Tracking and Synchronization of A Hyperchaotic Hyperjerk System With No Equilibria Using Backstepping Control

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Abstract: Deriving a simple and fast tracking and control law for a hyper-chaotic hyper-jerk system is one of the main existing issues in modern days control systems. In this paper, an attempt is made to synchronize two similar 4-dimensional hyper-chaotic systems by using backstepping control. In each synchronization process, a control function is designed to achieve synchronization between two systems from different initial conditions. To obtain these control functions via backstepping control, some coefficient of error dynamics is chosen such that the transient error dynamics reduces as much as possible. For the purpose of finding stability of these systems, Lyapunov's stability criteria is checked and as the criteria is full filled , as a result these systems are stabilized as well as synchronized. These systems are simulated for their initial condition and behavior is observed using MATLAB. The simulation and tracking of these systems are done by using Back-stepping technique. These systems are derived from a new set of four coupled first order ordinary differential equations for some rare 4-dimensional no equilibrium system.

Keywords: Back-stepping control, Hyper-chaotic System Synchronization, Lyapunov Stability Theory, Tracking control, Synchronization.

I. INTRODUCTION

Chaotic phenomenon has attracted much attention owing to its various applications in practical fields [1]. In last decades or so, many researchers have done chaos synchronization [2] by using the chaotic systems aperiodic long time behavior occurrence in deterministic dynamical system and sensitivity to initial conditional properties [3].

A three dimensional (3D) chaotic system [5] can be described by a set of three coupled first order ordinary differential equations (ODEs) in three phase space variables (x,y,z). Such coupled ODEs may be recast into a single third order ODE known as jerk ODE of the form $\dddot{x} = f(x, \dot{x}, \ddot{x})$. These systems can be expanded to an *n* dimensional chaotic systems [7] for n>3 is described by a set of n coupled first order ODEs , which may be recast into a single *n*th- order ODE known as hyperjerk ODE. For *n* = 4, a fourth order hyperjerk ODE may exhibit either chaos or hyperchaos, where chaos has one positive Lyapunov exponent (LE), and hyperchaos has (at least) two positive LEs. Occurrence of chaotic phenomenon is mostly undesirable but sometimes desirable. It is desirable when it is required to control the chaotic behavior of a system and to modify the systems desired response. Synchronization of chaotic system is an important use of controller. There are two methods for synchronization of chaotic system as drive-response scheme and coupling scheme. Drive-response also called master-slave schemes which are widely used. From literature survey it is revealed that backstepping control technique has been applied for control and synchronization and of identical chaotic or non-identical chaotic system with different ways [8], [9], [10] because of its inherent characteristics such easy realization, fast response and good transient performance [11].

A. Contribution of paper

Objective of this paper are (i) to design active backstepping control technique that can control and stabilize the hyperchaotic system at any desired position, and (ii) to design active backstepping control law to synchronize two identical systems. Lyapunov's Stability Theory [4] is used to show the stabilization and convergence of error dynamics.

B. Organization of paper

Rest of the paper organized as follows. In Section II, control of the hyperchaotic system using backstepping control is given. In Section III, synchronization of the system is given. In Section IV, MATLAB simulation results are shown for validation and verification. Finally, conclusions and future scopes are given in Section V.

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II. BACKSTEPPING CONTROL OF HYPERCHAOTIC SYSTEMS

In this section description of the taken hyperchaotic system and its control methodology using backstepping control technique is discussed.

A. Description of the taken system

A Lorenz based hyper-chaotic system [6] is taken-

$$\begin{cases} \dot{x} = y - x + w \\ \dot{y} = -axz \\ \dot{z} = xy - 1 \\ \dot{w} = -bx \end{cases} \quad (1)$$

Where, (x, y, z, w) are state variables and (a, b >0) are positive parameters and have values such as a=18.13, b=0.994 in which the system shows a hyper-chaotic behavior. The phase portrait of the system is shown in figure1. The system does not have any equilibrium point.

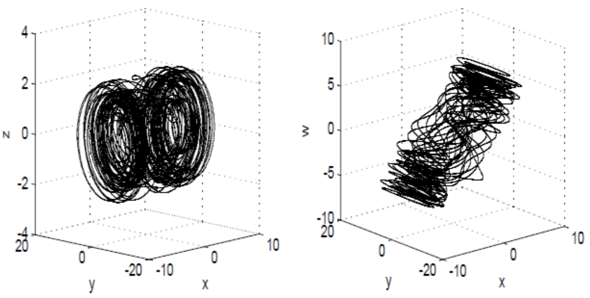

Figure1: Phase portraits of the system

Replacing $x_1 = x,\ x_2 = y,\ x_3 = z,\ x_4 = w$, and introducing controls u_1, u_2, u_3, u_4 in the system such that the system can take desired state values as $x_{1d}, x_{2d}, x_{3d}, x_{4d}$.

The system will be

$$\begin{cases} \dot{x}_1 = x_2 - x_1 x_4 + u_1 \\ \dot{x}_2 = -a x_1 x_3 + u_2 \\ \dot{x}_3 = x_1 x_2 - 1 + u_3 \\ \dot{x}_4 = -b x_1 + u_4 \end{cases} \quad (2)$$

Therefore the tracking error can be defined as

$$e = x_i - x_{id} \qquad \text{for i = 1, 2, 3, 4} \quad (3)$$

Where, $e = [e_1, e_2, e_3, e_4]^T$ is the tracking error vector.

Therefore the error dynamics,

$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d},\ \dot{e}_2 = \dot{x}_2 - \dot{x}_{2d},\ \dot{e}_3 = \dot{x}_3 - \dot{x}_{3d},\ \dot{e}_4 = \dot{x}_4 - \dot{x}_{4d}$,

Where, $x_{1d} = x_{2d} = x_{3d} = x_{4d} = x_d$ is obtained as follows:

$$\begin{cases} \dot{e}_1 = (e_2 + x_d) - (e_1 + x_d) + (e_4 + x_d) - \dot{x}_d + u_1 \\ \dot{e}_2 = -a(e_1 + x_d)(e_3 + x_d) - \dot{x}_d + u_2 \\ \dot{e}_3 = (e_1 + x_d)(e_2 + x_d) - \dot{x}_d - 1 + u_3 \\ \dot{e}_4 = -b(e_1 + x_d) - \dot{x}_d + u_4 \end{cases} \quad (4)$$

Now, our aim is to design back-stepping control law such that the resulting error dynamics should satisfy

$$\lim_{\substack{t\to\infty \\ i\to 4}} ||e_i(t)|| = 0$$

B. Design of active backstepping control for stabilization

Now, to establish the stability of error dynamics of the above system using Lyapunov stability theorem [1] a Lyapunov function candidate is defined as follows:

$$V(e) = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_3}^2 + {e_4}^2) \quad (5)$$

To satisfy the condition of stability such that $\dot{v} \leq 0$ for desired tracking, the control laws $u_i (i = 1,2,3,4)$ are designed as follows:-

$$\begin{cases} u_1 = x_d - (e_2 + x_d) - (e_4 + x_d) + \dot{x_d} \\ u_2 = a(e_1 + x_d)(e_3 + x_d) + \dot{x_d} \\ u_3 = -(e + x_d)(e_{2+} x_d) + 1 + \dot{x_d} \\ u_4 = b(e_1 + x_d) + \dot{x_d} \end{cases} \quad (6)$$

Therefore, the modified error dynamics is written as

$$\begin{cases} \dot{e_1} = -e_1 \\ \dot{e_2} = 0 \\ \dot{e_3} = 0 \\ \dot{e_4} = 0 \end{cases} \quad (7)$$

If derivative of the above system exist then using derivative of $V(e)$ gives

$$\dot{V}(e) = -{e_1}^2 \quad (8)$$

Since $\dot{V}(e) \leq 0$ and according to Lyapunov stability theory, states of the systems are stabilized and ensured the convergence of error dynamics i.e.

$$\lim_{t\to\infty} ||e_i(t)|| = 0 \quad (9)$$

Therefore, the state variables x_1, x_2, x_3, x_4 of the system track the state values at x_{1d}, x_{2d}, x_{3d}, x_{4d}.

In the next section, synchronization of the taken hyperchaotic system is discussed.

III. SYNCHRONIZATION OF THE HYPERCHAOTIC SYSTEM

The taken hyperchaotic system is considered as a master system as well as the slave system to achieve synchronization. The system is the master system and slave system is defined in

$$\begin{cases} \dot{y}_1 = y_2 - y_1 + y_4 + u_1 \\ \dot{y}_2 = -a y_1 y_3 + u_2 \\ \dot{y}_3 = y_1 y_2 - 1 + u_3 \\ \dot{y}_4 = -b y_1 + u_4 \end{cases} \quad (10)$$

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Where, y_1, y_2, y_3, y_4 are the states of the slave system and u_1, u_2, u_3, u_4 are the added control inputs such that states of the slave system synchronize to the master system.

Synchronization error is defined as follows:

$$e_{s_i} = y_i - x_i \text{ for i =1, 2, 3, 4} \qquad (11)$$

Therefore, the error dynamics is defined as:

$$\begin{cases} \dot{e}_{s_1} = e_{s_2} - e_{s_1} + e_{s_4} + u_1 \\ \dot{e}_{s_2} = a e_{s_1} e_{s_3} + u_2 \\ \dot{e}_{s_3} = e_{s_1} e_{s_2} - 1 + u_3 \\ \dot{e}_{s_4} = -b e_{s_1} + u_4 \end{cases} \qquad (12)$$

The objective is to find the control law u_i that can stabilize the error dynamics at origin.

A. Active backstepping controller and its control law

Let's stabilize the first equation in by considering e_{s_1} as controller. Let a Lyapunov function candidate be $v_1(e_{s_1}) = \frac{1}{2} {e_{s_1}}^2$ and differentiating with time we have

$$\dot{v_1}(e_{s_1}) = e_{s_1}\dot{e}_{s_1} = e_{s_1}[e_{s_2} - e_{s_1} + e_{s_4} + u_1]$$

$$= e_{s_1}[-e_{s_1}(-e_{s_2} - e_{s_4}) + u_1]. \qquad (13)$$

Considering, $u_1 = -e_{s_2} - e_{s_4}$, equation (13) can be written as : $\dot{v_1} = -{e_{s_1}}^2$ is negative definite and hence e_{s_1} is stabilized. Now we stabilize $\dot{e}_{s_2}$ i.e. second equation in system (12). Let a lyapunov function candidate $v_2(e_{s_2}) = \frac{1}{2} {e_{s_2}}^2$ and taking its time derivative, we get

$$\dot{v_2}(e_{s_2}) = e_{s_2}\dot{e}_{s_2} = e_{s_2}[a e_{s_1} e_{s_3} + u_2] \quad (14)$$

If we consider $u_2 = 0$, and estimating that $e_{s_3} = \alpha_1(e_{s_2})$ and $e_{s_1} = \alpha_2(e_{s_2})$ then, equation (15) can be rewritten if the estimated functions $\alpha_1(e_{s_2}) = 0$, and $\alpha_2(e_{s_2}) = 0, \dot{v_2}(e_{s_2}) = 0$ and hence $\dot{e}_{s_2}$ subsystem is stabilized. Now, the errors w_1, w_2 between e_{s_3} *and* $\alpha_1(e_{s_2})$, e_{s_1} *and* $\alpha_2(e_{s_2})$ respectively are given as follows:

$$\begin{cases} w_1 = e_{s_3} - \alpha_1(e_{s_2}) = e_{s_3} \\ w_2 = e_{s_1} - \alpha_2(e_{s_3}) = e_{s_1} \end{cases} \qquad (15)$$

Substituting $\dot{e}_{s_3}$ and e_{s_2} from equation (12) and (15) into time derivative of (15) yields,

$$\dot{w_2} = w_2 e_{s_2} - 1 + u_3 \qquad (16)$$

Let's stabilize the (w_2, e_{s_2}) subsystem given by equation (16) as follows:

Let a Lyapunov function candidate be

$$\dot{v_3}(w_2, e_{s_2}) = \dot{v_2}(e_{s_2}) + w_2\dot{w_2} \qquad (17)$$

From (14) and (16), (17) can be written as:

$$\dot{v_3}(w_2, e_{s_2}) = 0 + [e_{s_1}w_2 - 1 + u_3]\dot{w_2} \quad (18)$$

Estimating that $u_3 = -e_{s_1}w_2$, above equation (18) can be written as:

$$\dot{v_3}(w_2, e_{s_2}) = -1 < 0 \qquad (19)$$

Thus, the subsystem is stabilized. Now, for stabilization of $\dot{e}_{s_4}$ *i.e* fourth subsystem in (12), let us consider an error w_3 between e_{s_1} *and* $\alpha_3(e_{s_4})$ is w_3 and if $\alpha_3(e_{s_4}) = 0$, then

$$w_3 = e_{s_1} - \alpha_3(e_{s_4}) = e_{s_1} \qquad (20)$$

Now, substituting $\dot{e}_{s_4}$ and e_{s_1} from (12) and (20), into time derivative of (20) yields,

$$\dot{w_3} = -b w_3 + u_4 \qquad (21)$$

Let's stabilize the (e_{s_1}, w_3) subsystem given by (21) as follows:

Let a Lyapunov function candidate be $\dot{v_4}((e_{s_1}, w_3) = \dot{v_1}(e_{s_1}) + \frac{1}{2}{w_3}^2$ and time derivative is given by,

$$\dot{v_4}(e_{s_1}, w_3) = \dot{v_1}(e_{s_1}) + w_3\dot{w_3} \qquad (22)$$

From equation (14) and (21), equation (21) can be written as:

$$\dot{v_4}(e_{s_1}, w_3) = -{e_{s_1}}^2 + w_3[-b w_3 + u_4]$$

Therefore, if $u_4 = 0$, above equation (23) can be written as: $\dot{v_4}(e_{s_1}, w_3) = -{e_{s_1}}^2 \leq 0$ and hence the subsystem $\dot{e}_{s_4}$ is stabilized.

Thus, the synchronization goal is achieved with the controller defined as:

$$\begin{cases} u_1 = -e_{s_2} - e_{s_4} \\ u_2 = 0 \\ u_3 = -e_{s_1}w_2 \\ u_4 = 0 \end{cases} \qquad (24)$$

These control laws u_1, u_2, u_3, u_4 of equation (24) cause the synchronization between the master and the slave systems and satisfies

$$\lim_{\substack{t\to\infty \\ i\to 4}} ||e_{si}(t)|| = 0$$

Hence, the error dynamics converges to zero and those states are asymptotically synchronized.

IV. RESULTS AND DISCUSSION

MATLAB ode45 is used for solving the dynamics of control and synchronization. Results are simulated with time step $\Delta t = 0.05$ and runs for T = 8 second.

The initial conditions for the master (1) and slave systems (10) are considered as $x(0) = y(0) = [1, -1, 1, -1]^T$, respectively. Parameters of this system (1) for chaotic behavior are α = 18.13 and β = 0.994. Figure2 shows the time response of states of the system (1) and ensures the chaotic behavior of the system.

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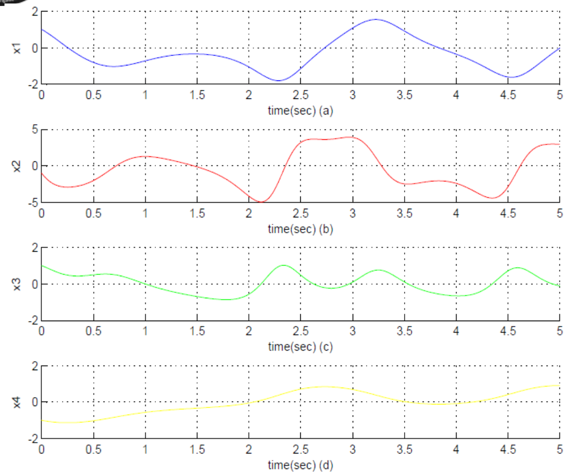

Figure2: Time response of the system

A. Control of the system

Taken chaotic system (1) is controlled at origin using the backstepping control law defined in (7). In the figure, the desired states x_d are taken as zero. States are converging quickly to zero after control input are applied at time t = 0. In figure3, the desired states are taken as $x_d = 10\sin(2.14t)$ and the states are converging to x_d quickly after control input is activated at time t = 0.

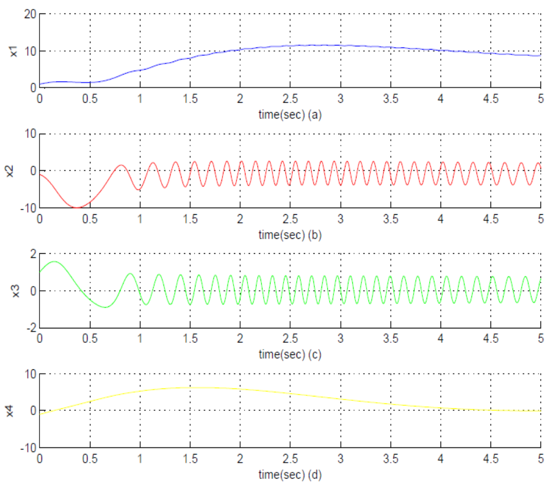

Figure3: Time response of the state variables when desired state is introduced.

B. Synchronization of the chaotic system

Figure4 show the synchronization between the master and slave systems for the first, second, third and fourth states respectively. Synchronization error between each state of master and slave system is shown in figure4. It may be observed that second and third states are taking more time to synchronize in comparison to other states.

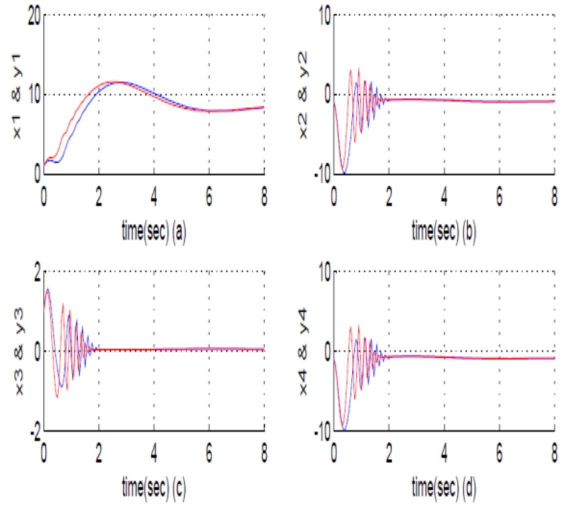

Figure4: Time response plot of master- slave synchronized state.

V. CONCLUSION

In this paper, tracking control and synchronization of a chaotic system is investigated. An active backstepping control law is designed to track the desired state as well as to achieve the synchronization between the chaotic systems as master and slave systems. This system is controlled at origin and tracking is shown for desired sine wave. Lyapunov stability condition is derived to ensure the stability of error dynamics. For the backstepping control technique, the error dynamics are chosen and derived in such a way that the results shows reduction is transient error response. The result also shows a fast transient error dynamics convergence and synchronization of those systems. An active backstepping control of this hyperchaotic hyperjerk system is derived and its MATLAB simulation is given in details. Tracking control and synchronization of this system using backstepping control is first time in literature.

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REFERENCES

[1] B. R. Andrievskii, and A. L. Fradkov, “Control of Chaos: Methos and Applications-II”,Automation and Remote Control, Vol. 65, No. 4, 2004, pp. 505-533.

[2] S. Boccalettia, J. Kurthsc, G. Osipovd, D. L. Valladaresb, C. S. Zhouc, “The synchronization of chaotic systems”, Physics Reports, vol. 366, pp.1-101, 2002.

 DOI : 10.35291/2454-9150.2019.0079

[3] G. Chen, and X. Dong, "From Chaos to Order, World scientific." Singapore 1998.

[4] H. K. Khalil, Nonlinear systems, 2nd edition, Prientice Hall, NJ, 1996.

[5] S. Vaidyanathan, A new 3-D jerk chaotic system with two cubic nonlinearities and its adaptive backstepping control, Archives of Control Sciences,Volume 27(LXIII), 2017,No. 3, pages 409–439.

[6] Irfan Ahmad, Banlue Srisuchinwong, and Wimol San-Um,On the First Hyperchaotic Hyperjerk System with No Equilibria, A Simple Circuit for Hidden Attractors ,2018.2850371, IEEE.

[7] S. Vaidyanathan, S. Jafari, V.T. Pham, A. T. Azar and F.E. Alsaadi, A 4-D chaotic hyperjerk system with a hidden attractor, adaptive backstepping control and circuit design, Archives of Control Sciences,Volume 28(LXIV), 2018.

[8] M.K. Shukla, and B.B. Sharma, "stabilization of a class of frictional order chaotic systems via backstepping approach", Chaos solitons and Fractals, vol. 98. Pp. 56-62, 2017.

[9] S. Bowong, F. M. M. Kakmeni,"Synchronization of uncertain chaotic systems via backstepping approach ", Chaos, Solitons and Fractals, vol. 21, pp. 999-1011, 2004.

[10] A. N. Njah, "Tracking control and synchronization of the new hyperchaotic Liu system via backstepping techniques", Nonlinear Dyn, vol. 61, pp. 1-9, 2010.

[11] U. E. Vincent,"Chaos Synchronization Using Active Control and Backstepping Control: A comparative Analysis ", Nonlinear Analysis: Modeling and Control, Vol. 13, No. 2, 253-261, 2008.

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