A New Similarity Measure on Interval Valued Intuitionistic Fuzzy Sets and Its Validation through Applications

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Abstract: The aim of this paper is to find a new distance measure on interval valued intuitionistic fuzzy numbers which computes the distance between interval valued intuitionistic fuzzy sets. Some properties of this new measure are also studied. Some relations and operations of interval-valued intuitionistic fuzzy numbers are suggested by which a ranking principle is established. Some examples are given to show the applicability of this measure in application problems.

Keywords* — *Distance Measure, Interval Valued Intuitionistic Fuzzy Set, Multi-criteria Fuzzy Decision-making, Pattern Recognition, Similarity Measure, TOPSIS Method.

I. INTRODUCTION

Fuzzy set was put forward by Zadeh [34] in the year 1965. This concept is successfully applied in different fields because of its usefulness. Fuzzy set was generalized to Intuitionistic fuzzy set (IFS) by Atanassov [1] in 1986 and further to vague set by Gau and Buehrer [8] in 1993. Both these sets are used to process imprecise or vague information. The IFS characterizes the degrees of belongingness and non-belongingness by membership and non-membershipfunctions respectively. Fuzzy set was further generalized to interval valued fuzzy sets (IVFS) by Gorzalaczany [9] and Turksen [27] and interval valued intuitionistic fuzzy sets ($IVIFS$) by Atanassov and Gargov [2].

Many inventors have analyzed IVFSs and its compatible topics, for example, Burillo and Bustince [4-6] explored entropy and distance for IVFSs, Grzegorzewski [10] exercised distance between IVFSsbased on the Hausdorff metric, Cheng and Li [18-19] studied the relationship between entropy and similarity measure of IVFSs.

Similarity measure and distance measure serve as a tool to solve practical applications. Both these measures, being counter parts of IFS, symbolize two expressions of the same measure. The similarity measure estimates the degree of similarity and hence the distance measure between IFSs. Hence similarity measures between two fuzzy sets have been defined by many authors [13-15], [18-20], [28]. Xu and Chen [31] have given a complete overview of the distance and similarity measures of IFSs and suggested additional continuous distance and similarity measures for IFSs.

Atanassov and Szmidt and Kacprzyk [24-26] have used Hamming distance and the Euclidean distance in various methods to calculate the distance between IFSs. Xu and Yager [32] recommended an improved degree of similarity between IFSs which is based on the method of Szmidt and Kacprzyk.

Hwang and Yoon [16] introduced TOPSIS method which is often used in multiple criteria decision making (MCDM) problem. Now extensive research involving TOPSIS theory and its applications is in progress. An algorithmic TOPSIS method for MCDM problem with interval data was developed by Jahanshaloo [17].

The approach of this paper is coordinated as follows: The definition of IFSs, IVIFSs, properties of distance measure and similarity measure, comparable, comparable by vagueness, comparable by impreciseness are briefly introduced in section 2. In section 3, new distance measure for IVIFS is introduced and analyzed. In section 4, new similarity measure for IVIFS is introduced and categorized. In section 5, the proposed method is studied by giving illustrative examples and is summarized by their counter intuitive examples. In section 6, a new ranking principle is established using the proposed similarity measure and applied in application problems by giving a

 DOI : 10.35291/2454-9150.2019.0094

numerical example. In section 7, conclusion and future scope are given.

II. PRELIMINARIES

A short review of preliminaries is given below.

Definition 2.1 [1]: An $IFSA$ of a non empty set X is defined as $A = \{(x, \mu_A(x), \vartheta_A(x))/x \in X\}$ where $\mu_A: X \to [0,1]$ and $\vartheta_A: X \to [0,1]$ define the degree of membership $\mu_A(x)$ and degree of non-membership $\vartheta_A(x)$ of x in X to lie in A, such that, $0 \le \mu_A(x) + \vartheta_A(x) \le 1$.

Definition 2.2 [2]: An $IVIFS$ on a nonempty set X is defined as $A = \{(x, \mu_A(x), \vartheta_A(x)) : x \in X\}$,

where $\mu_A(x) = [\underline{\mu}_A(x), \overline{\mu}_A(x)]$ and

$\vartheta_A(x) = [\underline{\vartheta}_A(x), \overline{\vartheta}_A(x)]$ are closed sub-intervals of $[0,1]$ which satisfy the condition $0 \le \overline{\mu}_A(x) + \overline{\vartheta}_A(x) \le 1$.

The collection of all $IVIFS$ on X is denoted by $IVIFS(X)$. An $IVIFS$ on singleton set is called $IVIF$ Number. The collection of all $IVIF$ Numbers is denoted by $IVIFN$.

Definition 2.3 [22]: Two $IVIFNs$, $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}]$, are said to be comparable, $\hat{A} \le_1 \hat{B}$, if $\mu_{a_1} \le \mu_{a_2}, \mu_{b_1} \le \mu_{b_2}, \nu_{c_1} \ge \nu_{c_2}$ and $\nu_{d_1} \ge \nu_{d_2}$.

Definition 2.4 [23]: Two $IVIFNs$, $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}]$, are said to be comparable by vagueness, $\hat{A} \le_2 \hat{B}$, if $\mu_{a_2} \le \mu_{a_1}, \mu_{b_1} \le \mu_{b_2}, \nu_{c_1} \le \nu_{c_2}$ and $\nu_{d_2} \le \nu_{d_1}$.

Definition 2.5 [23]: Two $IVIFNs$, $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}]$, are said to be comparable by impreciseness $\hat{A} \le_3 \hat{B}$, if $\mu_{a_2} \le \mu_{a_1}, \mu_{b_1} \le \mu_{b_2}, \nu_{c_2} \le \nu_{c_1}$ and $\nu_{d_1} \le \nu_{d_2}$.

Distance is a measure of the difference between two elements of a set. In the case of IVIFSs, the distance between two elements must satisfy the following axioms.

Definition 2.6 [31]: A mapping $D: IVIFS(X) \times IVIFS(X) \to [0,1]$ is called the distance measure on $IVIFS(X)$ if: For any $\hat{A}, \hat{B}, \hat{C} \in$ IVIFS(X)

(D1). $0 \le D(\hat{A}, \hat{B}) \le 1$.

(D2). $D(\hat{A}, \hat{B}) = 0$ if and only if $\hat{A} = \hat{B}$.

(D3). $D(\hat{A}, \hat{B}) = D(\hat{B}, \hat{A})$.

(D4). If $\hat{A} \le_1 \hat{B} \le_1 \hat{C}$, then $D(\hat{A}, \hat{B}) \le D(\hat{A}, \hat{C})$ and $D(\hat{B}, \hat{C}) \le D(\hat{A}, \hat{C})$.

(D5). $D(\hat{A}, \hat{C}) \le D(\hat{A}, \hat{B}) + D(\bar{B}, \bar{C})$.

The similarity measure is viewed as a complementary concept of distance measure which is defined as follows.

Definition 2.7 [31]: A function $S: IVIFS(X) \times IVIFS(X) \to [0,1]$ is called the similarity measure on $IVIFS(X)$ if: For any $\hat{A}, \hat{B}, \hat{C} \in$ IVIFS(X)

(S1). $0 \le S(\hat{A}, \hat{B}) \le 1$.

(S2). $S(\hat{A}, \hat{B}) = 1$ if and only if $\hat{A} = \hat{B}$.

(S3). $S(\hat{A}, \hat{B}) = S(\hat{B}, \hat{A})$.

(S4). If, $\hat{A} \le_1 \hat{B} \le_1 \hat{C}$, then $S(\hat{A}, \hat{B}) \ge S(\hat{A}, \hat{C})$ and $S(\hat{B}, \hat{C}) \ge S(\hat{A}, \hat{C})$.

Definition 2.8 [1]: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}] \in IVIFN$. Now, $\hat{A} \cap \hat{B}$, $\hat{A} \cup \hat{B}$ and $\hat{A}^c$ are defined by

$$\hat{A} \cap \hat{B} = \begin{pmatrix} [min\{\mu_{a_1}, \mu_{a_2}\}, min\{\mu_{b_1}, \mu_{b_2}\}], \\ [\max\{\nu_{c_1}, \nu_{c_2}\}, \max\{\nu_{d_1}, \nu_{d_2}\}] \end{pmatrix}$$

$$\hat{A} \cup \hat{B} = \begin{pmatrix} [max\{\mu_{a_1}, \mu_{a_2}\}, max\{\mu_{b_1}, \mu_{b_2}\}], \\ [\min\{\nu_{c_1}, \nu_{c_2}\}, \min\{\nu_{d_1}, \nu_{d_2}\}] \end{pmatrix},$$

$$\hat{A}^c = ([\nu_{c_1}, \nu_{d_1}], [\mu_{a_1}, \mu_{b_1}]).$$

Definition 2.9 [3]: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}]$ be two $IVIFNs$. Then, $\hat{A} + \hat{B} = ([\mu_{a_1} + \mu_{a_2} - \mu_{a_1}\mu_{a_2}, \mu_{b_1} + \mu_{b_2} - \mu_{b_1}\mu_{b_2}], [\nu_{c_1}\nu_{c_2}, \nu_{d_1}\nu_{d_2}])$.

Definition 2.10 [30]: The score function S of $IVIFN$ $\hat{A} = ([\mu_a, \mu_b], [\nu_c, \nu_d])$ is given by $S(\hat{A}) = (\mu_a + \mu_b - \nu_c - \nu_d)/2$.

Definition 2.11 [30]: An accuracy function H of $IVIFN$ $\hat{A} = ([\mu_a, \mu_b], [\nu_c, \nu_d])$ is expressed by $H(\hat{A}) = (\mu_a + \mu_b + \nu_c + \nu_d)/2$.

Definition 2.12 [33]: A novel accuracy function $M$of $IVIFN$ $\hat{A} = ([\mu_a, \mu_b], [\nu_c, \nu_d])$ is expressed by $M(\hat{A}) = \mu_a + \mu_b - 1 + (\nu_c + \nu_d)/2$.

Definition 2.13 [22]: A general accuracy function LG of $IVIFN$ $\hat{A} = ([\mu_a, \mu_b], [\nu_c, \nu_d])$, is expressed by $LG(\hat{A}) = ((\mu_a + \mu_b)(1 - \delta) + \delta(2 - \nu_c - \nu_d))/2$, where $\delta \in [0,1]$. We note that if $\delta = \frac{1}{2}$, $LG(\hat{A}) = \frac{1}{2} + \frac{\mu_a + \mu_b - \nu_c - \nu_d}{2} = \frac{1}{2} + S(\hat{A})$.

Definition 2.14 [31]: Let $A_j (j = 1, 2, \dots, n) \in IVIFS(X)$. The weighted arithmetic average operator

 DOI : 10.35291/2454-9150.2019.0094

$F_w(A_1, A_2, \dots, A_n)$ and weighted geometric operator $G_w(A_1, A_2, \dots, A_n)$ are defined by $F_w(A_1, A_2, \dots, A_n) =$

$$\left(\begin{array}{c} \left[\begin{array}{c} 1-\prod\left(1-\mu_{A_{j_L}}(x)\right)^{w_j}, \\ 1-\prod\left(1-\mu_{A_{j_U}}(x)\right)^{w_j} \end{array} \right], \\ \left[\prod\left(\gamma_{A_{j_L}}(x)\right)^{w_j}, \prod\left(\gamma_{A_{j_U}}(x)\right)^{w_j}\right] \end{array} \right)$$

$G_w(A_1, A_2, \dots, A_n) =$

$$\left(\begin{array}{c} \left[\prod\left(\mu_{A_{j_L}}(x)^{w_j}\right), \prod\left(\mu_{A_{j_U}}(x)^{w_j}\right)\right], \\ \left[1-\prod\left(1-\gamma_{A_{j_L}}(x)\right)^{w_j}, 1-\prod\left(1-\gamma_{A_{j_U}}(x)\right)^{w_j}\right] \end{array} \right),$$

w_j is the weight of $A_j (j = 1,2, \dots, n), w_j \in [0,1]$ and $\sum w_j = 1$. Especially, assume $w_j = 1/p \ (j = 1,2, \dots, p)$, then F_w and G_w are called arithmetic average operator and geometric operator for $IVIFSs$.

III. A New Distance Measure on IVIFN

Definition 3.1: A map $D: IVIFN \times IVIFN \rightarrow [0,1]$ between two $IVIFNs$ $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}]$ is defined by $D(\hat{A}, \hat{B}) = \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right| + \left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_2}}{2}\right| + \left|\frac{\nu_{d_1}-\nu_{d_2}}{2}\right|\right)$, where $\alpha \in [0, 1]$.

Proposition 3.1: $D: IVIFN \times IVIFN \rightarrow [0,1]$ is a distance measure

Proof: The conditions in definition 2.6, (D1), (D2), (D3) and (D4), are obvious. Let us prove (D5). Now $D(\hat{A}, \hat{C}) = \left(\left|\frac{\mu_{a_1}-\mu_{a_3}}{2}\right| + \left|\frac{\mu_{b_1}-\mu_{b_3}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_3}}{2}\right| + \left|\frac{\nu_{d_1}-\nu_{d_3}}{2}\right|\right)$
$\leq \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right| + \left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \left(\left|\frac{\mu_{a_2}-\mu_{a_3}}{2}\right| + \left|\frac{\mu_{b_2}-\mu_{b_3}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_2}}{2}\right| + \left|\frac{\nu_{d_1}-\nu_{d_2}}{2}\right|\right) + \alpha\left(\left|\frac{\nu_{c_2}-\nu_{c_3}}{2}\right| + \left|\frac{\nu_{d_2}-\nu_{d_3}}{2}\right|\right) = D(\hat{A}, \hat{B}) + D(\hat{B}, \hat{C})$.

Proposition 3.2: If $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = [\mu_{a_1} - \varepsilon_1, \mu_{b_1} + \varepsilon_2], [\nu_{c_1} + \varepsilon_3, \nu_{d_1} - \varepsilon_4]$, $\hat{C} = [\mu_{a_1} - \varepsilon_5, \mu_{b_1} + \varepsilon_6], [\nu_{c_1} - \varepsilon_7, \nu_{d_1} + \varepsilon_8] \in IVIFN, \varepsilon_i \geq 0$, then $D(\hat{A}, \hat{B}) = ((\varepsilon_1 + \varepsilon_2)(1-\alpha) + \alpha(\varepsilon_3 + \varepsilon_4))/2$ and $D(\hat{A}, \hat{C}) = ((\varepsilon_5 + \varepsilon_6)(1-\alpha) + \alpha(\varepsilon_7 + \varepsilon_8))/2$. (It is noted that $\hat{A} \leq_2 \hat{B}$ and $\hat{A} \leq_3 \hat{C}$).

Proposition 3.3: Let $\hat{A}, \hat{B}, \hat{C} \in IVIFN$, if $\hat{A} \leq_1 \hat{B} \leq_1 \hat{C}$, then $D(\hat{A}, \hat{C}) = D(\hat{A}, \hat{B}) + D(\hat{B}, \hat{C})$.

Proposition 3.4: Let $\hat{A} = (\mu_{a_1}, \nu_{c_1})$ and $\hat{B} = (\mu_{a_2}, \nu_{c_2})$ be two IFNs. Then $D(\hat{A}, \hat{B}) = |\mu_{a_1} - \mu_{a_2}|(1-\alpha) + \alpha|\nu_{c_1} - \nu_{c_2}|$.

Proposition 3.5: Let $\hat{A} = [\mu_{a_1}, \mu_{b_1}]$ and $\hat{B} = [\mu_{a_2}, \mu_{b_2}]$ be two IVFNs. Then $D(\hat{A}, \hat{B}) = \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right| + \left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \alpha\left|\frac{\mu_{a_2}-\mu_{a_1}}{2}\right| + \left|\frac{\mu_{b_2}-\mu_{b_1}}{2}\right|$.

Proposition 3.6: Let $\hat{A} = \mu_{a_1}$ and $\hat{B} = \mu_{a_2}$ be two Fuzzy numbers defined on singleton set. Then $D(\hat{A}, \hat{B}) = |\mu_{a_1} - \mu_{a_2}|(1-\alpha) + \alpha|\mu_{a_2} - \mu_{a_1}|$.

Proposition 3.7: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}] \in IVIFN$. Then

$$D(\hat{A} \cup \hat{B}, \hat{A}) = \left[\frac{|\mu_{a_1} - max\{\mu_{a_1}, \mu_{a_2}\}|}{2} + \frac{|\mu_{b_1} - max\{\mu_{b_1}, \mu_{b_2}\}|}{2}\right] (1-\alpha) + \alpha\left[\frac{|\nu_{c_1} - min\{\nu_{c_1}, \nu_{c_2}\}|}{2} + \frac{|\nu_{d_1} - min\{\nu_{d_1}, \nu_{d_2}\}|}{2}\right]$$

Proposition 3.8: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ and $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}] \in IVIFN$. Then

$$D(\hat{A} \cap \hat{B}, \hat{A}) = \left[\frac{|\mu_{a_1} - min\{\mu_{a_1}, \mu_{a_2}\}|}{2} + \frac{|\mu_{b_1} - min\{\mu_{b_1}, \mu_{b_2}\}|}{2}\right] (1-\alpha) + \alpha\left[\frac{|\nu_{c_1} - max\{\nu_{c_1}, \nu_{c_2}\}|}{2} + \frac{|\nu_{d_1} - max\{\nu_{d_1}, \nu_{d_2}\}|}{2}\right]$$

Proposition 3.9: The distance between two crisp numbers $\hat{A} = ([0, 0], [1, 1])$ and $\hat{B} = ([1, 1], [0, 0])$ is obtained as one $(D(\hat{A}, \hat{B}) = 1)$, which supports our existing crisp set theory.

Proposition 3.10: Let $\hat{A}, \hat{B}$ be two $IVIFNs$, if $\hat{A} \leq_1 \hat{B}$, then (i). $D(\bar{A} \cup \bar{B}, \bar{A}) = D(\hat{A}, \hat{B})$, (ii). $D(\hat{A} \cup \hat{B}, \hat{B}) = 0$ and (iii). $D(\hat{A} \cap \hat{B}, \hat{B}) = D(\hat{A}, \hat{B})$.

Proof: Since $\hat{A} \leq_1 \hat{B}$ we have $\hat{A} \cup \hat{B} = \bar{B}$ and $\hat{A} \cap \hat{B} = \hat{A}$ and hence the proposition.

Proposition 3.11: When $\hat{A} \leq_2 \hat{B}$ or $\hat{A} \leq_3 \hat{B}$, then $D(\hat{A} \cup \hat{B}, \bar{A}) = \left(\left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_2}}{2}\right|\right)$ and $D(\hat{A} \cup \hat{B}, \hat{B}) = \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{d_1}-\nu_{d_2}}{2}\right|\right)$ and hence

 DOI : 10.35291/2454-9150.2019.0094

$D(\hat{A} \cup \hat{B}, \hat{A}) \leq D(\hat{A}, \hat{B})$ and $D(\hat{A} \cup \hat{B}, \hat{B}) \leq D(\hat{A}, \hat{B})$.

Proposition 3.12: When $\hat{A} \leq_2 \hat{B}$ or $\hat{A} \leq_3 \hat{B}$, then $D(\hat{A} \cap \hat{B}, \hat{A}) = \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_2}}{2}\right|\right)$, $D(\hat{A} \cap \hat{B}, \hat{B}) = \left(\left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{d_1}-\nu_{d_2}}{2}\right|\right)$ and hence $D(\hat{A} \cap \hat{B}, \hat{A}) \leq D(\bar{A}, \bar{B})$ and $D(\hat{A} \cap \hat{B}, \hat{B}) \leq D(\bar{A}, \bar{B})$.

Proposition 3.13: Let $\hat{A}, \hat{B}$ be two $IVIFNs$. If $\hat{A} \leq_1 \hat{B}$, then $D(\hat{A} \cap \hat{B}, B) = D(\hat{A}, \hat{B})$ (since $\hat{A} \cap \hat{B} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}]) = \hat{A}$).

Proposition 3.14: Let $\hat{A}, \hat{B}, \hat{C}, \hat{D} \in IVIFNs$ and Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}])$, $\hat{C} = ([\mu_{a_3}, \mu_{b_3}], [\nu_{c_3}, \nu_{d_3}])$, $\hat{D} = ([\mu_{a_4}, \mu_{b_4}], [\nu_{c_4}, \nu_{d_4}])$. Then $D(\hat{A} + \hat{C}, \hat{B} + \hat{D}) \leq D(\hat{A}, \hat{B}) + D(\hat{C}, \hat{D})$.

Proof: Let $\hat{A}, \hat{B}, \hat{C}, \hat{D} \in IVIFNs$. $D(\hat{A} + \hat{C}, \hat{B} + \hat{D}) = [|\mu_{a_1} + \mu_{a_3} - \mu_{a_1}\mu_{a_3} - \mu_{a_2} - \mu_{a_4} + \mu_{a_2}\mu_{a_4}| + |\mu_{b_1} + \mu_{b_3} - \mu_{b_1}\mu_{b_3} - \mu_{b_2} - \mu_{b_4} + \mu_{b_2}\mu_{b_4}|]\left(\frac{1-\alpha}{2}\right) + \left(\frac{\alpha}{2}\right)[|\nu_{c_1}\nu_{c_3} - \nu_{c_2}\nu_{c_4}| + |\nu_{d_1}\nu_{d_3} - \nu_{d_2}\nu_{d_4}|] = (|(\mu_{a_1} - \mu_{a_2})(1 - \mu_{a_3}) + (\mu_{a_3} - \mu_{a_4})(1 - \mu_{a_2})| + |(\mu_{b_1} - \mu_{b_2})(1 - \mu_{b_3}) + (\mu_{b_3} - \mu_{b_4})(1 - \mu_{b_2})|)\left(\frac{1-\alpha}{2}\right) + \left(\frac{\alpha}{2}\right)(|\nu_{c_3}(\nu_{c_1} - \nu_{c_2}) + \nu_{c_2}(\nu_{c_3} - \nu_{c_4})| + |\nu_{d_3}(\nu_{d_1} - \nu_{d_2}) + \nu_{d_2}(\nu_{d_3} - \nu_{d_4})|) \leq (|(\mu_{a_1} - \mu_{a_2})(1 - \mu_{a_3})| + |(\mu_{a_3} - \mu_{a_4})(1 - \mu_{a_2})| + |(\mu_{b_1} - \mu_{b_2})(1 - \mu_{b_3})| + |(\mu_{b_3} - \mu_{b_4})(1 - \mu_{b_2})|)\left(\frac{1-\alpha}{2}\right) + \left(\frac{\alpha}{2}\right)(|\nu_{c_3}(\nu_{c_1} - \nu_{c_2})| + |\nu_{c_2}(\nu_{c_3} - \nu_{c_4})| + |\nu_{d_3}(\nu_{d_1} - \nu_{d_2})| + |\nu_{d_2}(\nu_{d_3} - \nu_{d_4})|) \leq \left(\left|\frac{\mu_{a_1}-\mu_{a_2}}{2}\right| + \left|\frac{\mu_{b_1}-\mu_{b_2}}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\nu_{c_1}-\nu_{c_2}}{2}\right| + \left|\frac{\nu_{d_1}-\nu_{d_2}}{2}\right|\right) = D(\hat{A}, \hat{B}) + D(\hat{C}, \hat{D})$.

Proposition 3.15: Let $\hat{A}, \hat{B}, \hat{C} \in IVIFNs$ and Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}])$, $\hat{C} = ([\mu_{a_3}, \mu_{b_3}], [\nu_{c_3}, \nu_{d_3}])$.Then $D(\hat{A} + \hat{C}, \hat{B} + \hat{C}) \leq D(\hat{A}, \hat{B})$.

Proof: By putting $\hat{D} = \hat{C}$ in Proposition (3.14), we have, $D(\hat{A} + \hat{C}, \ \hat{B} + \hat{C}) \leq D(\hat{A}, \hat{B})$.

Proposition 3.16: Let $\hat{A} \in IVIFN$. Then (i). $D[0, \hat{A}] = LG(\hat{A})$, (ii). $D[1, \hat{A}] = 1 - LG(\hat{A})$. Hence, $D[0, \hat{A}] + D[1, \ \hat{A}] = 1$.

Proposition 3.17: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}]) \in IVIFN$. Then $D(\hat{A}, \hat{A}^c) = (|\mu_{a_1} - \nu_{c_1}| + |\mu_{b_1} - \nu_{d_1}|)/2$.

IV. A New Similarity Measure on $IVIF$ Sets

Definition 4.1: A map $S: IVIFN \times IVIFN \rightarrow [0,1]$ between two $IVIFNs$, $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}])$ is defined as $S_p(\hat{A}, \hat{B}) = 1 - D(\hat{A}, \hat{B})$.

Proposition 4.1: The proposed map is a similarity measure on $IVIFN$.

Proof: The conditions (S1), (S2) and (S3) are obvious. Let us prove (S4). Since, $\hat{A} \leq_1 \hat{B} \leq_1 \hat{C}$, we know that, $D(\hat{A}, \hat{B}) \leq D(\hat{A}, \hat{C})$, implies that, $1 - D(\hat{A}, \hat{B}) \geq 1 - D(\hat{A}, \hat{C})$. Hence, $S_p(\hat{A}, \hat{B}) \geq S_p(\hat{A}, \hat{C})$ and similarly, $S_p(\hat{B}, \hat{C}) \geq S_p(\hat{A}, \hat{C})$.

Proposition 4.2: Let $\hat{A}, \hat{B}, \hat{C} \in IVIFNs$. If $\hat{A} \leq_1 \hat{B} \leq_1 \hat{C}$, then $S_p(\hat{A}, \hat{C}) = S_p(\hat{A}, \hat{B}) + S_p(\hat{B}, \hat{C})$.

Proposition 4.3: The similarity measure between two crisp numbers $\hat{A} = ([0,0],[1,1])$, $\hat{B} = ([1,1],[0,0])$ is obtained as zero $(S(\hat{A}, \hat{B}) = 0)$, which supports our existing crisp set theory.

Proposition 4.4: Let $\hat{A}, \hat{B}$ be two $IVIFNs$. If $\hat{A} \leq_1 \hat{B}$, then $S_p(\hat{A} \cup \hat{B}, \hat{A}) = S_p(\hat{A}, \hat{B})$, $S_p(\hat{A} \cup \hat{B}, \hat{B}) = 1$, also, $S_p(\hat{A} \cap \hat{B}, \hat{B}) = S_p(\hat{A}, \hat{B})$, ($\hat{A} \leq_1 \hat{B}$ implies that $\hat{A} \cup \hat{B} = \hat{B}$).

Proposition 4.5: Let $\hat{A}, \hat{B}$ be two $IVIFNs$. If $\hat{A} \leq_2 \hat{B}$ or $\hat{A} \leq_3 \hat{B}$, then $S_p(\hat{A} \cup \hat{B}, \hat{A}) \geq S_p(\hat{A}, \hat{B})$ and $S_p(\hat{A} \cup \hat{B}, \hat{B}) \geq S_p(\hat{A}, \hat{B})$.

Proposition 4.6: Let $\hat{A}, \hat{B}$ be two $IVIFNs$. If $\hat{A} \leq_2 \hat{B}$ or $\hat{A} \leq_3 \hat{B}$, then $S_p(\hat{A} \cap \hat{B}, \hat{A}) \geq S_p(\hat{A}, \hat{B})$ and $S_p(\hat{A} \cap \hat{B}, \hat{B}) \geq S_p(\hat{A}, \hat{B})$.

Proposition 4.7: Let $\hat{A}, \hat{B}$ be two $IVIFNs$. If $\hat{A} \leq_2 \hat{B}$, then $S_p(\hat{A} \cup \hat{B}, \hat{A}) + S_p(\hat{A} \cap \hat{B}, \hat{B}) = S_p(\hat{A}, \hat{B})$.

Proposition 4.8: Let $\hat{A}, \hat{B}, \hat{C}, \hat{D} \in IVIFNs$ and Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}])$, $\hat{C} = ([\mu_{a_3}, \mu_{b_3}], [\nu_{c_3}, \nu_{d_3}])$, $\hat{D} = ([\mu_{a_4}, \mu_{b_4}], [\nu_{c_4}, \nu_{d_4}])$. Then, $S_p(\hat{A} + \hat{C}, \hat{B} + \hat{D}) \geq S_p(\hat{A}, \hat{B}) + S_p(\hat{C}, \hat{D})$.

Proposition 4.9: Let $\hat{A}, \hat{B}, \hat{C} \in IVIFNs$ and Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$, $\hat{B} = ([\mu_{a_2}, \mu_{b_2}], [\nu_{c_2}, \nu_{d_2}])$, $\hat{C} = ([\mu_{a_3}, \mu_{b_3}], [\nu_{c_3}, \nu_{d_3}])$. Then $S_p(\hat{A} + \hat{C}, \hat{B} + \hat{C}) \geq S_p(\hat{A}, \hat{B})$.

 DOI : 10.35291/2454-9150.2019.0094

Proposition 4.10: Let $\hat{A} \in IVIFN$,
(i). $S_p[0,\hat{A}] = 1 - LG(\hat{A})$ (ii). $S_p[1,\hat{A}] = LG(\hat{A})$. Hence, $S_p[0,\hat{A}] + S_p[1,\hat{A}] = 1$.

Proposition 4.11: Let $\hat{A} = ([\mu_{a_1}, \mu_{b_1}], [\nu_{c_1}, \nu_{d_1}])$ be two $IVIFNs$. Now $S_p(\hat{A}, \hat{A}^c) = 1 - (|\mu_{a_1} - \nu_{c_1}| + |\mu_{b_1} - \nu_{d_1}|)/2$.

Definition 4.2: Let $X = \{x_i | i = 1,2,\dots,n\}$ and let $\hat{A} = \{x_i, ([\mu_{A_L}(x_i), \mu_{A_U}(x_i)], [\vartheta_{A_L}(x_i), \vartheta_{A_U}(x_i)])\}$ and $\hat{B} = \{x_i, ([\mu_{B_L}(x_i), \mu_{B_U}(x_i)], [\vartheta_{B_L}(x_i), \vartheta_{B_U}(x_i)])\}$ be two IVIFS(X).Then $S(\hat{A},\hat{B}) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left(\left|\frac{\mu_{A_L}(x_i)-\mu_{B_L}(x_i)}{2}\right| + \left|\frac{\mu_{A_U}(x_i)-\mu_{B_U}(x_i)}{2}\right|\right)(1-\alpha) + \alpha\left(\left|\frac{\vartheta_{A_L}(x_i)-\vartheta_{B_L}(x_i)}{2}\right| + \left|\frac{\vartheta_{A_U}(x_i)-\vartheta_{B_U}(x_i)}{2}\right|\right)$, where $\alpha \in [0,1] = 1 - \frac{1}{n}\sum_{i=1}^{n} D(\hat{A}_i, \hat{B}_i)$, where $\hat{A}_i$ and $\hat{B}_i$ are $IVIFNs$ defined on $\{x_i\}$.

V. Significance of the proposed method

In this section, the significance of the proposed method is shown by comparing with existing methods through numerical examples and the given method is validated through numerical examples in pattern recognition problem.

5.1. Drawbacks of existing similarity measures

5.1.1. Drawbacks of similarity measures Chen [7], Li and Cheng [18] and Hong and Kim [11]

Let $\hat{A} = \{x_i, (\mu_{\hat{A}}(x_i), \vartheta_{\hat{A}}(x_i))\}$ and $\hat{B} = \{x_i, (\mu_{\hat{B}}(x_i), \vartheta_{\hat{B}}(x_i))\}$ be two $IVIFS(X)$.
The similarity measures of (1). Chen- $S_C(\hat{A},\hat{B})$, (2). Li and Cheng- $S_{DC}(\hat{A},\hat{B})$ and (3). Hong and Kim- $S_H(\hat{A},\hat{B})$ given by $S_C(\hat{A},\hat{B}) = 1 - \sum_{i=1}^{n}\left|\frac{S_{\hat{A}}(x_i)-S_{\hat{B}}(x_i)}{2n}\right|$, where $S_{\hat{A}}(x_i) = \mu_{\hat{A}}(x_i) - \vartheta_{\hat{A}}(x_i)$, $S_{\hat{B}}(x_i) = \mu_{\hat{B}}(x_i) - \vartheta_{\hat{B}}(x_i)$. $S_{DC}(\hat{A},\hat{B}) = 1 - \sqrt[p]{\sum_{i=1}^{n}\left|\frac{\varphi_{\hat{A}}(x_i)-\varphi_{\hat{B}}(x_i)}{n}\right|^p}$, where p is a parameter and $\varphi_{\hat{A}}(x_i) = \frac{\mu_A(x_i)+1-\vartheta_A(x_i)}{2}$, $\varphi_{\hat{B}}(x_i) = \frac{\mu_{\hat{B}}(x_i)+1-\vartheta_{\hat{B}}(x_i)}{2}$.
$S_H(\hat{A},\hat{B}) = 1 - \sum_{i=1}^{n}\left|\frac{(\mu_{\hat{A}}(x_i)-\mu_{\hat{B}}(x_i))-(\vartheta_{\hat{A}}(x_i)-\vartheta_{\hat{B}}(x_i))}{2n}\right|$, are not reliable whenever $\mu_{\hat{A}}(x_i) = \vartheta_{\hat{A}}(x_i)$ and $\mu_{\hat{B}}(x_i) = \vartheta_{\hat{B}}(x_i)$ because $S_C(\hat{A},\hat{B}) = 1$, $S_{DC}(\hat{A},\hat{B}) = 1$, $S_H(\hat{A},\hat{B}) = 1$ and hence $\hat{A} = \hat{B}$. But the proposed measure $S_p(\hat{A},\hat{B}) = 1 - [|\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|(1-\alpha) + \alpha|\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|] = 1 - |\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)| \neq 1$ which supports human intuition. Hence $\hat{A} \neq \hat{B}$. The proposed similarity measure is proved better than the above existing similarity measures.

5.1.2. Drawbacks of Hung and Yang [13]

The similarity measures $S_{HY}^1(\hat{A},\hat{B})$, $S_{HY}^2(\hat{A},\hat{B})$ and $S_{HY}^3(\hat{A},\hat{B})$ in Hung and Yang are given by $S_{HY}^1(\hat{A},\hat{B}) = 1 - d_H(\hat{A},\hat{B})$, $S_{HY}^2(\hat{A},\hat{B}) = \frac{(e^{-d_H(\hat{A},\hat{B})} - e^{-1})}{(1-e^{-1})}$, $S_{HY}^3(\hat{A},\hat{B}) = \frac{(1-d_H(\hat{A},\hat{B}))}{(1+d_H(\hat{A},\hat{B}))}$, where $d_H(\hat{A},\hat{B}) = \frac{1}{n}\sum_{i=1}^{n} max(|\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|, |\vartheta_{\hat{A}}(x_i) - \vartheta_{\hat{B}}(x_i)|)$. The above measures are not reliable whenever $\hat{A} = (1,0)$, $\hat{B} = (0,0)$ since $S_{HY}^1(\hat{A},\hat{B}) = S_{HY}^2(\hat{A},\hat{B}) = S_{HY}^3(\hat{A},\hat{B}) = 0$. But, by the proposed similarity measure, whenever, $\hat{A} = ([1,1][0,0])$, $\hat{B} = ([0,0]\,[0,0])$, we have $S_p(\hat{A},\hat{B}) = 1 - \alpha$, where $\alpha \in (0,1)$. Hence the proposed similarity measure is better than the above existing similarity measures.

5.1.3. Drawbacks of similarity measures Hong and Kim [11], Liang and Shi [19] and Mitchell [21]

The similarity measures of (1). Hong and Kim-$S_H(\hat{A},\hat{B})$, (2). Li and Shi-$S_e^p(\hat{A},\hat{B})$ and (3). Mitchell- $S_{HB}(\hat{A},\hat{B})$ given by,
$S_H(\hat{A},\hat{B}) = 1 - \sum_{i=1}^{n}\frac{(|\mu_{\hat{A}}(x_i)-\mu_{\hat{B}}(x_i)| + |\vartheta_{\hat{A}}(x_i)-\vartheta_{\hat{A}}(x_i)|)}{2n}$,
$S_e^p(\hat{A},\hat{B}) = 1 - \sqrt[p]{\sum_{i=1}^{n}\frac{(\varphi_t(x_i)+\varphi_f(x_i))^p}{n}}$, where $\varphi_t(x_i) = \frac{|\mu_{\hat{A}}(x_i)-\mu_{\hat{B}}(x_i)|}{2}$, $\varphi_f(x_i) = \frac{|\vartheta_{\hat{A}}(x_i)-\vartheta_{\hat{B}}(x_i)|}{2}$,
$S_{HB}(\hat{A},\hat{B}) = \frac{1}{2}\left(\rho_\mu(\hat{A},\hat{B}) + \rho_\vartheta(\hat{A},\hat{B})\right)$, where $\rho_\mu(\hat{A},\hat{B}) = 1 - \sqrt[p]{\sum_{i=1}^{n}\frac{|\mu_{\hat{A}}(x_i)-\mu_{\hat{B}}(x_i)|^p}{n}}$,
$\rho_\vartheta(\hat{A},\hat{B}) = 1 - \sqrt[p]{\sum_{i=1}^{n}\frac{|\vartheta_{\hat{A}}(x_i)-\vartheta_{\hat{B}}(x_i)|^p}{n}}$ leads to anti-intuitive result since whenever $A = (1, 0), B = (0,0)$ and $C = (0.5, 0.5)$, we have, $S_H(\hat{A},\hat{B}) = 0.5 = S_H(\hat{A},\hat{C})$, $S_e^p(\hat{A},\hat{B}) = 0.5 = S_e^p(\hat{A},\hat{C})$ and $S_{HB}(\hat{A},\hat{B}) = 0.5 = S_{HB}(\hat{A},\hat{C})$. But for the proposed similarity measure when $\alpha \in [0, 0.5)$, $S_P(\hat{A},\hat{B}) > S_P(\hat{A},\hat{C})$ and when $\alpha \in (0.5, 1]$, $S_P(\hat{A},\hat{B}) < S_P(\hat{A},\hat{C})$. Only when $\alpha = 0.5$, we have $S_P(\hat{A},\hat{B}) = S_P(\hat{A},\hat{C})$. This supports our intuition. Further, whenever $\hat{A} = (\epsilon, 2\epsilon)$, $\hat{B} = (2\epsilon, 3\epsilon)$, $\hat{C} = (3\epsilon, 4\epsilon)$, $S_H(\hat{A},\hat{B}) = 1 = S_H(\hat{A},\hat{C})$, which is not reliable. But, $S_P(\hat{A},\hat{B}) = 1 - \epsilon$, $S_P(\hat{A},\hat{C}) = 1 - 2\epsilon$. That is, $S_P(\hat{A},\hat{B}) \geq S_P(\hat{A},\hat{C})$ which supports our intuition.

5.1.4. Drawbacks of cosine similarity measure [29]

Definition [29]: Let $X = \{x_1, x_2, \dots, x_n\}$. Let $\hat{A} = (x_i, A_i = [a_{A_i}, b_{A_i}], [c_{A_i}, d_{A_i}])$, $\hat{B} = (x_i, B_i = [a_{B_i}, b_{B_i}], [c_{B_i}, d_{B_i}])$, $i = 1,2,\dots,n$ be two $IVIFNs$ in $X = \{x_1, x_2, \dots, x_n\}$. Then the weighted similarity measure between two IVIFNs $\hat{A}$ and $\hat{B}$ is defined as $S(\hat{A},\hat{B}) = \sum_{i=1}^{n} w_i\, S(\hat{A}_i, \hat{B}_i)$, where w_i is the weight vector with

 DOI : 10.35291/2454-9150.2019.0094

$0 < w_i \le 1$ and $\sum w_i = 1$. When $w_i = \frac{1}{n}$, $S(\hat{A}, \hat{B}) = \frac{1}{n}\sum_{i=1}^{n} S(\hat{A}_i, \hat{B}_i)$.

The cosine similarity measure $C_{IFS}(\hat{A}, \hat{B})$ is given by

$C_{IFS}(\hat{A}, \hat{B}) = \frac{1}{n}\frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i) + \vartheta_A(x_i)\vartheta_B(x_i)}{\sqrt{\mu_A^2(x_i)+\vartheta_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\vartheta_B^2(x_i)}}$.

Let $X = \{x_1, x_2, \dots, x_n\}$, $\hat{A} = (x_i, A_i[\mu_A(x_i), \vartheta_A(x_i)])$, $\hat{B} = (x_i, A_i[\mu_A(x_i), \vartheta_A(x_i)])$, $i = 1, 2, \dots, n$.

(i). Clearly $C_{IFS}(\hat{A}, \hat{B})$ cannot be determined whenever either $A_i$or B_i equals $(0, 0)$. But $S_p(\hat{A}, \hat{B})$ is determinable and is given by $S_p(\hat{A}, \hat{B}) = 1 - \left[\left(\frac{\mu_{A_1}+\mu_{A_2}}{2}\right)(1-\alpha) + \alpha\left(\frac{\vartheta_{A_1}+\vartheta_{A_2}}{2}\right)\right]$.

(ii).Whenever $\mu_{\hat{A}}(x_i) = \vartheta_{\hat{A}}(x_i)$ and $\mu_{\hat{B}}(x_i) = \vartheta_{\hat{B}}(x_i)$, we have, $C_{IFS}(\hat{A}, \hat{B}) = \frac{1}{n}\frac{\sum_{i=1}^{n} 2\mu_{\hat{A}}(x_i)\mu_{\hat{B}}(x_i)}{\sqrt{2\mu_{\hat{A}}^2(x_i)}\sqrt{2\mu_{\hat{B}}^2(x_i)}} = 1$. Hence $\hat{A} = \hat{B}$, which is not reliable. But $S_p(\hat{A}, \hat{B}) = 1 - \left[|\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|\frac{(1-\alpha)}{2} + \frac{\alpha}{2}|\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|\right] = 1 - |\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)| \ne 1$, which supports human intuition. Hence, $\hat{A} \ne \hat{B}$.

(iii). Whenever, $\mu_{\hat{A}}(x_i) = 0 = \mu_{\hat{B}}(x_i)$. Then $C_{IFS}(\hat{A}, \hat{B}) = 1$ implies that, $\hat{A} = \hat{B}$, which is anti-intuitive. Similar result is obtained, when $\vartheta_{\hat{A}}(x_i) = 0 = \vartheta_{\hat{B}}(x_i)$. But $S_p(\hat{A}, \hat{B}) = 1 - |\vartheta_{\hat{A}}(x_i) - \vartheta_{\hat{B}}(x_i)|\alpha \ne 1$. Hence $\hat{A} \ne \hat{B}$. Whenever, $\vartheta_{\hat{A}}(x_i) = 0 = \vartheta_{\hat{B}}(x_i)$, $S_p(\hat{A}, \hat{B}) = 1 - |\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)|(1-\alpha) \ne 1$. Hence $\hat{A} \ne \hat{B}$.

(iv). Whenever, $\hat{A} = (0, 1)$, $\hat{B} = (0, 0)$. We have $C_{IFS}(\hat{A}, \hat{B}) = 0$. But the proposed measure $S_p(\hat{A}, \hat{B}) = 1 - \alpha$, where $\alpha \in (0, 1)$. For all the above cases the proposed similarity measure is proved better than the above existing similarity measure.

5.2 Application of the Proposed Similarity Measure to Pattern Recognition

Assume that three IFS on $A = \{x_1, x_2, x_3\}$ representing three patterns which are given by

$X_1 = \{(0.4, 0.4), (0.3, 0.3), (0.2, 0.2)\}$,

$X_2 = \{(0.3, 0.3), (0.3, 0.3), (0.3, 0.3)\}$,

$X_3 = \{(0.5, 0.5), (0.5, 0.5), (0.5, 0.5)\}$.

Assume that a sample $Y = \{(0.4, 0.4), (0.3, 0.3), (0.2, 0.2)\}$ is to be identified.

Table 5.1: The similarity measure between the known pattern and the unknown pattern in example (Patterns not discriminated are in bold type) (**Refer Page 12**).

The similarity degrees of $S(X_1, Y)$, $S(X_2, Y)$ and $S(X_3, Y)$ calculated for all similarity measure are shown in table 1.

The proposed similarity measure S_p can be calculated by above example as:

$S_p(X_1, Y) = 1$, $S_p(X_2, Y) = 0.93$, $S_p(X_3, Y) = 0.8$.

It is clear that Y is equal to X_1, which indicates that sample Y is indistinguishable from X_1. However, the similarity degree of $S(X_1, Y)$, $S(X_2, Y)$ and $S(X_3, Y)$ are equal to each other when S_C, S_H, S_{DC}, and C_{IFS} are employed. These four similarity measures will not be enough to discriminate the difference between the three patterns. This means that the proposed similarity measure is more applicable and useful.

VI. Ranking of Interval Valued Intuitionistic Fuzzy Sets Using the Proposed Similarity Measure by TOPSIS Method

In this section, the significance of the proposed distance measure is illustrated by a numerical example using the ranking in principle [16]. Here the performance of six alternatives on four attributes is given by IVFDM which is evaluated by TOPSIS method. The IVFDMD in step 1 is given below: $D =$

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{array}\begin{array}{c} V_1 \quad\quad\quad V_2 \quad\quad\quad V_3 \quad\quad\quad V_4 \\ \begin{bmatrix} [0.22, 0.65] & [0.43, 0.51] & [0.22, 0.56] & [0.23, 0.41] \\ [0.28, 0.59] & [0.70, 0.74] & [0.26, 0.58] & [0.31, 0.72] \\ [0.48, 0.77] & [0.50, 0.54] & [0.19, 0.53] & [0.11, 0.66] \\ [0.39, 0.70] & [0.64, 0.77] & [0.43, 0.71] & [0.44, 0.76] \\ [0.37, 0.44] & [0.34, 0.61] & [0.37, 0.65] & [0.34, 0.75] \\ [0.25, 0.66] & [0.29, 0.65] & [0.30, 0.72] & [0.45, 0.52] \end{bmatrix} \end{array}$$

The interval-valued fuzzy decision matrix D' in step 2 with is given below (**Refer Page 12**).

Step3, In this case, Here we consider v_1, v_2, and v_4 as benefit attributes and v_3 as cost attribute. Here, $J_1 = \{v_1, v_2, v_4\}$ and $J_2 = \{v_3\}$. The interval-valued positive-ideal solution and interval-valued negative ideal solution A^*, A^- are found as:

$$A^* = \begin{bmatrix} (0.48, 0.23, 0.29)(0.70, 0.23, 0.07) \\ (0.19, 0.47, 0.34)(0.45, 0.24, 0.31) \end{bmatrix}.$$

$$A^- = \begin{bmatrix} (0.22, 0.56, 0.22)(0.29, 0.49, 0.22) \\ (0.43, 0.24, 0.33)(0.11, 0.59, 0.30) \end{bmatrix}.$$

Applying step4, the separation measure based on the Distance measure and their normalized versions are depicted in table 6.1.

The distance measure is, $D(\hat{A}, \hat{B}) = \sum_{i=1}^{n}(\mu_{\hat{A}_{1i}} - \mu_{\hat{B}_{1i}})(1-\alpha) + \alpha(v_{\hat{A}_{1i}} - v_{\hat{B}_{1i}})$.

Table 6.1

Distance measures for the numerical example

	$D(T, A^*)$	$D(T, A^-)$
T_1	$0.78 - (0.02)\alpha$	$0.47 + (0.08)\alpha$
T_2	$0.41 - (0.11)\alpha$	$0.84 + (0.03)\alpha$
T_3	$0.54 - (0.21)\alpha$	$0.71 + (0.13)\alpha$
T_4	$0.40 - (0.15)\alpha$	$0.85 + (0.07)\alpha$
T_5	$0.76 - (0.14)\alpha$	$0.49 + (0.06)\alpha$
T_6	$0.75 - (0.09)\alpha$	$0.50 + (0.01)\alpha$

Separation measure based on the proposed distance measure (IFS) $D(T, A^*)$:

$D(T_1, A^*) = 0.78 - (0.02)\alpha > D(T_3, A^*) = 0.54 - (0.21)\alpha > D(T_2, A^*) = 0.41 - (0.11)\alpha > D(T_4, A^*) = 0.40 - (0.15)\alpha$, for all α. But $D(T_5, A^*) = 0.76 - (0.14)\alpha < D(T_6, A^*) = 0.75 - (0.09)\alpha$ provided

 DOI : 10.35291/2454-9150.2019.0094

$\alpha \in [0,0.2)$ and $D(T_5, A^*) = 0.76 - (0.14)\alpha \geq D(T_6, A^*) = 0.75 - (0.09)\alpha$ provided $\alpha \in [0.2, 1]$.
Hence $T_4 < T_2 < T_3 < T_5 < T_6 < T_1$ whenever $\alpha \in [0,0.2)$ and $T_4 < T_2 < T_3 < T_6 \leq T_5 < T_1$ whenever $\alpha \in [0.2, 1]$ depending on the individuals intention toward hesitation.

Separation measure based on the proposed distance measure (IFS), $D(T, A^-)$:
$D(T_4, A^-) = 0.85 + (0.07)\alpha > D(T_2, A^-) = 0.84 + (0.03)\alpha > D(T_3, A^-) = 0.71 + (0.13)\alpha$, for all α. Also $D(T_5, A^-) = 0.49 + (0.06)\alpha < D(T_6, A^-) = 0.50 + (0.01)\alpha$ provided $\alpha \in [0, \alpha')$ and $D(T_5, A^-) = 0.49 + (0.06)\alpha \geq D(T_6, A^-) = 0.50 + (0.01)\alpha$ for $\alpha \in [\alpha', 1]$ where $\alpha' = 0.2$. We also have $D(T_1, A^-) = 0.47 + (0.08)\alpha < D(T_6, A^-) = 0.50 + (0.01)\alpha$ provided $\alpha \in [0, \alpha")$ and $T_1 = 0.47 + (0.08)\alpha \geq T_6 = 0.50 + (0.01)\alpha$ provided $\alpha \in [\alpha", 1]$ where $\alpha" = 3/7$.

Hence $T_4 < T_2 < T_3 < T_6 < T_5 < T_1$ whenever $\alpha \in [0, \alpha')$, $T_4 < T_2 < T_3 < T_5 \leq T_6 < T_1$ whenever $\alpha \in [\alpha', \alpha")$ and $T_4 < T_2 < T_3 < T_5 < T_1 \leq T_6$ whenever $\alpha \in [\alpha", 1]$ with $\alpha' = 0.2, \alpha" = 3/7$ depending on the individuals intention toward hesitation.

6.1. Application of the Proposed Similarity Measure in Multicriteria Fuzzy Decision-Making ($MCDM$) Problem

A fuzzy MCDM problems with weights is given in this section. Let the set of alternatives be $X = \{X_1, X_2, \ldots, X_m\}$ and let the corresponding weights of the criteria $A_1, A_2, \ldots, A_n$ be $w_1, w_2, \ldots, w_n$, where $\sum_{j=1}^{n} w_j = 1$.

Let the IVIFN
$X_i = \{\langle X_j, [\mu_{X_iL}(A_j), \mu_{X_iU}(A_j)], [\gamma_{X_iL}(A_j), \gamma_{X_iU}(A_j)] \rangle | A_j \in A\}$, where $0 \leq \mu_{X_iU}(A_j) \leq \gamma_{X_iU}(A_j) \leq 1$, $\mu_{X_iL}(A_j) \geq 0$, $\gamma_{X_iL}(A_j) \geq 0$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m$ represent the performance of alternative X_j. The above $IVIFN$ is denoted by $\alpha_{ij} = ([x_{ij}, y_{ij}], [z_{ij}, w_{ij}])$, where the alternative X_i satisfies the criterion A_j with degree $[x_{ij}, y_{ij}]$ and the alternative X_i does not satisfy the criterion A_j at with degree $[z_{ij}, w_{ij}]$ as given by the decision maker. Therefore we can elicit a decision matrix $D = (\alpha_{ij})_{m \times n}$. We obtain the aggregating IVIFN α_i for $A_i (i = 1, 2, \ldots, m)$ as $\alpha_i = ([x_i, y_i], [z_i, w_i]) = F_{iw}(\alpha_{i1}, \ldots, \alpha_{in})$ or
$\alpha_i = ([x_i, y_i], [z_i, w_i]) = G_{iw}(\alpha_{i1}, \ldots, \alpha_{in})$ by applying definition (2.14) to the decision matrix.

The similarity measures between $A_i (i = 1, 2, \ldots, m)$ and $\cup_{i=1}^{m} \alpha_i$ is calculated using definition 4.1 and proposition 4.5 and the alternative X_j with higher similarity measure is considered priority. If the similarity measures would be equal, then they may be ranked for which the similarity measure with $\cup_{i=1}^{m} \alpha_i$ is lower.

6.1.1 Illustrative Example

We have utilised the example from Herrera and Herrera-Viedma [12] for a $MCDM$ problem along with minor corrections to demonstrate the application of the given similarity measure in a realistic scenario, and to validate its effectiveness. There is a panel with four possible alternatives to invest the money: (1). V_1 is a motor firm; (2). V_2 is a mobile firm; (3). $V_3$is a Bike firm; (4). V_4 is anelectronic firm. The investment company must take a decision according to the following three criteria: (1). D_1 is the risk analysis; (2). D_2 is the growth analysis; (3). D_3 is the environmental impact analysis. The four possible alternatives are to be evaluated using the interval valued intuitionistic fuzzy information by the decision maker under the above three criteria as listed in the following decision matrix. (**Refer Page 12-Table-3**)

Assuming the weights of D_1, D_2 and D_3 as $0.35, 0.25$ and 0.40, we obtain the weighted geometric average value α_i for $V_i (i = 1, 2, 3, 4)$ using definition (2.14) as follows
$\alpha_1 = ([0.2297, 0.4266], [0.3674, 0.4898])$,
$\alpha_2 = ([0.5102, 0.6581], [0.2416, 0.3419])$,
$\alpha_3 = ([0.5384, 0.7335], [0.1000, 0.2263])$,
$\alpha_4 = ([0.4181, 0.6000], [0.2260, 0.3618])$.
By applying definition (2.8), we get $\cup_{i=1}^{m} \alpha_i$ as $\cup_{i=1}^{m} \alpha_i = ([0.5384, 0.7335], [0.1000, 0.2263])$.
Now $S(\alpha_i, \cup_{i=1}^{m} \alpha_i)$ $(i = 1, 2, 3, 4)$ is found as
$S(\alpha_1, \cup_{i=1}^{m} \alpha_i) = 1 - 0.3078\,(1 - \alpha) - 0.26545\alpha = 0.6922 + 0.04235\alpha$,
$S(\alpha_2, \cup_{i=1}^{m} \alpha_i) = 1 - 0.0518\,(1 - \alpha) - 0.1286\alpha = 0.9482 - 0.0768\alpha$,
$S(\alpha_3, \cup_{i=1}^{m} \alpha_i) = 1 - 0(1 - \alpha) - 0\alpha = 1 + 0\alpha$,
$S(\alpha_4, \cup_{i=1}^{m} \alpha_i) = 1 - 0.1269\,(1 - \alpha) - 0.13075\alpha = 0.8731 - 0.00385\alpha$. Here all alternatives are ranked in accordance with the similarity measures with $\cup_{i=1}^{m} \alpha_i$ as follows $V_3 > V_2 > V_4 > V_1$, for any value of $\alpha \in [0, 1]$. We have,

$\alpha_2 = ([0.5102, 0.6581], [0.2416, 0.3419]) <_1 ([0.5384, 0.7335], [0.1000, 0.2263]) = \alpha_3$. But by the definition 2.12, we get $V_2 > V_4 > V_3 > V_1$ and by definition 2.11, we get $V_2 > V_3 > V_4 > V_1$ which are contradictions. This contradiction arises from the non-applicability and illogicality.

VII. Conclusions and Future Scope

In this paper a new similarity measure between IVIFSs is given and is applied. The new proposed similarity measure has been verified by comparison with the existing similarity measure in the illustrative examples. The usefulness is shown by applying the proposed method in a pattern recognition problem and in MCDM problems. In near future, the distance measure can be developed to any triangular, trapezoidal, IFNs or any two generalized IFNs by using proposed method in this paper which will

 DOI : 10.35291/2454-9150.2019.0094

opening of new research in pattern recognition and clustering by developing new algorithms.

ACKNOWLEDGMENT

The authors thank the valuable suggestions from anonymous referees for the betterment of the quality of the article.

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Table 5.1

Existing measure	$S(X_1,Y)$	$S(X_2,Y)$	$S(X_3,Y)$
$S_C = 1 - \sum_{i=1}^{n} \left\lvert \frac{S_{\hat{A}}(x_i) - S_{\hat{B}}(x_i)}{2n} \right\rvert$	**1**	**1**	**1**
$S_H = 1 - \sum_{i=1}^{n} \left\lvert \frac{(\mu_{\hat{A}}(x_i) - \mu_{\hat{B}}(x_i)) - (\vartheta_{\hat{A}}(x_i) - \vartheta_{\hat{B}}(x_i))}{2n} \right\rvert$	**1**	**1**	**1**
$S_{DC} = 1 - \sqrt[p]{\sum_{i=1}^{n} \left\lvert \frac{\varphi_{\hat{A}}(x_i) - \varphi_{\hat{B}}(x_i)}{n} \right\rvert^p}$	**1**	**1**	**1**
$S_{HY}^1 = 1 - d_H(\hat{A},\hat{B})$	1	0.96	0.8
$S_{HY}^2 = \frac{(e^{-d_H(\hat{A},\hat{B})} - e^{-1})}{(1 - e^{-1})}$	1	0.89	0.71
$S_{HY}^3 = \frac{(1 - d_H(\hat{A},\hat{B}))}{(1 + d_H(\hat{A},\hat{B}))}$	1	0.87	0.67
$C_{IFS} = \frac{1}{n} \frac{\sum_{i=1}^{n} \mu_{\hat{A}}(x_i)\mu_{\hat{B}}(x_i) + \vartheta_{\hat{A}}(x_i)\vartheta_{\hat{B}}(x_i)}{\sqrt{\mu_{\hat{A}}^2(x_i) + \vartheta_{\hat{A}}^2(x_i)}\sqrt{\mu_{\hat{B}}^2(x_i) + \vartheta_{\hat{B}}^2(x_i)}}$	**1**	**1**	**1**

The interval-valued fuzzy decision matrix D' in step 2:$D =$

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{array} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \\ \end{array}$$

$$\begin{bmatrix} (0.22, 0.35, 0.43) & (0.43, 0.49, 0.08) & (0.22, 0.44, 0.34) & (0.23, 0.59, 0.18) \\ (0.28, 0.41, 0.31) & (0.70, 0.26, 0.04) & (0.26, 0.42, 0.32) & (0.31, 0.28, 0.41) \\ (0.48, 0.23, 0.29) & (0.50, 0.46, 0.04) & (0.19, 0.47, 0.34) & (0.11, 0.34, 0.55) \\ (0.39, 0.30, 0.31) & (0.64, 0.23, 0.13) & (0.43, 0.29, 0.28) & (0.44, 0.24, 0.32) \\ (0.37, 0.56, 0.07) & (0.34, 0.39, 0.27) & (0.37, 0.35, 0.28) & (0.34, 0.25, 0.41) \\ (0.25, 0.34, 0.41) & (0.29, 0.35, 0.36) & (0.30, 0.28, 0.42) & (0.45, 0.48, 0.07) \end{bmatrix}$$

Table 6.2: Decision Matrix

	D_1	D_2	D_3
V_1	([0.4, 0.5], [0.3, 0.4])	([0.4, 0.6], [0.2, 0.4])	([0.1, 0.3], [0.5, 0.6])
V_2	([0.6, 0.7], [0.2, 0.3])	([0.6, 0.7], [0.2, 0.3])	([0.4, 0.6], [0.3, 0.4])
V_3	([0.7, 0.8], [0.1, 0.2])	([0.6, 0.7], [0.1, 0.3])	([0.4, 0.7], [0.1, 0.2])
V_4	([0.3, 0.6], [0.3, 0.4])	([0.5, 0.6], [0.3, 0.4])	([0.5, 0.6], [0.1, 0.3])

 DOI : 10.35291/2454-9150.2019.0094