

Application of Linear Algebra in Conogethes Punctiferails

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ABSTRACT - The present paper is focussing on the mathematical concepts from Linear Algebra, which can be applied into entomology. In order to perform such an analysis, by using several critical concepts from Linear Algebra, including eigen values, eigen vectors and diagonalization of a matrix. Throughout the study of entomology, there are many factors that takepart into study by analysing the life cycles of the species of conogethespunctiferails. These species undergo four different stages in one life cycle. The Stages are (egg, larva, pupa, adult) and we call it as first generation. Totally seven generations are possible in one year. In this paper, we discuss the changes happening in each stage in upcoming generations. Using real time data of these species we are able to develop the mathematical data by using Linear Algebra Techniques.

Key words: *ConogethesPunctiferails* , *ConogethesPunctiferails life cycle*, *Diagonalization*, *Eigen values*, *Eigen vectors*.

- Real time data of conogethespunctiferails is obtained from GKVKlaboratory under the guidance of B.DoddaBasappa,Asst.Prof.(Entomology),collage of Horticulture.

I. INTRODUCTION

Algebra is a mathematical approach of solving variables by using equations. Linear Algebra takes that to another level by manipulating these variables using mathematical structures. In Linear Algebra, we study the vector spaces and linear mapping between the Vector Spaces. That will explain, how to represent the entire system of equations in the form of matrices and vectors. These eigen values and eigen vectors explain linear transformations in an easy way, so that we need to have more linearly independent eigen vectors associated in a single linear transformation and hence will be useful in variety of real-world problems. Here we are focussing on Entomology. Entomology is the study of insects, a branch of zoology. Conogethespunctiferails is one of the insects and conogethespunctiferails is also called as yellow moth or castor shoot and capsule borer. The pest has four life stages that is egg stage, larva stage, pupa stage and adult stage. The pest has seven generations in a year. The adult conogethespunctiferails were medium sized moth, brownish yellow body with yellow wings having number of dark spots.

Now each age group has certain mortality rate and reproduction rate and the death rate at different stages, depending upon the genetics, plant morphological and anatomical characteristics, plant biochemical constituents (phenon, total soluble sugar, total soluble protein, trypsin inhibitor units and silica) and natural enemies, abiotic factors etc. Temperature and relative humidity also

influence growth and development of different stages of pest.



Figure-1

II. DEFINITIONS

2.1 Eigen values

A scalar λ is called an eigen value of $n \times n$ matrix A if there is a nontrivial solution X of $AX = \lambda X$. Such an X is called an eigen vector corresponding to the eigen value λ .

2.2 Eigen vectors

Eigen vectors are a special set of vectors associated with a linear system of equations (i.e., matrix equation) that are sometimes also known as characteristic vectors, proper vectors, or latent vectors. Diagonalization for the given transition square matrix A is said to be diagonalizable $A = PDP^{-1}$.

III. RESULT

Let us observe certain *Conogethespunctiferalis* population from the Real Time data from GKVK laboratory of Bangalore.

These species of *Conogethespunctiferalis* only have 7 generations in one year and 4 stages (life cycle) in each generation namely

- i) Egg stage (ii) Larva stage (iii) Pupa stage (iv) Adult stage.

Let us Model the problem in the following method

P_0 = stage 0 = any *Conogethespunctiferalis* species in egg stage.

P_1 = stage 1 = any stage 0 *Conogethespunctiferalis* species which are survived to the end of the days and move to the next stage (larva stage).

P_2 = stage 2 = any stage 1 *Conogethespunctiferalis* species that is survived to the end of the days and move to the next stage (pupa stage).

We will assume that the life cycle of this species of *Conogethespunctiferalis* of first generation ends in these 4 stages (Egg stage, Larva stage, Pupa stage and Adult stage) and enter into the second generation.

The life cycle of these species of *Conogethespunctiferalis* of second generation ends in these 4 stages (Egg stage, Larva stage, Pupa stage and Adult stage) and enter into the third generation.

The life cycle of these species of *Conogethespunctiferalis* of third generation ends in these 4 stages (Egg stage, Larva stage, Pupa stage and Adult stage) and enter into the fourth generation, and so on up to 7 generation in a year. Now each stage of *Conogethespunctiferalis* in all 7 generations has certain mortality rate, certain reproductive rate and death rate.

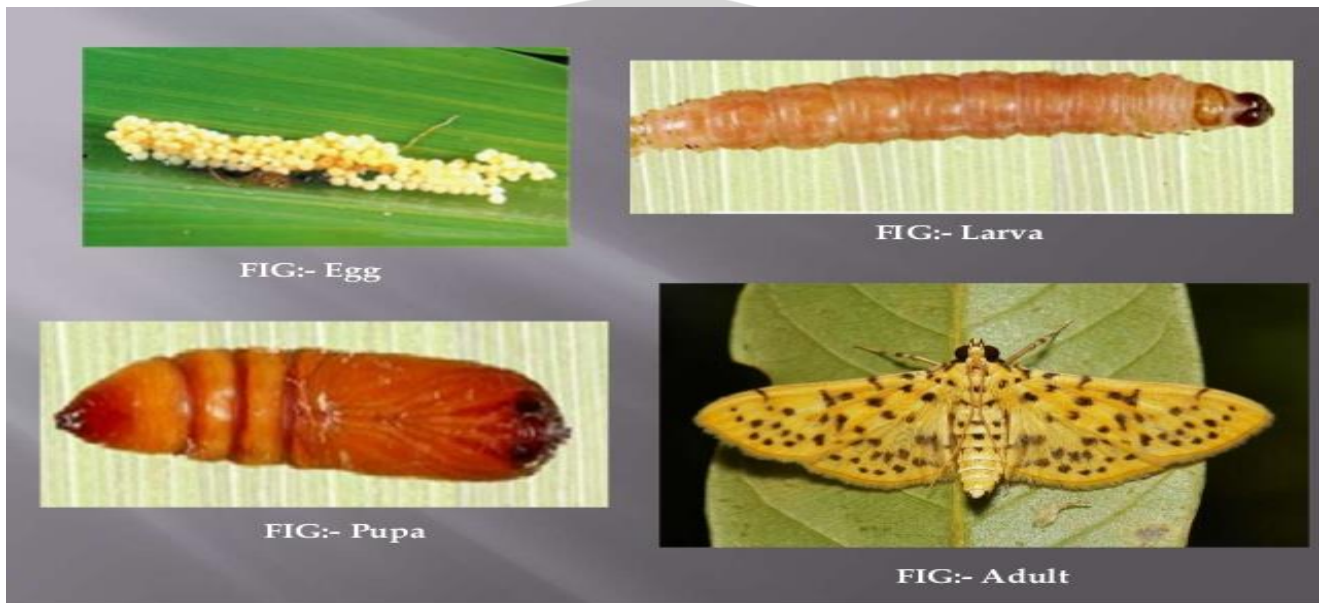


Figure-2

The following table shows the Real time data of certain *Conogethespunctiferalis* population in the GKVK laboratory of Bangalore.

Table-1

GENERATIONS	EGG STAGE		LARVA STAGE			
	Egg numbers	Egg duration	Number of larva hatch	Survivability in (%)	Death rate in(%)	Larva duration in days
First generation	48	2.76	37	77.08	22.92	30
Second generation	36	3.56	30	83.33	16.67	28
Third generation	22	2.66	17	77.27	22.73	21

Table-2

GERNERATIONS	PUPA STAGE				ADULT STAGE			
	Pupation	Survivability in (%)	Death rate in(%)	Pupa duration in days	Adult	Survivability in (%)	Death rate in(%)	Adult duration in days
First generation	28	75.68	24.32	9.62	20	71.43	28.57	6.42
Second generation	22	73.33	26.67	9.78	17	77.27	22.73	6.05
Third generation	13	76.47	23.53	9.36	9	69.23	30.77	5.83

Above table contains each age group has certain mortality rate and reproduction rate and the death rate at different stage up to third generation.

Now, in these particular species we are concentrating on survivability rate. We will describe this survivability rate below:

FIRST GENERATION

P_0 = This group has 100% survivability rate at beginning (egg stage)

(5 pairs of *Conogethes punctiferalis* produces 48 eggs and they will die, these 48 eggs we are taking as 100% survivable rate in the beginning and these species will take 2.76 days for egg duration)

P_1 = These species after finishing p_0 , it will enter into p_1 , here the survivability rate is 77.08% (larva stage)

(Out of 48 eggs, only 37 eggs will hatch, so we got 77.08% of survivability rate, 22.92% of death rate and it will take 30 days for larva duration)

P_2 = These species after finishing p_1 , it will enter into p_2 , here the survivability rate is 75.68% (pupa stage)

(Out of 37, only 28 will move to pupa stage, so we got 75.68% of survivability rate, 24.32% of death rate and it will take 9.62 days for pupa duration)

P_3 = These species after finishing p_2 , it will enter into p_3 , here the survivability rate is 71.43% (larva stage)

(Out of 28, we got only 20 adults so we got 71.43% of survivability rate, 28.57% of death rate, it will take 6.42 days for adult duration)

SECOND GENERATION

P_0 = This group has 100% survivability at beginning (egg stage)

(Here in that 20 adults of First generation we will take 5 pairs of *Conogethes punctiferalis* produces 36 eggs and they will die, these 36 eggs we are taking as 100% survivable rate in the beginning of second generation and these species will take 3.56 days for egg duration)

P_1 = These species after finishing p_0 , it will enter into p_1 , here the survivability rate is 83.33% (larva stage)

(Out of 36 eggs only 30 eggs will hatch, so we got 83.33% of survivability rate, 16.67% of death rate and it will take 28 days for larva duration)

P_2 = This species after finishing p_1 , it will enter into p_2 , here the survivability rate is 73.33% (pupa stage)

(Out of 30 only 22 will move to pupa stage, so we got 73.33% of survivability rate, 26.67% of death rate and it will take 8.78 days for pupa duration)

P_3 = This species after finishing p_2 , it will enter into p_3 , here the survivability rate is 77.27% (larva stage)

(Out of 22 we got only 17 adults so we got 77.27% of survivability rate, 22.73% of death rate, it will take 6.05 days for adult duration)

THIRD GENERATION

P_0 = This group has 100% survivability at beginning (egg stage)

(Here in that 17 adults of second generation we will take 4 pairs of *Conogethes punctiferalis* produces 22 eggs and they will die, these 22 eggs we are taking as 100% survivable rate in the beginning of third generation and these species will take 2.66 days for egg duration)

P_1 = This species after finishing p_0 , it will enter into p_1 , here the survivability rate is 77.27% (larva stage)

(Out of 22 eggs only 17 eggs will hatch, so we got 77.27% of survivability rate, 22.73% of death rate and it will take 21 days for larva duration)

P_2 = This species after finishing p_1 , it will enter into p_2 , here the survivability rate is 76.47% (pupa stage)

(Out of 17 only 13 will move to pupa stage, so we got 76.47% of survivability rate, 23.53% of death rate and it will take 9.36 days for pupa duration)

P_3 = This species after finishing p_2 , it will enter into p_3 , here the survivability rate is 69.23% (larva stage)

(Out of 13, we got only 09 adults so we got 69.23% of survivability rate, 30.77% of death rate, it will take 5.83 days for adult duration)

IV. MATHEMATICAL MODELLING

We will also make the following assumption about the surviving probability of each stage to next stage as follows:

77.08% of p_0 population survive from t to $t+1$

75.6% of p_1 population survive from t to $t+1$

71.43% of p_2 population survive from t to $t+1$

0% of p_3 population survive from t to $t+1$ (remember the stage stops at p_3 and enters into next generation)

In any given time, we will represent the total population as a vector:

$$P(t) = \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

This represents the population in time “t” of each of the stages.

We want to find p(t+1) from p(t). This will simply given by following matrix multiplication (using all the information above):

(Note: since the 4th generation data is not available let us neglect Adult stage)

$$\begin{bmatrix} P_0(t+1) \\ P_1(t+1) \\ P_2(t+1) \end{bmatrix} \begin{bmatrix} 1 & 0.81 & 0.56 \\ 1 & 0.78 & 0.59 \\ 1 & 0.85 & 0.52 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \end{bmatrix} = P(t+1)$$

After solving matrix, $A = \begin{bmatrix} 1 & 0.81 & 0.56 \\ 1 & 0.78 & 0.59 \\ 1 & 0.85 & 0.52 \end{bmatrix}$

We get eigen values:

$$\lambda_1 = 0, \lambda_2 = -0.07, \lambda_3 = 2.37$$

And corresponding eigen vectors for these corresponding values are

For $\lambda_1 = 0$, Eigen vectors are: $\begin{bmatrix} -0.0548 \\ 0.0400 \\ 0.0400 \end{bmatrix}$

For $\lambda_2 = -0.07$, Eigen vectors are: $\begin{bmatrix} 0.0019 \\ -0.0713 \\ 0.0995 \end{bmatrix}$

For $\lambda_3 = 2.37$, Eigen vectors are: $\begin{bmatrix} -1.9745 \\ -1.9745 \\ -1.9745 \end{bmatrix}$

Diagonalization:

Result: If a Matrix A is diagonalized, then $A = PDP^{-1}$ implies $A^n = PD^nP^{-1}$

Proof: By considering $A = PDP^{-1}$, we get

$$A^2 = (PDP^{-1})(PDP^{-1}) = PD(P^{-1}P)DP^{-1} = PD(I)DP^{-1} = PDDP^{-1} = PD^2P^{-1}$$

Similarly, $A^n = PD^nP^{-1}$

$$A = \begin{bmatrix} -0.05480 & 0.0019 & -1.9745 \\ 0.0400 & -0.0713 & -1.9745 \\ 0.0400 & 0.0995 & -1.9745 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.0713 & 0 \\ 0 & 0 & 2.37 \end{bmatrix}$$

$$\begin{bmatrix} 10.5705 & 6.0407 & 4.5298 \\ 0 & -5.8683 & 5.8683 \\ -0.2132 & -0.1724 & -0.1191 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.2922131 & 0 \\ 0 & 0 & 0.5607787 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.08538849 & 0 \\ 0 & 0 & 0.3144727 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.2495163 & 0 \\ 0 & 0 & 0.1763493 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.0729119 & 0 \\ 0 & 0 & 0.098892 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.0213058 & 0 \\ 0 & 0 & 0.0554564 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.006225 & 0 \\ 0 & 0 & 0.0310987 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.0018192 & 0 \\ 0 & 0 & 0.0174394 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.000531 & 0 \\ 0 & 0 & 0.009779 \end{bmatrix}$$

By the above calculations, we can estimate and predict that, up to 8th generation, this species “Conogethes punctiferalis” may exist according to the data available with us.

V. CONCLUSION

Results of this study of Entomology, we could assess that a relative importance was usually given to many factors that take part into study by analysing the life cycles of the species of conogethespunctiferails. These species undergo four different stages in one life cycle i.e. one Generation. By Modelling the Problem, we are able to identify that there are totally seven generations i.e. 8th generation will be the last generation, possible in one year. In this paper, we discuss the changes happening in each stage in upcoming generations. Using Realtime data of these species, we are able to develop the mathematical Model by using Diagonalization.

VI. FUTURE WORK

In Future, this study can be given preference in agricultural activities in different regions in spite of the system exist in present systems. If the Data is obtained by analyzing the region, some observations can be made. We can strongly decide that the risk factors have several effects depends upon different activities. Hence, we cannot generalize the outcome. Hence, we can give importance on priority levels by using Fuzzy Techniques and hence obtain a better prediction.

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