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The Dufour and Thermal Diffusion Effects of an Unsteady Magneto hydrodynamic Free Convection Casson Fluid Flow past an Exponentially Accelerated Plate via Laplace Transform

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Abstract— The influence of the Dufour and thermal diffusion effects in the Unsteady MHD natural convection flow past an conducting exponentially, incompressible, reacting chemically, viscous and heat absorbing Casson fluid flow past a accelerated plate. Using Laplace Transform technique to find the closed form of the solutions of velocity, temperature and concentration equations is found. The expression for Nusselt number, Sherwood number is also derived. In this paper, we find that the velocities of the Casson fluid decreasing with are increases values of Dufour effect and the temperature decreases on increasing value dufour effect. The applications of the Casson fluid are processing of food, operations in drilling and operations in bio-engineering.

Keywords— Dufour effects, Casson flow, hydrodynamic, Thermal diffusion.

I. INTRODUCTION

In the Newtonian fluid model many flow characteristics are not understandable. Hence the study of the non-Newtonian fluid model is useful. Non Newtonian fluid exerts non-linear relationships between of shear strain and rate of the shear stress. It has a many applications in engineering and industry, especially in the extraction of crude oil from petroleum products. The Casson fluid is one of the important non fluid models. Casson fluid is several applications in processing of food, operations in drilling and operations in bio-engineering.

The investigation of MHD flow of non Newtonian fluid in a flow past an exponentially conducting, incompressible, viscous and heat absorbing fluid past an accelerated plate. Jithender et al. [1] has discussed influence of viscous dissipation on unsteady MHD natural convective flow of Casson fluid over an oscillating vertical plate via FEM. Kkhalil ur Rehman et al. [2] analyzed numerical analysis of MHD Casson navier's slip nano fluid flow yield by rigid rotating disk. Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium engineering science and technology have made by Asma Khalid et al. [3]. Hari et al. [4] analyzed soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium. Prasad et al. [5] obtained thermal and species concentration of MHD Casson fluid at a vertical sheet in the presence variable fluid properties. Hari et al. [6] also investigated radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium. Bilal et al. [7] proposed mixed convection flow of casson fluid over a stretching sheet with convective boundary conditions and hall effect. Animasaun et al. [8] considered Casson fluid flow with variable thermo-physical property along exponentially stretching sheet with suction and exponentially decaying internal heat generation using the homotopy analysis method. Swati mukhopadhyay et al.[9] have examined Casson fluid flow over an unsteady stretching surface. Pramanik [10] have presented Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Omowaye et al. [11] dufour and soret effects on steady MHD convective flow of a fluid in a porous medium with temperature dependent viscosity using homotopy analysis method. Ramana Reddy et al. [12] effect of cross diffusion on MHD non-Newtonian fluids flow past a stretching sheet with non-uniform heat source/sink a comparative study. Kashif Ali Khan et al. [13] have studied effects of heat and mass transfer on unsteady boundary layer flow of a chemical reacting Casson fluid. Mariam Sheikh et al. [14] homogeneous–heterogeneous reactions in stagnation point flow of Casson fluid due to a stretching/shrinking sheet with uniform suction and slip effects. Sravanthi et al. [15] homotopy analysis solution of MHD slips flow past an

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exponentially stretching inclined sheet with soret-Dufour effects. Nawaz et al. [16] Magneto hydrodynamic axisymmetric flow of Casson fluid with variable thermal conductivity and free stream. Mahanthesh et al. [17] have made Convection in Casson liquid flow due to an infinite disk with exponential space dependent heat source and cross-diffusion effects. Rajuet al. [18] defines MHD Casson fluid in a suspension of convective conditions and cross diffusion across a surface of paraboloid of revolution. Alao et al. [19] extended effects of thermal radiation, Soret and Dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. Vijayaragavan et al. [20] have examined influence of thermal diffusion effects on unsteady MHD free convection flow past an exponentially accelerated inclined plate with ramped wall temperature. Vijayaragavan et al. [21] also investigated chemical reaction and thermal diffusion effects on unsteady MHD free convection with exponentially accelerated inclined plate. Jagdish Prakash et al. [22] have made dufour effects on unsteady hydromagnetic radiative fluid flow past a vertical plate through porous medium. Rajput et al. [23] deal with Dufour Effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion. Muhammad Faisal Javed et al. [24] have discussed axisymmetric flow of Casson fluid by a swirling cylinder. Sadia Siddiqaet al. [25] investigated heat transfer analysis of Casson dusty fluid flow along a vertical wavy cone with radiating surface. Veeresh et al. [26] considered joule heating and thermal diffusion effects on MHD radiative and convective Casson fluid flow past an oscillating semi-infinite vertical porous plate.Srinivasa Raju et al. [27] proposed MHD Casson viscous dissipative fluid flow past a vertically inclined plate in presence of heat and mass transfer: a finite element technique. Hiranmoy Mondal et al. [28] defined thermo phoresis and soret-dufour on MHD mixed convection mass transfer over an inclined plate with non-uniform heat source/sink and chemical reaction. Swetha et al. [29] obtained diffusion-thermo and radiation effects on MHD free convection flow of chemically reacting fluid past an oscillating plate embedded in porous medium. Bala Anki Reddy [30] who presented magneto hydrodynamic flow of a Casson fluid over an exponentially inclined permeable stretching surface with thermal radiation and chemical reaction. Hari et al. [31] have examined heat and mass transfer in magneto hydrodynamic Casson fluid flow past over an oscillating vertical plate embedded in porous medium with ramped wall temperature propulsion. Das et al. [32] have described Newtonian heating effect on unsteady hydro magnetic casson fluid flow past a flat plate with heat and mass transfer. Hayat et al. [33] have studied mixed convection stagnation point flow of Casson fluid with convective boundary conditions. In this paper we find that the velocities of the Casson fluid decreasing with are increases values of Dufour effect and the temperature decreases on increasing value Dufour effect.

II. MATHEMATICAL ANALYSIS

The Unsteady MHD natural convection flow with heat and mass transfer of an exponentially conducting, chemically reacting, incompressible, viscous and heat absorbing fluid past an accelerated inclined plate with ramped temperature is studied in the presence of Dufour and mass diffusions effect are analyzed. We consider the co-ordinate system in such a process that x' -axis is along the plate in upward direction, y'-axis normal to the plane of the plate and z'-axis perpendicular to $x'y'$-plane. The fluid is allowed by uniform transverse magnetic field B_0 applied parallel to y axis. Consider at time $t' \le 0$, both the plate and fluid are at rest and maintained at uniform temperature T'_∞ and uniform surface concentration C'_∞. At time $t' \ge 0$, plate starts moving in x'-direction against the gravitational field with time dependent velocity u' .Temperature of the plate is lowered or raised to $T'_\infty + (T'_w - T'_\infty)\dfrac{u_0^2 t'}{\nu}$ at $t' \ge 0$ and concentration of the plate is lowered or raised to $C'_\infty + (C'_w - C'_\infty)\dfrac{u_0^2 t'}{\nu}$ at $t' \ge 0$.It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible. As the plate is infinite in extent, so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only. Thus the motion is one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied. In this paper discuss the manipulate of the dufour and thermal diffusion effects on the Unsteady MHD natural convection Casson flow past an exponentially conducting, chemically reacting, incompressible, viscous and heat absorbing fluid past an exponentially accelerated plate. The dimensionless governing equations are solved using Laplace transform technique and the solutions are expressed in terms of complementary error and exponential functions. The rheological equation of state for the Cauchy stress tensor of Casson fluid is defined as.

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$$\tau_{ij} = \begin{cases} 2e_{ij}(\mu_B + \frac{py}{\sqrt{2\pi}}) & \pi > \pi_c \\ 2e_{ij}(\mu_B + \frac{py}{\sqrt{2\pi_c}}) & \pi < \pi_c \end{cases} \qquad (1)$$

where rate, $\pi = e_{ij}$ and e_{ij} is the (i, j) th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μB is plastic dynamic viscosity of the non-Newtonian fluid and Py is yield stress of fluid. $p_y = \frac{\mu_B\sqrt{2\pi}}{\lambda}$ Denote the yield stress of fluid. Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called rheopectic, in the case of Casson fluid flow where $\pi > \pi_c$ Substituting p_y into the following equation we get

$$\mu = \mu_B + \frac{p_y}{\sqrt{2\pi}}$$

Then, the kinematic viscosity can be written as

$$\nu = \frac{\mu}{\rho} = \frac{\mu_B}{\rho}\left(1 + \frac{1}{\lambda}\right)$$

To conclude λ is the Casson fluid parameter and as $\lambda \to \infty$ .Under above conditions we obtain the following partial differential equation with initial and boundary conditions are specified below.

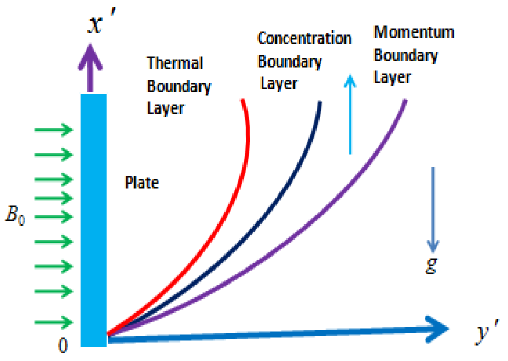

Figure 1 Projected physical model of the problem

$$\frac{\partial u'(y,t)}{\partial t'} = \frac{\mu_B}{\rho}\left(1+\frac{1}{\lambda}\right)\frac{\partial^2 u'(y,t)}{\partial y'^2} - \frac{\mu u'(y,t)}{\rho k_1'} - \frac{\sigma B_0^2 u'(y,t)}{\rho} + g\beta(T' - T_\infty') + g\beta_c(C' - C_\infty'), \qquad (2)$$

$$\frac{\partial T'(y,t)}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'(y,t)}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{D_m k_T}{\rho c_p C_s}\frac{\partial^2 C'(y,t)}{\partial y'^2}, \qquad (3)$$

$$\frac{\partial C'(y,t)}{\partial t'} = D\frac{\partial^2 C'(y,t)}{\partial y'^2} \qquad (4)$$

and the boundary conditions for the flow are given by

$$\left.\begin{aligned} & u' = 0,\ T' = T_\infty', C' = C_\infty' \ \ for\ t' \le 0\ and\ \ y' \ge 0, \\ & u' = u_0 \exp\left(\frac{u_0^2 t'}{\nu}\right),\ at\ y' = 0\ \ for\ t' \ge 0, \\ & T' = T_\infty' + (T_w' - T_\infty')\frac{u_0^2 t'}{\nu}\ \text{at}\ y' = 0\ \ for\ t' > 0, \\ & C' = C_\infty' + (C_w' - C_\infty')\frac{u_0^2 t'}{\nu}\ \text{at}\ y' = 0\ for\ \ t' > 0, \\ & u' \to 0,\ T' \to T_\infty',\ C' \to C_\infty'\ \ as\ y' \to \infty\ for\ t' > 0 \end{aligned}\right] \qquad (5)$$

The local gradient for the case of an optically slim gas is expressed as in the following form

$$\frac{\partial q_r}{\partial y'} = -4a'\sigma(T_\infty'^4 - T'^4), \qquad (6)$$

We assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T_∞' and neglecting the higher order terms, thus, we have

$$T'^4 = 4T_\infty'^3 T' - 3T_\infty'^4, \qquad (7)$$

Substituting equations (6) and (7) in (3) we get

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial t'^2} - \frac{16a'\sigma}{\rho c_p}T_\infty'^3(T' - T_\infty') + \frac{D_m k_T}{c_p c_s}\frac{\partial^2 C'}{\partial y'^2}. \qquad (8)$$

On introducing the following non dimensional parameters and variables

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$$y=\frac{y'u_0}{v},u=\frac{u'}{u_0},t=\frac{u_0^2t'}{v},T=\frac{T'-T'_\infty}{T'_w-T'_\infty},$$
$$C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\mu=\rho v,G_r=\frac{g\beta v(T'_w-T'_\infty)}{u_0^3},$$
$$G_m=\frac{g\beta_c v(C'_w-C'_\infty)}{u_0^3},P_r=\frac{\mu c_p}{k},k_1=\frac{u_0^2k'_1}{v^2}$$
$$S_c=\frac{v}{D},\ M=\frac{\sigma B_0^2 v}{\rho u_0^2},\ R=\frac{16a'\sigma v^2 T'^3_\infty}{ku_0^2},$$
$$Du=\frac{D_m k_T(C'_w-C'_\infty)}{c_s c_p v(T'_w-T'_\infty)}, \tag{9}$$

We have the following governing equation which is dimensionless form

$$\frac{\partial u}{\partial t}=\left(\frac{1}{1+\lambda}\right)\frac{\partial^2 u}{\partial y^2}-\frac{(Mk_1+1)}{k_1}u+G_rT+G_mC, \tag{10}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-\frac{R}{P_r}T+D_u\frac{\partial^2 C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{12}$$

and corresponding boundary becomes

$$\left.\begin{array}{l}u=0,\ T=0,\ C=0 \text{ for } y\ge 0 \quad and\ t\le 0\\ u=\exp(t),C=t,T=t \text{ at } y=0 \quad for\ t>0\\ u\to 0,\ T\to 0,\ C\to 0,\ as\ y\to\infty \ for\ t>0\end{array}\right] \tag{13}$$

The above equation can written in the form

$$\frac{\partial u}{\partial t}=a\frac{\partial^2 u}{\partial y^2}-bu+G_rT+G_mC, \tag{14}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-\frac{R}{P_r}T+D_u\frac{\partial^2 C}{\partial y^2}, \tag{15}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{16}$$

and corresponding boundary condition are

$$\left.\begin{array}{l}u=0,\ T=0,\ C=0 \text{ for } y\ge 0\ and\ t\le 0\\ u=\exp(t), \text{at } y=0\ for\ t>0\\ T=t\ ,C=t\quad at\ y=0\ for\ \ t>0\\ u\to 0,\ T\to 0,\ C\to 0,\ as\ y\to\infty\ for\ t>0\end{array}\right] \tag{17}$$

Where $a=\dfrac{1}{1+\lambda},\ b=\dfrac{(M\,k_1+1)}{k_1}$

III. THE METHOD OF SOLUTION

To find the accurate solution for fluid velocity, fluid temperature and concentration by solving the dimensionless governing equations from (14) to (16) subject to the boundary conditions (17) using Laplace transforms method and after simplification the fluid velocity, fluid temperature and concentration is presented in error function, complementary error function and exponential form.

The concentration of the fluid is given by

$$C(y,t)=\varphi_1(y,t) \tag{18}$$

Where

$$\varphi_1(y,t)=\left(\frac{y^2 S_c}{2}+t\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right)-\frac{y\sqrt{tS_c}}{2\sqrt{\pi}}\exp\left(\frac{-y^2S_c}{4t}\right)$$

and temperature of the fluid is given by

$$T(y,t)=\varphi_2(y,t) \tag{19}$$

Where,

$$\varphi_2(y,t)=\left(\frac{t}{2}+\frac{yp_r}{4\sqrt{R}}\right)\exp(y\sqrt{R})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{\frac{R}{p_r}t}\right)+\left(\frac{t}{2}-\frac{yp_r}{4\sqrt{R}}\right)\exp(-y\sqrt{R})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{\frac{R}{p_r}t}\right)$$
$$+\frac{a_3}{2}\left[\begin{array}{l}\exp(y\sqrt{R})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{\frac{R}{p_r}t}\right)\\ +\exp(-y\sqrt{R})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{\frac{R}{p_r}t}\right)\end{array}\right]$$
$$-\frac{a_3\exp(a_2t)}{2}\left[\begin{array}{l}\exp(y\sqrt{R+a_2P_r})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{\frac{Rt}{p_r}+a_2t}\right)\\ +\exp(-y\sqrt{R+a_2P_r})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{\frac{Rt}{p_r}+a_2t}\right)\end{array}\right]$$
$$-a_3\,erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right)$$
$$+\frac{\exp(a_2t)}{2}\left[\begin{array}{l}\exp(y\sqrt{a_2S_c})\,erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}+\sqrt{a_2t}\right)\\ +\exp(-y\sqrt{a_2S_c})\,erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}-\sqrt{a_2t}\right)\end{array}\right]$$

The velocity of the fluid is given by

$$u(y,t)=\varphi_3(y,t) \tag{20}$$

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Where,

$$\phi_3(y,t)=\frac{\exp(t)}{2}\left[\begin{array}{l}\exp(y\sqrt{(1+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{(1+b)t}\right)\\+\exp(-y\sqrt{(1+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{(1+b)t}\right)\end{array}\right]$$

$$+\frac{b_1}{2}\left[\begin{array}{l}\exp(y\sqrt{bc})\ erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{bt}\right)\\+\exp(-y\sqrt{bc})\ erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{bt}\right)\end{array}\right]$$

$$+\frac{b_2}{2}\left[\begin{array}{l}\left(\frac{t}{2}+\frac{yc}{4\sqrt{b}}\right)\exp(y\sqrt{bc})\ erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{bt}\right)\\+\left(\frac{t}{2}-\frac{yc}{4\sqrt{b}}\right)\exp(-y\sqrt{bc})\ erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{bt}\right)\end{array}\right]$$

$$+\frac{b_4\exp(a_2t)}{2}\left[\begin{array}{l}\exp(y\sqrt{(a_2+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{(a_2+b)t}\right)\\+\exp(-y\sqrt{(a_2+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{(a_2+b)t}\right)\end{array}\right]$$

$$+\frac{b_3\exp(-a_5t)}{2}\left[\begin{array}{l}\exp(y\sqrt{(-a_5+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{(-a_5+b)t}\right)\\+\exp(-y\sqrt{(-a_5+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{(-a_5+b)t}\right)\end{array}\right]$$

$$-\frac{a_{12}\exp(a_{11}t)}{2}\left[\begin{array}{l}\exp(y\sqrt{(a_{11}+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{(a_{11}+b)t}\right)\\+\exp(-y\sqrt{(a_{11}+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{(a_{11}+b)t}\right)\end{array}\right]$$

$$-\frac{a_{16}\exp(a_{15}t)}{2}\left[\begin{array}{l}\exp(y\sqrt{(a_{15}+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}+\sqrt{(a_{15}+b)t}\right)\\+\exp(-y\sqrt{(a_{15}+b)c})\,erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}}-\sqrt{(a_{15}+b)t}\right)\end{array}\right]$$

$$+\frac{b_4}{2}\left[\begin{array}{l}\exp(y\sqrt{P_rR})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{\frac{R}{p_r}t}\right)\\+\exp(-y\sqrt{P_rR})\,erfc\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{\frac{R}{p_r}t}\right)\end{array}\right]$$

IV. NUSSELT NUMBER

The Nusselt number is defined and denoted by the formula

$$Nu=-\left[\frac{\partial T}{\partial y}\right]_{y=0},$$

Using equation (19), we get Nusselt number.

$$=\left[\begin{array}{l}\frac{p_r}{2\sqrt{R}}(1-erfc(\sqrt{\frac{R}{p_r}t}))+t\sqrt{R}(1-erfc\left(\sqrt{\frac{R}{p_r}t}\right))\\+\frac{2\sqrt{p_rt}}{\sqrt{\pi}}\exp\left(-\sqrt{\frac{R}{p_r}t}\right)\end{array}\right]$$

$$+\frac{a_3}{2}\left[\sqrt{R}(1-erfc\left(\sqrt{\frac{R}{p_r}t}\right))+\frac{\sqrt{p_rt}}{\sqrt{\pi}}\exp\left(-\sqrt{\frac{R}{p_r}t}\right)\right]$$

$$-\frac{a_3\exp(a_2t)}{2}\left[\begin{array}{l}\sqrt{R+a_2P_r}\,)(1-erfc\left(\sqrt{\left(\frac{R}{p_r}+a_2\right)t}\right)\\+\frac{\sqrt{p_rt}}{\sqrt{\pi}}\exp\left(-\sqrt{(\frac{R}{p_r}+a_2)t}\right)\end{array}\right]$$

$$-a_3\left(\frac{\sqrt{S_c}}{\sqrt{\pi t}}\right)+\frac{\exp(a_2t)}{2}\left[\begin{array}{l}\sqrt{a_2S_c}\,)(1-erfc(\sqrt{a_2t}))\\+\frac{\sqrt{S_c}}{\sqrt{\pi t}}\exp(-a_2t)\end{array}\right] \quad (21)$$

V. SHERWOOD NUMBER

The Sherwood number obtained by concentration field and is given in non-dimensional form as,

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$$S_h = 2\sqrt{\frac{S_c t}{\pi}} \tag{22}$$

The utilized constant expressions are described in the Appendix section.

VI. NUMERICAL RESULTS AND DISCUSSION

In order to study the control of dufour effect, magnetic field, species buoyancy force, thermal buoyancy force, Casson fluid parameter and chemical reaction in the boundary layer region, the numerical values of fluid velocity, concentration, temperature, Nusselt number and Sherwood number calculated from the analytical solutions reported in Sections 2 and 5 are exhibited graphically. Using Laplace Transform technique to find the closed form of the solutions of velocity, temperature and concentration equations is found. The expression for Nusselt number, Sherwood number is also derived. In this paper, we find that the velocities of the Casson fluid decreasing with are increases values of Dufour effect and the temperature decreases on increasing value dufour effect.

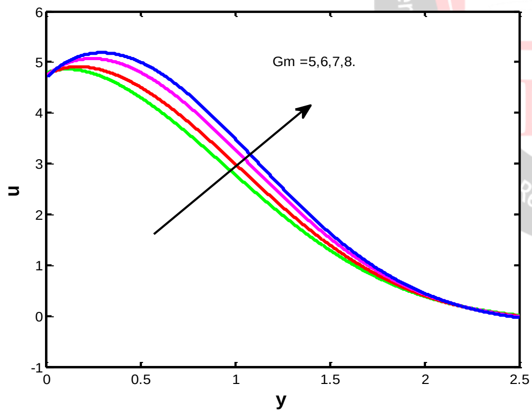

Figure 1 Velocity profiles for different values Gm with M=0.6, $Gr = 0.1$, $\lambda = 0.35$, $R = 4$, $Du = 0.3$, $\Pr = 0.5$, $k = 0.5$, $Sc = 2.01$ and $t = 0.5$.

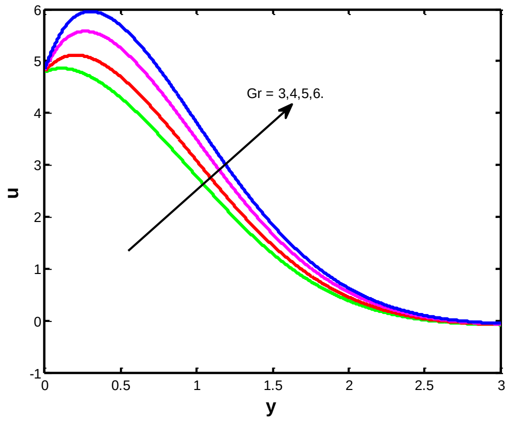

Figure 2 Velocity profiles for different values Gr with M=0.6, $Gm = 5$, $\lambda = 0.35$, $R = 4$, $Du = 0.3$, $\Pr = 0.5$, $k = 0.5$, $Sc = 2.01$ and $t = 0.5$.

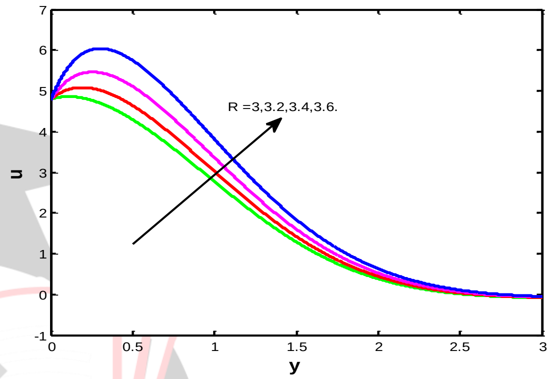

Figure 3 Velocity profiles for different values R with M=0.6, $Gm = 5$, $Gr = 0.1$, $\lambda = 0.35$, $R = 4$, $Du = 0.3$, $\Pr = 0.5$, $k = 0.5$, $Sc = 2.01$ and $t = 0.5$.

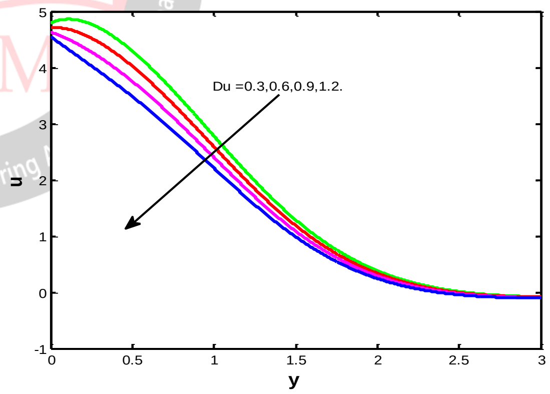

Figure 4 Velocity profiles for different values Du with M=0.6, $Gm = 5$, $\lambda = 0.35$, $Gr = 0.1$, $R = 4$, $\Pr = 0.5$, $k = 0.5$, $Sc = 2.01$ and $t = 0.5$.

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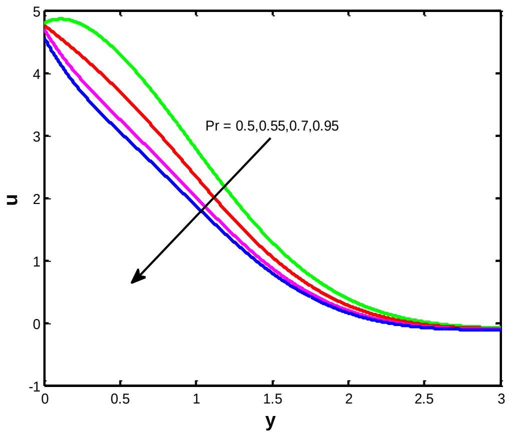

Figure 5 Velocity profiles for different values $\Pr$ with M=0.6, $Gm=5$, $\lambda=0.35$, $Gr=0.1$, $R=4$ $Du=0.3$, $Sc=2.01$ and $t=0.5$.

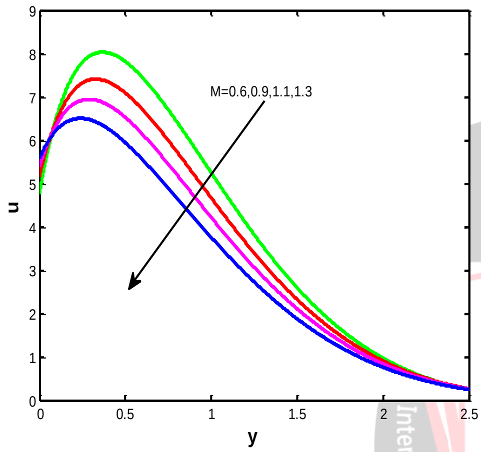

Figure 6 Velocity profiles for different values M=0.6 with $Gm=5$, $\lambda=0.35$, $Gr=0.1$, $R=4$ $\Pr=0.5$, $k=0.5$, $Du=0.3$, $Sc=2.01$ and $t=0.5$.

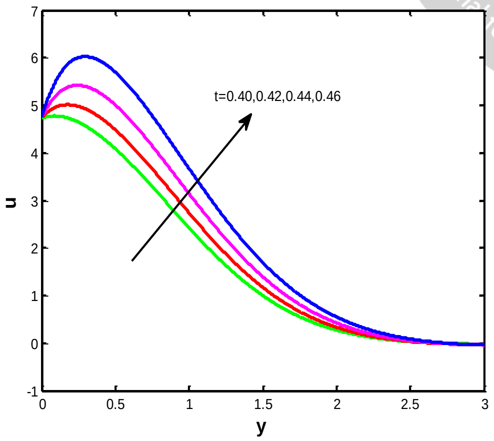

Figure 7 Velocity profiles for different values t with M=0.6, $Gm=5$, $\lambda=0.35$, $Gr=0.1$ $R=4$, $\Pr=0.5$, $k=0.5$, $Du=0.3$ and $Sc=2.01$

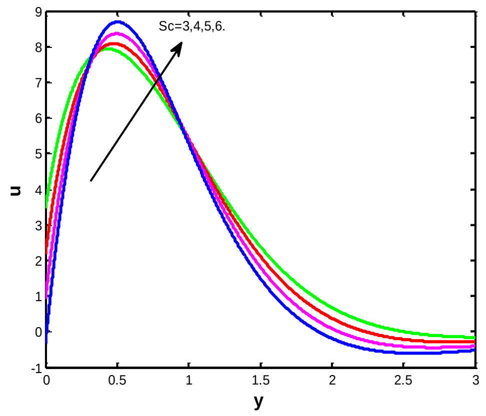

Figure 8 Velocity profiles for different values Sc with M=0.6, $Gm=5$, $\lambda=0.35$, $Gr=0.1$ $R=4$, $\Pr=0.5$, $k=0.5$, $Du=0.3$ and $t=0.5$.

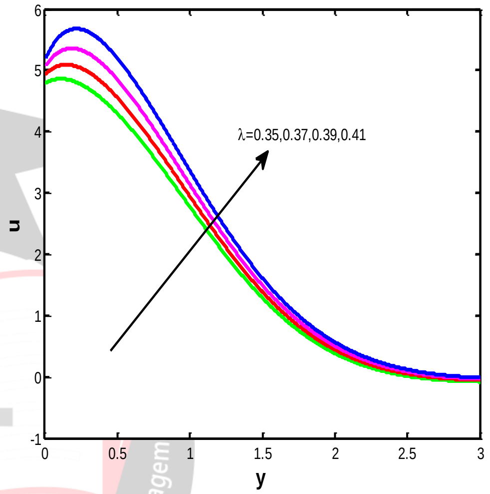

Figure 9 Velocity profiles for different values Sc with M=0.6, $Gm=5$, $\lambda=0.35$, $Gr=0.1$ $R=4$, $\Pr=0.5$, $k=0.5$, $Du=0.3$ and $t=0.5$.

6 .1 Velocity profile

We have easy to get to the non-dimensional fluid velocity, fluid temperature and concentration of different values of Prandtl number, thermal Grashof number, mass Grashof number, Casson parameter, magnetic parameter, and Dufour effect Figures 2–9. The thermal Grashof number Gr indicates the ratio of thermal buoyancy force to viscous hydrodynamic force and mass Grashof number Gm means ratio of buoyancy force and viscous hydrodynamic force. Figure 1 and Figure 2 we found that velocity raises with increase in Gr or Gm .Thus, we have that motion of fluid accelerated due to development in either mass buoyancy force or temperature buoyancy force. Physically, raise in Gr indicates raise in the strength of the flow, small viscous effects in the momentum equation and thus, reasons the raise in velocity profiles. Figure 4 shows the Dufour effect Du on fluid velocity. It is mentioned that the velocities of fluid decreasing with are increases values of Dufour effect Du. Figure 5 exhibit the velocity profiles for different values of Prandtl number $\Pr$. We found that motion of the fluid velocity decreases with increasing Prandtl number $\Pr$. Figure 6 represents the effect of magnetic parameter M on the velocity profiles. It is

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experiential that the fluid velocity as well as the boundary layer thickness decreases when M is increased. In Figure7 the influence of dimensionless time t on the velocity profiles is shown. It is found that the velocity is an increasing function of time t. In figure 8 shown the graphical results of Sc. It is observed that the fluid velocity increase with increase in Sc. Figure 9 shows that influence of the Casson fluid parameter λ on velocity profiles. It is seen that velocity increase with increase within the Casson fluid parameter λ.It is observed that with the Casson fluid parameter λ is great an adequate amount the non Newtonian fluid disappear and that the fluid behaves like Newtonian fluid. Therefore large value of Casson fluid parameter the velocity boundary layer thickness is large compare to Newtonian fluid.

6.2 Temperature profile

We have found that Figure 10 for different values of radiation parameter R. It is observed that fluid temperature decreases on increasing value radiation parameter R. It is depicting from Figure 11 that, the temperature decreases as the Prandtl number $\Pr$ increases. It is suitable due to the fact that thermal conductivity of the fluid decreases with increasing Prandtl number $\Pr$ and we conclude that Prandtl number $\Pr$ increases with the thermal boundary layer thickness decreases. Figure 12 is plotted to demonstrate the effects of the dimensionless time t on the temperature profiles. Clearly the temperature increases with increasing time t. Figure 13 shows that the temperature decreases on increasing value Dufour effect Du. Fig. 13 shows the effects of Duffer effect Du on fluid temperature .It is noted that the temperatures of fluid increasing with are increasing values of Dufour effect Du.

6.3 Concentration profile

Figure 14 shows that the concentration profiles for different values of t.We have to found that concentration increases with increasing values of time t. Figure 15 shows that the concentration profiles for different values of t.We have to found that concentration decreases with increasing values of Sc.

6.4 Nusselt number and Shear wood number profile

Figure 16 exhibits the Nusselt number for different values of Sc. It is observed that Nusselt number decreases with increase in Sc. Figure 17 shows the effect of Du on Nusselt number. It is observed that Nusselt number increasing with increasing Du. Figure 18 shows the effect of $\Pr$ on Nusselt number. It is seen that Nusselt number decreases propensity with $\Pr$. Figure 19 shows the effect of Radiation effect R on Nusselt number. It is seen that Nusselt number increases with increases values of Radiation effect R. Figure 20 shows the effect of Sc on Sherwood number. It is observed that Sherwood number increases with increases values Sc.

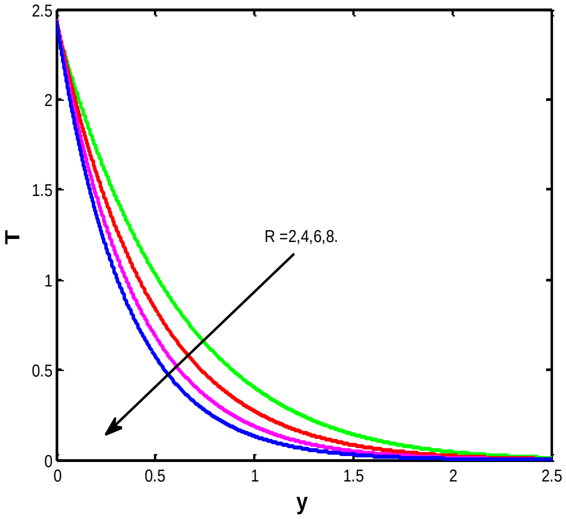

Figure 10 Temperature profiles for different values R with $Du = 0.2$, $\Pr = 0.71$, $Sc = 2.01$ and $t = 0.4$.

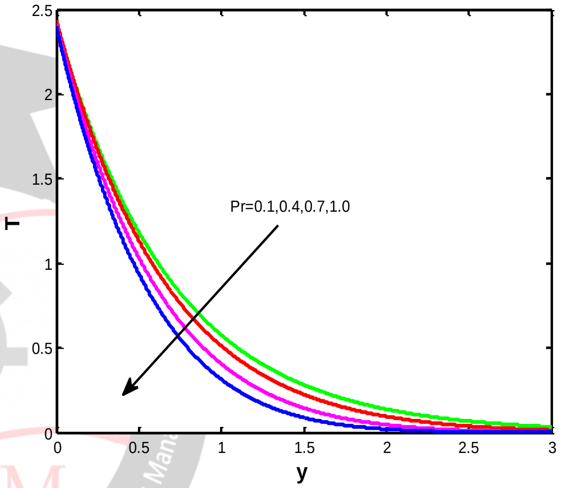

Figure 11 Temperature profiles for different values $\Pr$ with $Du = 0.2$, $R = 2$, $Sc = 2.01$ and $t = 0.4$.

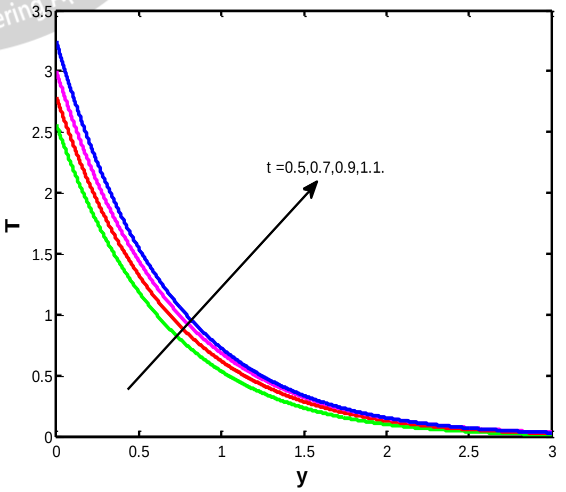

Figure 12 Temperature profiles for different values t with $Du = 0.2$, $R = 2$, $Sc = 2.01$ and $\Pr = 0.71$

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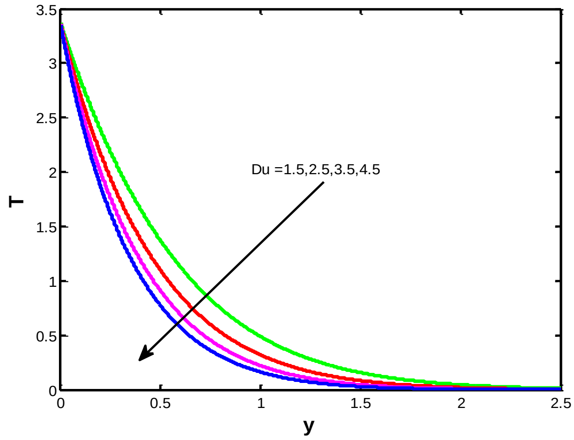

Figure 13 Temperature profiles for different values Du with, $\Pr = 0.71$, $R = 2$, $Sc = 2.01$ and $t = 0.4$.

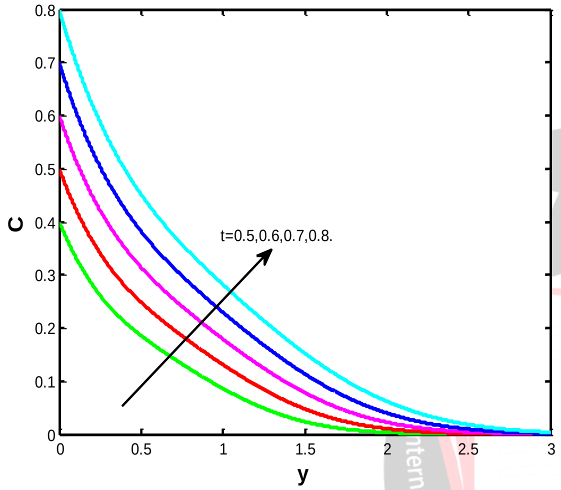

Figure 14 Concentration profiles for different values t with, $\Pr = 0.71$, $R = 2$, $Sc = 2.01$ and $t = 0.4$.

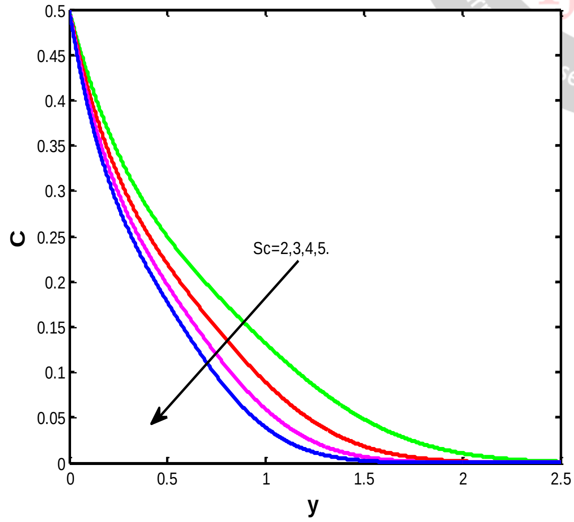

Figure 15 Concentration profiles for different values Sc with, $\Pr = 0.71$, $R = 2$, $Sc = 2.01$ and $t = 0.4$.

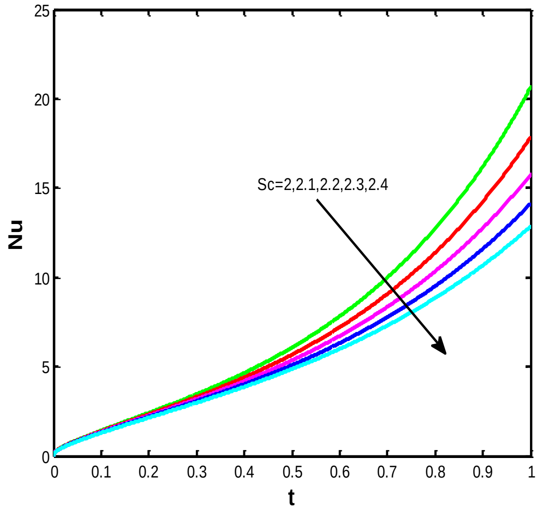

Figure 16 Nusselt number of different values Sc

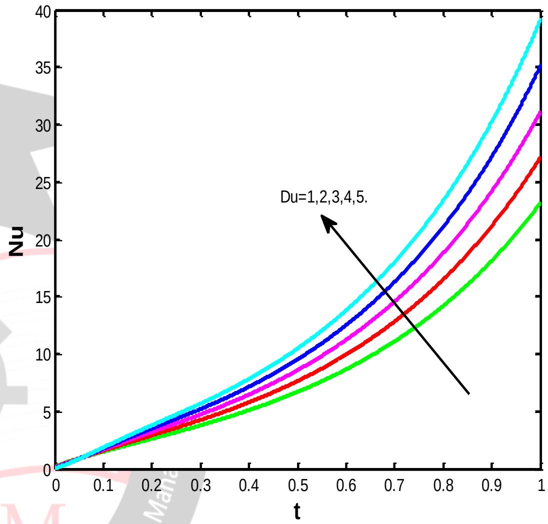

Figure 17 Nusselt number of different values Du

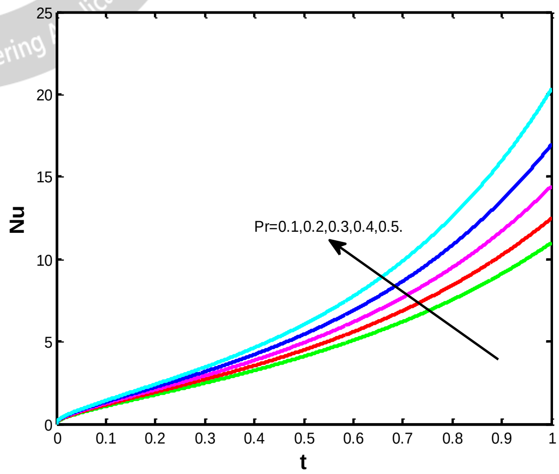

Figure 18 Nusselt number of different values $\Pr$

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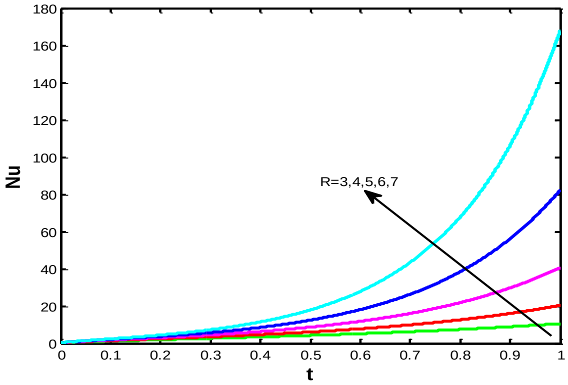

Figure 19 Nusselt number of different values R

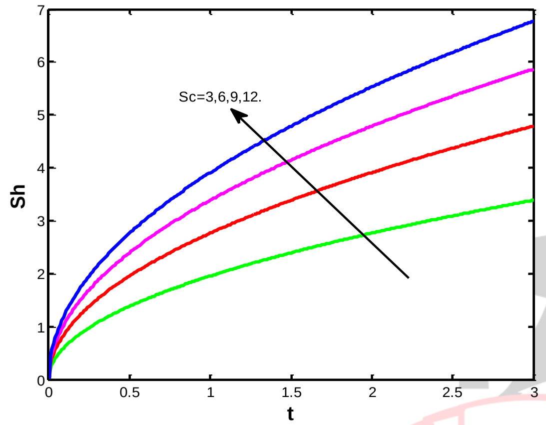

Figure 20 Sherwood number of different values Sc

VII. CONCLUSION

In this study we obtain exact solutions as well as numerical result for the unsteady natural convective Casson fluid flow past over an oscillating vertical plate in the presence of a transverse uniform magnetic field. Using Laplace transform technique to find the numerical solution of the velocity, the temperature and concentration in closed form. The effects of the applicable parameters on velocity, temperature, concentration, Nusselt number and Sherwood number profiles are presented graphically. In this paper we conclude that the following.

- ❖ The fluid Velocity increases with increasing values of thermal Grashof number Gr.and Mass Grashof number Gm.
- ❖ Velocity of the fluid increases with increasing values of Radiation parameter R
- ❖ The fluid Velocity decreases with increasing value of the Dufour effect Du.
- ❖ Velocity of the fluid with decreases with increasing Prandtl number $\Pr$.
- ❖ The fluid velocity as well as the boundary layer thickness decreases when magnetic parameter M is increased.
- ❖ For large values of permeability parameter k, velocity and boundary layer thickness increase.
- ❖ The fluid velocity increase with increase in Schmidt number Sc.
- ❖ Velocity increases with increasing values of Casson fluid parameter λ and time t.
- ❖ Temperature decreases as the Prandtl number $\Pr$ increases.
- ❖ Temperature and Concentration increases with increasing time t.
- ❖ Concentration decreases with increase in Schmidt number Sc.
- ❖ Nusselt number is decrease trend with Sc and t.
- ❖ Nusselt number is increases with increasing values of $\Pr$, Du and R.
- ❖ Sherwood number is increases with increasing in Schmidt number Sc.
- ❖ Thus we find that the velocities of the Casson fluid decreasing with are increases values of Dufour effect and the temperature decreases on increasing value Dufour effect.

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NOMENCLATURE

B_0 -Uniform magnetic field	T -dimensionless fluid temperature
k_0 - permeability parameter	T'_w -Temperature of the plate
C' - Species concentration	T'_∞ -Temperature of the fluid far away from the plate
C'_w -Concentration of the plate	t -Time
C'_∞ - Concentration of the fluid far away from the plate	t' - Dimensionless time
C -Dimensionless concentration	u' -Velocity of the fluid in the y' direction
C_p -Specific heat at constant pressure	u_0 -Velocity of the plate
Du - Dufour effect	u -Dimensionless velocity
g -Acceleration due to gravity	y' -Coordinate axis normal to the plate
G_r -Thermal Grash of number	y -Dimensionless Coordinate axis normal to the plate
G_m - Mass Grash of number	k_1 -Thermal conductivity of the fluid
M -Magnetic field parameter	α -Thermal diffusivity
Nu - Nusselt number	β -Volumetric coefficient of thermal expansion
P_r - Prandtl number	β_c -Volumetric coefficient of expansion with concentration
q_r - Radiative heat fluxes in the – y' direction	μ - Coefficient of viscosity
D_m -Coefficient of mass diffusivity	ν - Kinematic viscosity
R -Radiation parameter	ρ -Density of the fluid
Sc -Schmidt number	
λ -Casson fluid parameter	
σ - Electric conductivity	

APPENDIX

$$a_1 = \frac{-DuP_rS_c}{S_c - P_r}, a_2 = \frac{R}{S_c - P_r}, a_3 = \frac{a_1}{a_2}, a_4 = \frac{-G_r}{aP_r - 1}, a_5 = \frac{aR - b}{aP_r - 1}, a_6 = \frac{a_4}{a_5{}^2}$$

$$, a_7 = \frac{a_4}{a_5}, a_8 = \frac{a_1 a_4}{a_5(a_2 + a_5)}, a_9 = \frac{a_1 a_4}{a_2(a_2 + a_5)}, a_{10} = \frac{-G_r}{aS_c - 1}, a_{11} = \frac{b}{aS_c - 1},$$

$$a_{12} = \frac{-a_1 a_{10}}{a_{11}(-a_2 + a_{11})}, a_{13} = \frac{a_1 a_{10}}{a_2(a_2 - a_{11})}, a_{14} = \frac{-G_m}{aS_c - 1}, a_{15} = \frac{a_{14}}{a_{11}^2}, a_{16} = \frac{a_{14}}{a_{11}},$$

$$b_1 = (a_6 + a_8 + a_{12} + a_{15}),\ b_2 = (a_{16} - a_7), b_3 = (-a_9 - a_{13}), b_4 = (-a_6 - a_8),$$

$$b_5 = (-a_{12} - a_{16}).$$

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