Nonlinear Viscoelastic Dynamic and Modeling of Electrically Stimulated Cardiac Muscles using Control Theory

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Abstract : **Viscoelastic properties of human cardiac muscle makes complication by its own. It is not so easy to describe the force-velocity relation of cardiac muscle without considering viscoelastic model which led to Hill's force-velocity relation. This work investigates on the time-varying elasticity concept to describe the viscoelastic properties of cardiac muscle. It has been observed that, a time-varying elasticity has to be extended to a time, length, and velocity. Simulation result shows how a generalized force velocity description of cardiac muscle can be used to develop a realistic model. Human cardiac muscle can be made to contract by electrical stimulation. There are so many well established technique has already been used for this stimulation and new applications. In the recent twenty years, a number of dynamic muscle models based approach on force measurements taken from isometric contracting muscles have been developed and then stimulated by irregular pulse train. The results of experimental simulation have shown that a simple linear transfer-function model of muscle response may be in accurate over the full operational range, whereas nonlinear models based on local model networks (LMNs) are able to capture the nonlinear effects and also provides accuracy over a wide operational range. Here, in this paper, the use of an LMN with linear models have been investigated along with the type of network model used and some typical experimental results based on real measured data has also been depicted to compare the performance of designed model with previous research work.**

Keywords — LMN Model, Stimulated Cardiac Muscle, Force Model, Viscous Property , Stress-Strain Curve, Non-Linearity

I. INTRODUCTION

Mayow described muscle as an elastic material that changes due to metabolic processes in 1674 [1] whereas Weber analyzed muscle as an elastic spring whose stiffness varies depending on whether it is in a passive or active mode [2].After that,Chauveau and Laulani´e has also represented muscle as an elastic spring with time-dependent stiffness, and proved that shortening velocity affects force generation [3],[4].Then viscoelastic model has been proposed but stress relaxation by cardiac muscle are not explained by a simple viscoelastic model [5]. Hill reintroduced the viscoelastic muscle models concept after observing that external work done accelerating an inertial load was inversely related to reducing velocity [6]. Lumped muscle models are commonly based on Hill's equation and he experimentally have shown that, the measured force-velocity relation varies with different loading conditions [7], [8]. Like muscle, the muscle dynamic varies with loading conditions and muscle elasticity has been calculated in this paper by taking the partial derivative with respect to muscle length, the traditional definition of elasticity. This study attempts to identify the underlying mechanisms on the time-varying elasticity concept to describe the viscoelastic properties of cardiac muscle. It has been observed that, a time-varying elasticity has to be extended to a time, length, and velocity. Simulation result also shows how a generalized force velocity description of cardiac muscle can be used to develop a realistic model. In order to apply artificial electrical stimulation into the cardiac muscle, it is mandatory to take some basic physiological properties of the neuromuscular system into account. The experiments with linear system provide results which may be used to choose structure parameters of a dynamic model, but nonlinear models based on local model networks (LMNs) are able to capture the nonlinear effects and to provide accuracy over a wide operational range.

II. STRESS - STRAIN CURVE FOR CARDIAC MUSCLE

We must start with stress-strain relationships to understand the role of cardiac muscle in movement. Here the stress strain curves of stimulated muscle have been considered into the discussion. Unlike most of the uniform materials , muscle is a comparatively complex tissue and different component may contribute in different ways to the stress-strain curve. Passive muscle has a non-linear stress-strain curve. From the previous research work, it is observed that the rate of stress change as a function of strain change in

 DOI : 10.35291/2454-9150.2019.0097

unstipulated muscle is proportional to stress. Let μ is the stress and Ɛ is the strain of the cardiac muscle.

$$\frac{\partial \mu}{\partial \varepsilon} = a.(\mu + \alpha) \quad (1)$$

Where α *is a constant*. One solution of the differential equation is given below.

$$\mu = \sigma e^{ax} - \alpha \quad (2)$$

Where σ *is a free parameter*. Let y= $\mu + \boldsymbol{\alpha}$, means taking derivatives dy = d**μ.**

Again, $\frac{\partial y}{\partial \varepsilon} = a.y$, means $\frac{\partial y}{y} = \partial \varepsilon . a$ (3)

$$ln(y) = \boldsymbol{a}.\varepsilon \quad (4)$$

Y=$\sigma e^{\alpha x}$ and $\mu = \sigma e^{\alpha x} - \alpha$ (5)

Now if length verses tension is plotted for each muscle using above equation, the graph shows non linearity and shown in the Fig 1 given below.

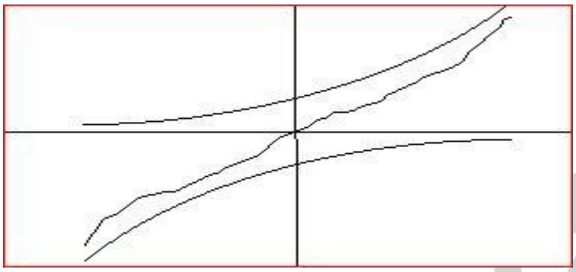

Figure 1: Represents the length verses tension for each cardiac muscle

III. Applied Method

Cardiovascular muscle senses the force generated due to the contraction and expansion of muscle wall .This can be well understood by the analytical approach of the transfer function generated by using a mechanical model of force displacement analogy[9],[10]. The efficiency of the work also lies in the measure of the movement of cardiovascular factors in the system. The input force causing the displacement of cardiac muscle is generated by the electrical and electrochemical activity in the cells. This input force is also known as muscle force which is represented by fm whereas the muscle length is denoted by fl. Now the time varying elasticity has been represented by the following equation given below.

$$\boldsymbol{\lambda_m(t)} = \frac{\boldsymbol{f_m(t)}}{\boldsymbol{f_l(t) - l_o}} \quad (6)$$

where l_o denotes the muscle length when no force is applied. Hence, cardiac muscle is thereby considered as a nonlinear spring whose stiffness varies with time and the corresponding mathematical equation is analogous to the time-varying elasticity. Solving equation 6 for muscle length fl and taking the derivative with respect to time gives the velocity of length shortening of cardiac muscle in the following equation.

$$\frac{df_l}{dt} = \frac{1}{\lambda_m}\frac{df_m}{dt} - \frac{f_m}{\lambda_m{}^2}\frac{d\lambda_m}{dt} + \frac{dl_o}{dt}. \quad (7)$$

The broadening concept of a time-varying elasticity to a time and length-varying elasticity can be expressed as follows

$$\boldsymbol{\lambda_m(t, f_l)} = \frac{\boldsymbol{f_m}}{\boldsymbol{m_l - l_0}} \quad (8)$$

IV. Generalized Force Model

As muscle force fm and muscle length fl are described here as a function of time t, the generalized force model equation can be represented as given below.

fm(fl,t) =a(fl-b)2+c(fl-d)2 (9)

Where, a is a measure of passive muscle elasticity and b represents the muscle length when force is zero .Parameters c and d describe muscle's active force generation. c, the length dependent component, is directly related to the muscle's contractile state. The model exhibits muscle's force-length relation. Let f(t) describes the time course of active force generation and if TC , TD are time constants characterizing the contraction (force increase) and relaxation (force decrease) processes respectively, then R is a measure of the overall rate of these processes.

$$f(t) = \frac{(1-e^{-\frac{t}{T_D}R})e^{-\frac{t-T_D}{T_C}}}{(1-e^{-\frac{t}{T_D}R})e^{-\frac{t-T_C}{T_C}}} \quad (10)$$

The activation function f(t) was modified to include velocity dependence by introducing k1 and k2 which describe how the number of bonds varies with muscle velocity at time t.

F(t,Vm)=f(t)+k1Vm(t)+K2 Vm(t-d) (11)

Where d represents the delay unit. Table 1 represents the extracted parameter from the muscle stripe.

Constants	Value [units]
a	1.9 [mN=mm²]
b	7.08 [mm]
c	17.2 [mN=mm]
d	150 [mN]
T_C	.19 [s]
T_D	.45 [s]
K_1,k_2	0.01 [s/mm]

Table 1: Represents the extracted parameter from the muscle stripe

V. Nonlinear Modeling of Cardiac Muscle

The simulation results obtained considering cardiac muscle as a linear system show that the muscle behavior is significantly nonlinear [11], [12]. In this work, we apply a nonlinear modeling approach, where information about the model structure is used from the linear experiments [13].The general architecture of a local model network (LMN) is shown in the Fig 2 given below.

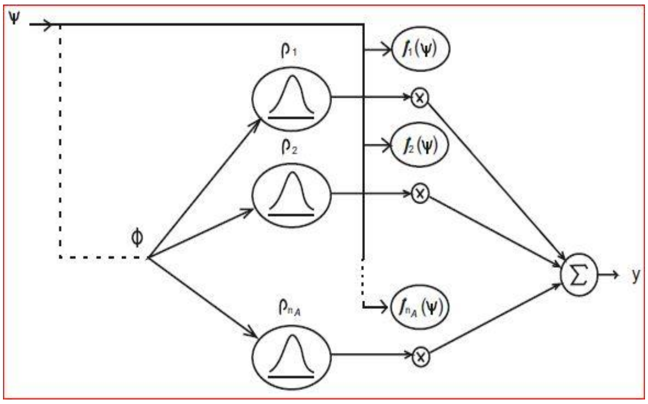

Figure 2: Represents the structure of LMN(Local model network)

 DOI : 10.35291/2454-9150.2019.0097

ISSN : 2454-9150 Vol-05, Issue-02, May 2019

Here ψ is the input data vector, Y is the output of the model and σ is the validity function which depends on some schedule vector Φ(t). The number of validity functions is given by VA. So the mathematical equation can be expressed as follows.

$$Y(t) = \sum_{i=1}^{V_A} \sigma_i \Phi(t) + f_i \psi(t) \qquad (12)$$

In general, the local models denoted by Fi in can be of any form, for example nonlinear or linear, state-space or input/output description, and discrete or continuous time. Using physical models of the system for operating conditions where they are available, and parametric models for conditions where there is no physical description available. However, for input/output modeling of dynamic systems, one often restricts oneself to a discrete-time with data vector ψ(t)[14],[15]. In the previous section the LMN was introduced as a general nonlinear model structure. Now we describe how this approach can be used for our application, mainly for the modeling of muscle contraction. To describe the very initial network structure the results of the linear system are used. Using the design parameters like second order, sampling period is 5ms,from previous research the input data vector ψ(t) becomes [u(t-1), u(t-3),y(t-1), y(t-2)]. Now using an LMN to model the dynamic input/output characteristic of a cardiac muscle during contraction and then comparing this equations with the response of the linear model, LMN model is able to approximate the muscle behavior much more accurately for all kind of situations. Here the transfer function of contracted cardiac muscle with respect to nonlinear approach using above model equations are tabulated in the table 2.

Model number	Transfer function	Gain
1	$\frac{s^{-2}(0.06 + 0.42s^{-1})}{1 - 1.30s^{-1} + .45s^{-2}}$	.74
2	$\frac{s^{-2}(0.86 + 0.112s^{-1})}{1 - 1.50s^{-1} + .24s^{-2}}$	.84
3	$\frac{s^{-2}(0.76 + 0.37s^{-1})}{1 - 1.35s^{-1} + .15s^{-2}}$	.68
4	$\frac{s^{-2}(0.96 + 0.82s^{-1})}{1 - 1.30s^{-1} + .11s^{-2}}$	.72
5	$\frac{s^{-2}(0.78 + 0.65)}{1 - 1.30s^{-1} + .83s^{-2}}$	.96
6	$\frac{s^{-2}(0.63 + 0.33s^{-1})}{1 - 1.30s^{-1} + .66s^{-2}}$	.51
7	$\frac{s^{-2}(0.17 + 0.02s^{-1})}{1 - 1.23s^{-1} + .49s^{-2}}$	.88

Table 2: Transfer function of contracted cardiac muscle

VI. SIMULATION RESULTS

In this work, quick release experiment has been performed on the time-varying elasticity model of cardiac muscle (length of 9 mm was released 5% over 9 msec). No force deactivation occurs for this model, which is depicted in the figure given below.

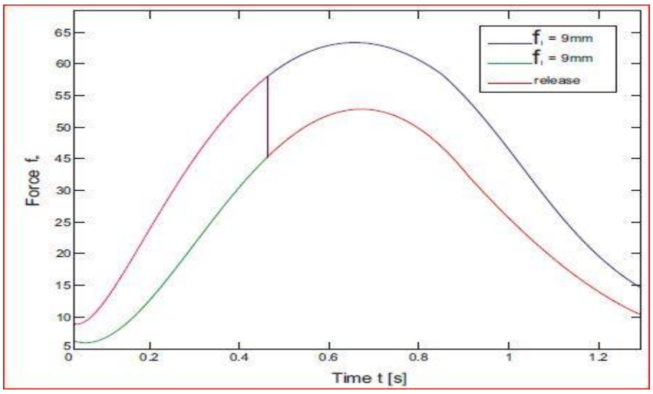

Figure 3: Quick release of cardiac muscle.(length 8 mm)

Similar experiment is done with length of 8.5 and 10 mm and simulation result is given below in Fig4.

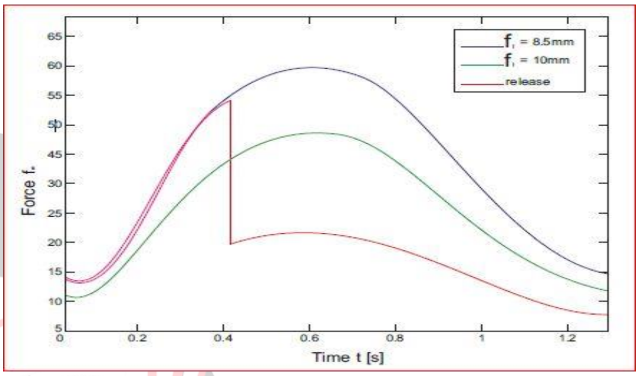

Figure 4: Quick release of cardiac muscle.(length 10 mm)

It is observed that, quick stretch experiment performed on the time and length-varying elasticity model of eq. 3 using eq. 4. Muscle length of 10 mm was stretched 5% over 10 msec. Muscle force overshoot is present, but stress relaxation is not for this model. Fig5 represents the simulation result.

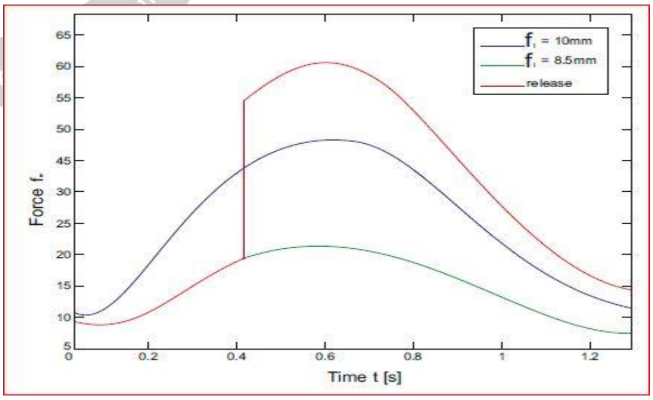

Figure 5: Quick stretch of cardiac muscle.(length 10 mm)

Finally, the quick release and stretch experiments computed for 5% changes in muscle length has been performed over 0.1s. Initial muscle length is 9mm. The dashed curves describe isometric force at the shorter (8.5mm) and longer (11.5mm) muscle lengths. The following Fig shows the simulation result.

 DOI : 10.35291/2454-9150.2019.0097

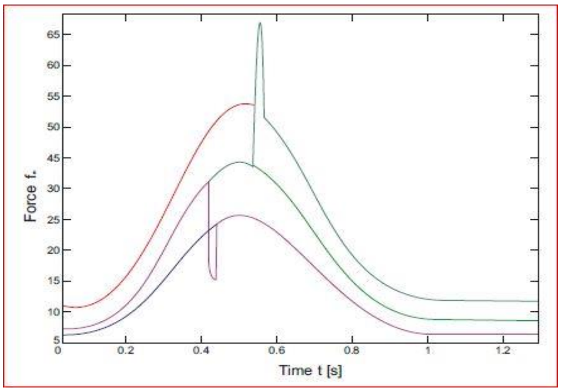

Figure 6: Quick release and stretch of cardiac muscle.

Another objective of the work is to design the proposed nonlinearity using LMN model. Comparing the response of the linear model as discussed earlier shows that the LMN model is able to approximate the muscle behavior much more accurately. Shape of the validity functions for the chosen model structure and gains are plotted and shown below in Fig 7.

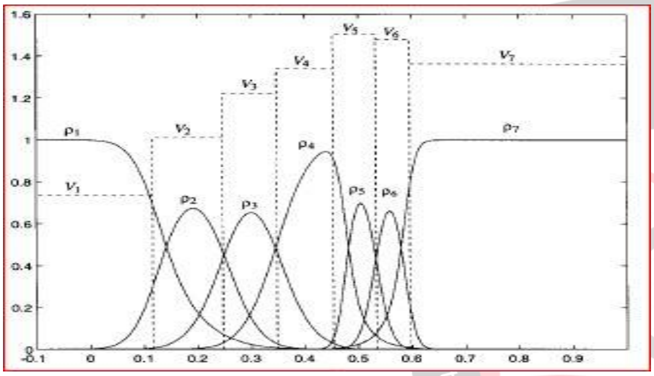

Figure 7: Validity functions and gains plots.

VII. Conclusion

This work investigates on the time-varying elasticity concept to describe the viscoelastic properties of cardiac muscle. It has been observed that, a time-varying elasticity has to be extended to a time, length, and velocity. Simulation result shows how a generalized force velocity description of cardiac muscle can be used to develop a realistic model. Human cardiac muscle can be made to contract by electrical stimulation. The experiments with linear system provide results which may be used to choose structure parameters of a dynamic model, but nonlinear models based on local model networks (LMNs) are able to capture the nonlinear effects and to provide accuracy over a wide operational range. Different simulation results show the verities in release and stretch of nonlinear viscoelastic Cardiac Muscle in different situations which will be beneficial for future research work.

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