(s+\mu)(s+2\mu)(s+3\mu)(s+4\mu)(s+5\mu)\ldots\ldots(s+11\mu)}\right]$$

Tables shown below gives the average probabilities of acceptance for different values of 'μ' and the figures show the discriminating power of operating characteristic curves for different values of ' i ' and for fixed values of s=1,2,3,4,5.

s / μ	1	2	3	4	5
0.1	0.9577	0.9644	0.9670	0.9684	0.9692
0.2	0.8714	0.8839	0.8892	0.8922	0.8940
0.3	0.7691	0.7823	0.7881	0.7913	0.7934
0.4	0.6606	0.6714	0.6759	0.6783	0.6798
0.5	0.5500	0.5571	0.5595	0.5606	0.5612
0.6	0.4390	0.4425	0.4429	0.4426	0.4423
0.7	0.3282	0.3290	0.3280	0.3268	0.3259
0.8	0.2181	0.2173	0.2157	0.2143	0.2132
0.9	0.1087	0.1076	0.1064	0.1054	0.1047
1.0	0	0	0	0	0

Table 1: μ values for Two Sided Complete Bayesian Chain Sampling Plan for given average probability of acceptance for i=j=1

DOI : 10.35291/2454-9150.2019.0158

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s / μ	1	2	3	4	5
0.1	0.9429	0.9496	0.9523	0.9539	0.9548
0.2	0.8444	0.8527	0.8563	0.8584	0.8597
0.3	0.7382	0.7440	0.7462	0.7473	0.7479
0.4	0.6308	0.6333	0.6337	0.6336	0.6333
0.5	0.5238	0.5238	0.5227	0.5218	0.5210
0.6	0.4176	0.4161	0.4143	0.4129	0.4119
0.7	0.3123	0.3101	0.3082	0.3069	0.3060
0.8	0.2076	0.2056	0.2041	0.2032	0.2026
0.9	0.1036	0.1023	0.1015	0.1010	0.1008
1.0	0	0	0	0	0

Table 2: μ values for Two Sided Complete Bayesian Chain Sampling Plan for given average probability of acceptance for i=j=2

s / μ	1	2	3	4	5
0.1	0.9331	0.9390	0.9415	0.9429	0.9438
0.2	0.8303	0.8353	0.8373	0.8384	0.8391
0.3	0.7242	0.7262	0.7265	0.7264	0.7263
0.4	0.6186	0.6182	0.6172	0.6163	0.6156
0.5	0.5139	0.5121	0.5105	0.5093	0.5085
0.6	0.4100	0.4077	0.4061	0.4050	0.4042
0.7	0.3068	0.3046	0.3032	0.3024	0.3019
0.8	0.2042	0.2016	0.2015	0.2010	0.2007
0.9	0.1019	0.1010	0.1005	0.1003	0.1002
1.0	0	0	0	0	0

Table 3: μ values for Two Sided Complete Bayesian Chain Sampling Plan for given average probability of acceptance for i=j=3

s / μ	1	2	3	4	5
0.1	0.9263	0.9312	0.9333	0.9345	0.9353
0.2	0.8220	0.8248	0.8257	0.8261	0.8263
0.3	0.7167	0.7168	0.7163	0.7157	0.7153
0.4	0.6124	0.6110	0.6096	0.6087	0.6080
0.5	0.5091	0.5070	0.5055	0.5045	0.5039
0.6	0.4065	0.4043	0.4030	0.4022	0.4017
0.7	0.3044	0.3025	0.3015	0.3010	0.3007
0.8	0.2026	0.2013	0.2007	0.2004	0.2003
0.9	0.1012	0.1005	0.1002	0.1001	0.1000
1.0	0	0	0	0	0

Table 4: μ values for Two Sided Complete Bayesian Chain Sampling Plan for given average probability of acceptance for i=j=4

s / μ	1	2	3	4	5
0.1	0.9214	0.9254	0.9271	0.9280	0.9286
0.2	0.8167	0.8180	0.8183	0.8182	0.8182
0.3	0.7122	0.7115	0.7105	0.7098	0.7093
0.4	0.6089	0.6071	0.6058	0.6050	0.6043
0.5	0.5064	0.5044	0.5032	0.5024	0.5019
0.6	0.4045	0.4026	0.4016	0.4011	0.4008
0.7	0.3030	0.3015	0.3008	0.3004	0.3003
0.8	0.2018	0.2008	0.2004	0.2002	0.2001
0.9	0.1008	0.1003	0.1001	0.1001	0.1000
1.0	0	0	0	0	0

Table 5: μ values for Two Sided Complete Bayesian Chain Sampling Plan for given average probability of acceptance for i=j=5

V. INTERPRETATION OF TABLES

Above tables discloses that

i. Increase in $\bar{p}$ decrease in μ for fixed $i = j$.
ii. Increase in $\bar{p}$ increase in s for fixed μ.
iii. Increase in $i = j$ decrease in $\bar{p}$ for fixed s and μ.

VI. SELECTION OF SAMPLING PLAN

To design a plan for the given s=3, i=j=2, μ_1=0.1015, μ_2=0.9523 select the respective values from Table 2.3.2. Compute the nearest values of μ_2/μ_1 corresponding to s=3, i=j=2, μ_1 =0.1015, μ_2 =0.9523. Then μ_2/μ_1 =9.3823 approximately equal to 9, Corresponding to s=3, i=j=2. One may additionally achieve the values of μ_2/μ_1 from Table 2.3.2. Hence the desired plan has parameters n=10, s=3, i=j=2. This plan found above for the same AQL and LQL requires a sample of size 10 only. This again confirms the suitability of the plan for small sample situations.

Example 1

For $i = j = 1, s = 1, \mu = 0.1$ the average probability of acceptance is 0.9577

Example 2

For $i = j = 2, s = 1, \mu = 0.1$ the average probability of acceptance is 0.9429

Example 3

For $i = j = 3, s = 1, \mu = 0.1$ the average probability of acceptance is 0.9331

Example 4

For $i = j = 4, s = 1, \mu = 0.1$ the average probability of acceptance is 0.9263

Example 5

For $i = j = 5, s = 1, \mu = 0.1$ the average probability of acceptance is 0.9214

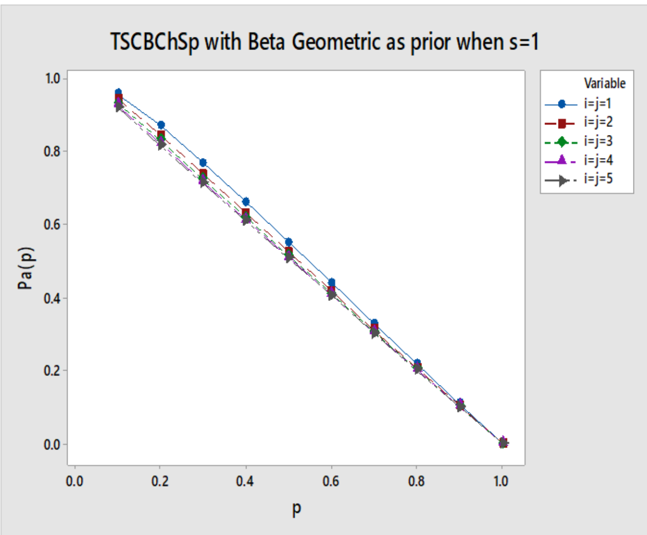

Figure 1: Comparison of OC curve for various values of i with s=1

 DOI : 10.35291/2454-9150.2019.0158

ISSN : 2454-9150 Vol-05, Issue-02, May 2019

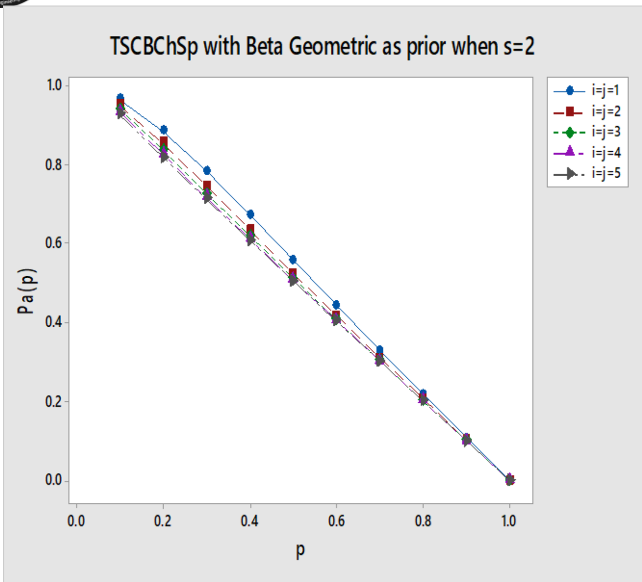

Figure 2: Comparison of OC curve for various values of i with s=2

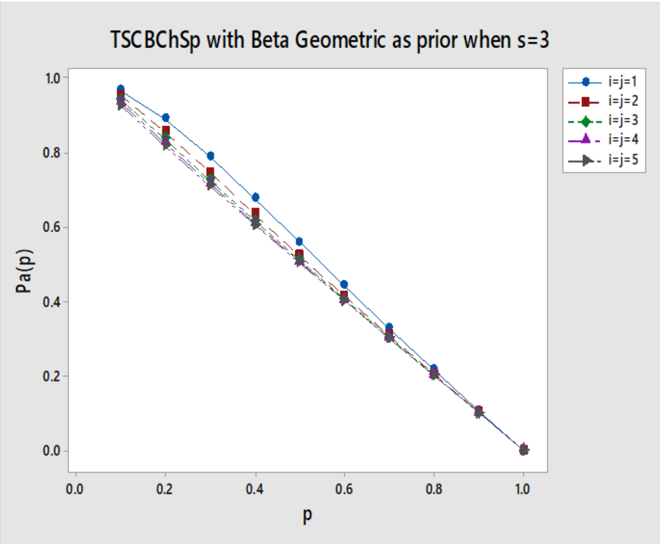

Figure 3: Comparison of OC curve for various values of i with s=3

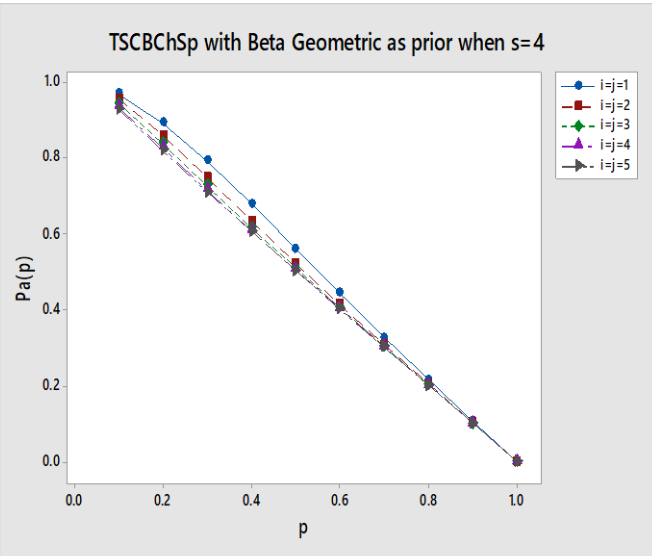

Figure 4: Comparison of OC curve for various values of i with s=4

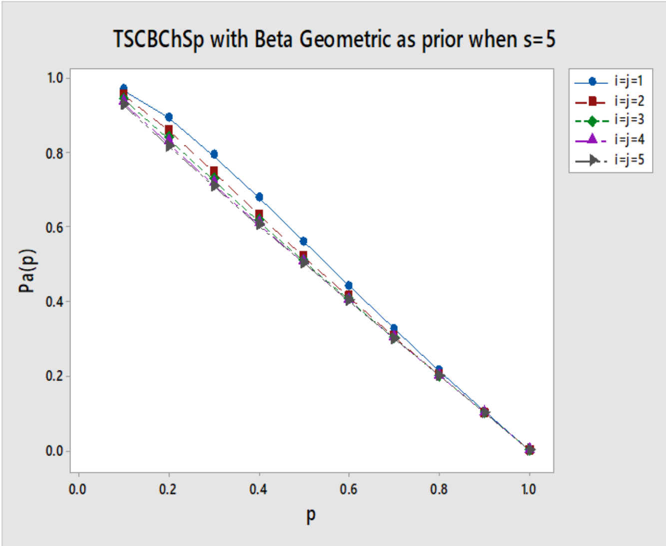

Figure 5: Comparison of OC curve for various values of i with s=5

VII. Interpretation of Graphs

OC curves display that after the lot quality is weakened, the probability of acceptance is minimized for growing values of $i = j$, securing the customer's interest. For example, the plan with $i = j = 5$, gives high probabilities at smaller p values and low probabilities at larger p values.

VIII. Conclusion

The Two-Sided Complete Bayesian Chain sampling plan gives more protection to the consumer while giving more pressure on the producer and shows better discrimination in confining the good lot against the bad one. These plans provide small sample sizes by ensuring protection for the producers and at the same time assuring the consumer with the better quality level after the inspection. The performance measures and tables develop during this paper will be used for the selection of the Two Sided Complete Bayesian Chain Sampling Plan with Beta Geometric as prior distribution. This sampling plan is recommended during testing or inspection for inexpensive or non-destructive items.

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 DOI : 10.35291/2454-9150.2019.0158