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Reverse Super Edge – BI Magic Labeling of Star Related Graphs

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ABSTRACT **-A reverse edge – bi magic labeling on a graph with p vertices and q edges is a one – to – one map taking the vertices and edges onto the integers 1, 2,…, p + q with the property that satisfies for every edge e, the sum of all vertex labels incident on edge e is subtracted from edge label f (e) is either a constant k_1 or k_2. The reverse edge – bi magic labeling is said to be reverse super edge – bi magic labeling if $f(v)=\{1,2,....p\}$ and $f(e)=\{p+1,p+2,....p+q\}$.**

In this paper, we investigate the reverse super edge -bi magic labeling of star related graphs.

Key Words: Degree splitting graph, Jelly fish graph, Jewel graph, Reverse super edge –bi magic labeling, shadow graph.

AMS Subject Classification (2010) : 05C78

I. INTRODUCTION

All graphs considered in this paper are finite, simple and undirected. The graph G has vertex set V = V(G) and the edge set E = E(G) and let $p=|V|$, $q=|E|$. The concept of reverse edge - magic labeling and reverse super edge – magic labeling was introduced by S. Sharif Basha [10]. Motivated by these notions, we have introduced the concept of reverse super edge – bi magic labelling in [5].In this paper we investigate reverse super edge – bi magic labelling for Jelly fish graph , Shadow graph of star , Degree splitting graph of star and Jewel graph.

Definition 1.1 [6]

The Shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 1.2 [6]

Let $G=(V(G),E(G))$ be a graph with $V=S_1\cup S_2\cup S_3\cup........\cup S_i\cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T=V/\cup S_i$ .The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1,w_2,....w_t$ and joining to each vertex of S_i for $1\le i\le t$.

Definition 1.3 [4]

The Jewel J_n is the graph with vertex set $V(J_n)=\{u,v,w,x,y_i:1\le i\le n\}$ and the edge set $E(J_n)=\{wu,ux,vx,vw,wx,uy_i,vy_i:1\le i\le n\}$

Definition 1.4 [4]

For integer $n\ge 0$ we consider the graph Jelly Fish $J_{n,n}$ with vertex set $V(J_{n,n})=\{u,v,x,w,u_i,v_i:1\le i\le n\}$ and edge set $E(J_{n,n})=\{wu,xv,ux,vw,wx,uu_i,vv_i:1\le i\le n\}$

II. MAIN RESULTS

Definition 2.1 [5]

A reverse edge – bi magic labeling of a graph G is a one – to – one map f from $V(G)\cup E(G)$ onto the integers $\{1,2,...,|V(G)\cup E(G)|\}$ with the property that, there is an integer constants k_1 and k_2 such that $f(xy)-\{f(x)+f(y)\}=k_1$ *or* k_2 for any $(x,y)\in E(G)$.

A reverse edge - bi magic labeling f is called reverse super edge – bi magic if $f(V)=\{1,2,...,p\}$ and $f(E)=\{p+1,p+2,...,p+q\}$. A graph admits

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reverse super edge – bi magic labeling is called a reverse super edge – bi magic graph.

Theorem 2.1

The Shadow graph of star $D_2\left(k_{1,n}\right)$ is a reverse super edge – bi magic graph for $n \geq 2$.

Proof:

Let $v_1, v_2, v_3, ..., v_n$ be the pendant vertices and v be the apex vertex of first copy of $k_{1,n}$ and $v_1^{'}, v_2^{'}, v_3^{'}, ...v_n^{'}$ be the pendant vertices and $v^{'}$ be the apex vertex of second copy of $k_{1,n}$

$$V\left[D_2\left(k_{1,n}\right)\right] = \left\{v, v', v_i, v_i' : 1 \leq i \leq n\right\}$$

$$E\left[D_2\left(k_{1,n}\right)\right] = \left\{vv_i, vv_i', v'v_i, v'v_i' : 1 \leq i \leq n\right\}$$

Then $\left|V\left[D_2\left(k_{1,n}\right)\right]\right| = 2n+2$

$$\left|E\left[D_2\left(k_{1,n}\right)\right]\right| = 4n$$

Define $f : V \cup E \to \{1, 2, ...6n+2\}$ as follows

The vertex labels are defined by

$$f(v) = 2n+1$$

$$f(v_i) = i, 1 \leq i \leq n$$

$$f(v') = 2n+2$$

$$f\left(v_i'\right) = n+i, 1 \leq i \leq n$$

The edge labels are defined by

$$f(vv_i) = 2n+2+i, 1 \leq i \leq n$$

$$f\left(vv_1'\right) = 3n+3$$

$$f\left(vv_i'\right) = 5n+2+i, 2 \leq i \leq n$$

$$f(v'v_i) = 4n+3+i, 1 \leq i \leq n$$

$$f\left(v'v_i'\right) = 3n+3+i, 1 \leq i \leq n$$

Then the constants k_1 and k_2 of the reverse super edge- bi magic labeling are obtained as follows

To find k_1:

$$(2n+1)k_1 = \sum_{i=1}^{n}\left[f(vv_i) - \{f(v) + f(v_i)\}\right] + \sum_{i=1}^{n}\left[f\left(v'v_i'\right) - \left\{f(v') + f\left(v_i'\right)\right\}\right] + \left[f\left(vv_1'\right) - \left\{f(v) + f\left(v_1'\right)\right\}\right]$$

$$k_1 = 1$$

To find k_2:

$$(2n-1)k_2 = \sum_{i=1}^{n}\left[f(v'v_i) - \{f(v') + f(v_i)\}\right] + \sum_{i=2}^{n}\left[f\left(vv_i'\right) - \left\{f(v) + f\left(v_i'\right)\right\}\right]$$

$$k_2 = 2n+1$$

Thus f is a reverse super edge – bi magic labeling

Hence, $D_2\left(k_{1,n}\right)$ is a reverse super edge – bi magic graph for $n \geq 2$.

Theorem 2.2:

The Jelly fish $J_{n,n}$ $(n \geq 1)$ is a reverse super edge – bi magic graph.

Proof:

Let $V\left(J_{n,n}\right) = \{u, v, x, w, u_i, v_i : 1 \leq i \leq n\}$

and

$$E\left(J_{n,n}\right) = \{wu, xv, ux, vw, wx, uu_i, vv_i : 1 \leq i \leq n\}$$

Then $\left|V\left(J_{n,n}\right)\right| = 2n+4$

$$\left|E\left(J_{n,n}\right)\right| = 2n+5$$

Define $f : V \cup E \to \{1, 2, ..., 4n+9\}$ as follows

The vertex labels are defined by

$$f(w) = 1$$

$$f(u) = 2$$

$$f(v) = 3$$

$$f(x) = 4$$

$$f(u_i) = 4+i, 1 \leq i \leq n$$

$$f(v_i) = 4+n+i, 1 \leq i \leq n$$

The edge labels are defined by

$$f(wu) = 2n+5$$

$$f(ux) = 2n+8$$

$$f(xv) = 2n+9$$

$$f(vw) = 2n+6$$

$$f(wx) = 2n+7$$

$$f(uu_i) = 2n+9+i, 1 \leq i \leq n$$

$$f(vv_i) = 3n+9+i, 1 \leq i \leq n$$

Then the constants k_1 and k_2 of the reverse super edge- bi magic labeling are obtained as follows

To find k_1:

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$$(n+5)k_1=\sum_{i=1}^{n}\left[f(vv_i)-\left(f(v)+f(v_i)\right)\right]$$

$$+\left[f(wu)-\left(f(w)+f(u)\right)\right]$$

$$+\left[f(ux)-\left(f(u)+f(x)\right)\right]$$

$$+\left[f(vx)-\left(f(v)+f(x)\right)\right]$$

$$+\left[f(vw)-\left(f(v)+f(w)\right)\right]$$

$$+\left[f(wx)-\left(f(w)+f(x)\right)\right]$$

$$k_1=2(n+1)$$

To find k_2:

$$nk_2=\sum_{i=1}^{n}\left[f(uu_i)-\left(f(u)+f(u_i)\right)\right]$$

$$k_2=2n+3$$

Thus f is a reverse super edge – bi magic labeling.

Hence, $J_{n,n}$ $(n\geq 1)$ is a reverse super edge– bi magic graph.

Theorem 2.3 :

The Degree Splitting graph of star $DS(k_{1,n})$ is a reverse super edge-bi magic graph for $n\geq 3$

Proof:

Let $V(k_{1,n})=\{u,v_i:1\leq i\leq n\}$ where v_i are pendant vertices

$$V(k_{1,n})=V_1\cup V_2$$

Where $V_1=u$

$$V_2=\{v_i:1\leq i\leq n\}$$

Now in order to obtain $DS(k_{1,n})$ from G we add w_1, w_2 corresponding to V_1, V_2.

Then $V(DS(K_{1,n}))=\{u,v_i,w_1,w_2:1\leq i\leq n\}$

$$E(DS(K_{1,n}))=\{uw_1,uv_i,w_2v_i:1\leq i\leq n\}$$

$$|V(DS(k_{1,n}))|=n+3$$

$$|E(DS(k_{1,n}))|=2n+1$$

Define $f:V\cup E\rightarrow\{1,2,.....3n+4\}$ as follows

The Vertex labels are defined as

$$f(u)=1$$

$$f(w_1)=n+3$$

$$f(w_2)=n+2$$

$$f(v_i)=i+1, 1\leq i\leq n$$

The edge labels are defined as

$$f(uv_i)=n+3+i, 1\leq i\leq n$$

$$f(w_2v_i)=2n+4+i, 1\leq i\leq n$$

$$f(uw_1)=2n+4$$

Then the constants k_1 and k_2 of the reverse super edge - bi magic labeling are obtained as follows

To find k_1 :

$$2nk_1=\sum_{i=1}^{n}\left[f(v_iw_2)-\{f(v_i)+f(w_2)\}\right]$$

$$+\sum_{i=1}^{n}\left[f(uv_i)-\{f(u)+f(v_i)\}\right]$$

$$k_1=n+1$$

To find k_2 :

$$k_2=f(uw_1)-\left[f(u)+f(w_1)\right]$$

$$k_2=n$$

Thus f is a reverse super edge – bi magic labelling.

Hence, $DS(k_{1,n})$ is a reverse super edge- bi magic graph for $n\geq 3$.

Theorem2.4:

The Jewel graph J_n is a reverse super edge - bi magic graph for $n\geq 1$ is odd.

Proof:

Let $V(J_n)=\{u,v,w,x,y_i:1\leq i\leq n\}$

$$E(J_n)=\{wu,ux,vx,vw,wx,uy_i,vy_i:1\leq i\leq n\}$$

$$|V(J_n)|=n+4$$

$$|E(J_n)|=2n+5$$

Define $f:V\cup E\rightarrow\{1,2,.....3n+9\}$ as follows

The Vertex labels are defined by

$$f(w)=1$$

$$f(u)=2$$

$$f(v)=3$$

$$f(x)=4$$

$$f(y_i)=\begin{cases}4+2i, & 1\leq i\leq\left\lfloor\frac{n}{2}\right\rfloor\\ 4+2\left(i-\frac{n}{2}\right), & \left\lceil\frac{n}{2}\right\rceil\leq i\leq n\end{cases}$$

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The Edge labels are defined by

$f(wu) = n+5$

$f(ux) = n+8$

$f(vx) = n+9$

$f(vw) = n+6$

$f(wx) = n+7$

$f(uy_i) = n+2i+8, 1 \le i \le n$

$f(vy_i) = n+2i+9, 1 \le i \le n$

Then the constants k_1 and k_2 of the reverse super edge - bi magic labeling are obtained as follows

To find k_1 :

$$\left\{5+2\left\lfloor\frac{n}{2}\right\rfloor\right\}k_1 = \left[f(wu) - \{f(w)+f(u)\}\right] + \left[f(ux) - \{f(u)+f(x)\}\right] + \left[f(vx) - \{f(v)+f(x)\}\right] + \left[f(vw) - \{f(v)+f(w)\}\right] + \left[f(wx) - \{f(w)+f(x)\}\right] + \sum_{i=1}^{\left\lfloor\frac{n}{2}\right\rfloor}\left[f(uy_i) - \{f(u)+f(y_i)\}\right] + \sum_{i=1}^{\left\lfloor\frac{n}{2}\right\rfloor}\left[f(vy_i) - \{f(v)+f(y_i)\}\right]$$

$$k_1 = n+2$$

To find k_2 :

$$2\left\lceil\frac{n}{2}\right\rceil k_2 = \sum_{i=\left\lceil\frac{n}{2}\right\rceil}^{n}\left[f(uy_i) - \{f(u)+f(y_i)\}\right] + \sum_{i=\left\lceil\frac{n}{2}\right\rceil}^{n}\left[f(vy_i) - \{f(v)+f(y_i)\}\right]$$

$$k_2 = 2(n+1)$$

Thus f is a reverse super edge – bi magic labeling.

Hence, J_n is a reverse super edge - bi magic graph when $n \ge 1$ is odd.

Theorem2.5:

The Jewel graph J_n is a reverse super edge bi - magic graph for $n \ge 1$ is even.

Proof:

Let $V(J_n) = \{u, v, w, x, y_i : 1 \le i \le n\}$

$E(J_n) = \{wu, ux, vx, vw, wx, uy_i, vy_i : 1 \le i \le n\}$

$|V(J_n)| = n+4$

$|E(J_n)| = 2n+5$

Define $f : V \cup E \to \{1, 2, ...3n+9\}$ as follows

The Vertex labels are defined by

$f(w) = 1$

$f(u) = 2$

$f(v) = 3$

$f(x) = 4$

$$f(y_i) = \begin{cases} 4+2i, & 1 \le i \le \frac{n}{2} \\ 4+2\left(i-\frac{n}{2}\right)-1, & \frac{n}{2}+1 \le i \le n \end{cases}$$

The Edge labels are defined by

$f(wu) = n+5$

$f(ux) = n+8$

$f(vx) = n+9$

$f(vw) = n+6$

$f(wx) = n+7$

$f(uy_i) = n+2i+8, 1 \le i \le n$

$f(vy_i) = n+2i+9, 1 \le i \le n$

Then the constants k_1 and k_2 of the reverse super edge - bi magic labeling are obtained as follows

To find k_1 :

$$\{n+5\}k_1 = \left[f(wu) - \{f(w)+f(u)\}\right] + \left[f(ux) - \{f(u)+f(x)\}\right] + \left[f(vx) - \{f(v)+f(x)\}\right] + \left[f(vw) - \{f(v)+f(w)\}\right] + \left[f(wx) - \{f(w)+f(x)\}\right] + \sum_{i=1}^{\frac{n}{2}}\left[f(uy_i) - \{f(u)+f(y_i)\}\right] + \sum_{i=1}^{\frac{n}{2}}\left[f(vy_i) - \{f(v)+f(y_i)\}\right]$$

$$k_1 = n+2$$

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To find k_2 :

$$nk_2 = \sum_{i=\frac{n}{2}+1}^{n} \left[f(uy_i) - \{ f(u) + f(y_i) \} \right] + \sum_{i=\frac{n}{2}+1}^{n} \left[f(vy_i) - \{ f(v) + f(y_i) \} \right]$$

$$k_2 = 2n+3$$

Thus f is a reverse super edge – bi magic labeling.

Hence, J_n is a reverse super edge bi - magic graph for $n \geq 1$ is even.

III. CONCLUSION

The concept of reverse super edge – bi magic labeling of several classes of graphs are discussed here. The reverse super edge – bi magic labeling of Shadow graph of star $D_2(k_{1,n})$, Degree splitting graph of star $DS(k_{1,n})$, Jellyfish graph $J_{n,n}$ and Jewel graph J_n are investigated.

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