International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-02, May 2019

Outer Sum Labeling Of Middle Graph Of Path And Cycle

Vishu Kumar M, Professor, School of Applied Science, REVA University, Bangalore, Karnataka, India, vishukumarm@reva.edu.in

Shravani Y, Student, REVA University, Bangalore, Karnataka, India, yshravani4@gmail.com

Abstract - Outer Sum labeling of a graph G is a labeling of a graph G is an injective function f from vertex set of G to Z^+ with the property that for every vertex $v \in V(G)$, there exist a vertex $w \in V(G)$ such that $f(w) = \sum_{u \in N(v)} f(u)$, where $N(v) = \{x : vx \in E(G)\}$. A graph G which admits an outer sum labeling is called an outer sum graph. If the graph G is not an outer sum graph then the minimum of isolated vertices required to make a graph G an outer sum graph is called outer sum number of G, and is denoted by $on(G)$. In this paper we obtain outer sum number of a middle graph of path and cycle.

AMS Subject classification : 05C78, 05C12, 05C15

Keywords: Sum labeling, Outer sum labeling, Outer sum number, Middle graph

I. INTRODUCTION

The graph $G(V,E)$ considered here be a simple connected, finite graph. For any $u, v \in V(G)$, $d(u,v)$ represents the shortest path between $u \& v$. A Sum labeling λ of a graph is a mapping of the vertices of G into distinct positive integers such that for any $u, v \in V(G)$, $uv \in E(G)$ if and only if the sum of the labels assigned to $u \& v$ equals the label of a vertex w of G. In such case w is called a working vertex. A graph which admits sum labeling is called a sum graph.

Sum graphs are originally introduced by Harary[2] and later extended to include al integer in [3]. Sum graphs cannot be connected graphs. Graphs which are not sum graphs can made a sum labeling by introducing number of isolated vertices which can bare the labels required by the graph. The minimum number of isolated vertices required by the graph G to support a sum labeling is called sum number of a graph G and is denoted by $\sigma(G)$.

A labeling of a graph G is an injective mapping $f : V(G) \to Z^+$. An outer sum labeling of a graph G with an added property that for every vertex $v \in V(G)$, there exists a $w \in V(G)$ such that $f(w) = \sum_{u \in N(v)} f(u)$, where $N(v) = \{x : vx \in E(G)\}$, A graph G which admits outer sum labeling is called an outer sum graph. If G is not outer sum graph then by adding isolated vertices to G we can make a resultant an outer sum graph is called the outer sum number of G and is denoted by $on(G)$.

II. DEFINITIONS.

1.1 Middle graph

The middle graph of a connected graph G denoted by $M(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if

i. They are adjacent edges of G or
ii. One is a vertex of G and the other is an edge incident with it.

III. SOME KNOWN RESULTS

Theorem 3.1. [4]
A Connected graph G is an outer sum graph if and only if $G \cong K_{1,n}$.

Theorem 3.2. [4]
For any $n \geq 3$, $on(C_n) = \begin{cases} 1 & if \quad n = 4 \\ 2 & otherwise \end{cases}$

Theorem 3.3. [4]
For any tree T on n vertices,
$$on(T) = \begin{cases} 0 & if \quad T \text{ is } star \\ 1 & otherwise \end{cases}$$

Theorem 3.4. [4]

For any positive integer n,

$$on(K_n)=\begin{cases}0 & if \quad n\le 2\\ n-1 & otherwise\end{cases}$$

Theorem 3.5. [4]

$$on(P_n+K)=\begin{cases}0 & if \quad n=1\\ 1 & if \quad n=3\\ 2 & otherwise\end{cases}$$

IV. MAIN RESULTS

Theorem 4.1 For any integer $on\left[M(P_n)\right]=1$,

Proof: By Theorem 3.1,

A connected graph G is an outer sum graph if and only if $G\cong K_{1,n}$

$$M(P_n)\ne K_{1,n} \text{ and } on\left(M(P_n)\right)\ge 1 \qquad (1)$$

To prove the reverse inequality, we define a following labeling procedure

$f(u_1)=1,\ f(v_1)=2,\ f(v_2)=3.$

$f(u_i)=f(v_{i-1})+f(v_i),\ 2\le i\le n-1$

$f(u_n)=\sum N(v_{n-2})$

$f(v_i)=\sum N(v_{i-2}),\ 3\le i\le n-1$

Also, we define a neighbourhood sum as follows

$N(u_1)=f(v_1)$

$N(u_i)=f(v_{i-1})+f(v_i)=f(u_i),\ 2\le i\le n-1$

$N(u_n)=f(v_n)$

$N(v_1)=f(u_1)+f(u_2)+f(v_2)=f(v_3)$

$N(v_i)=f(v_{i-1})+f(v_{i+1})+f(u_i)+f(u_{i+1})$
$=f(v_{i+2}),\ 2\le i\le n-3$

$N(v_{n-1})=f(u_n)+f(u_{n-1})+f(v_{n-2})=f(u)$

$N(v_{n-2})=f(v_{n-3})+f(v_{n-1})+f(u_{n-2})+f(u_{n-1})$

$N(v_n)=f(v_{n-1})$

From the above labeling procedure, we require one isolated vertex to make $M(P_n)$, an outer sum graph and hence

$$on\left(M(P_n)\right)\le 1 \qquad (2)$$

Thus $on\left(M(P_n)\right)=1$

Theorem 4.1 For positive integer n, $on\left[M(C_n)\right]\le 3$.

Proof: We define a following labeling procedure

$f(v_1)=1,\ f(v_2)=2,\ f(v_3)=4$

$f(u_i)=f(v_{i-1})+f(v_i),\ 2\le i\le n$

$f(u_1)=f(v_n)+f(v_1)$

$f(v_i)=f(v_{i-3})+f(u_{i-2})+f(u_{i-1})+f(v_{i-1}),$
$4\le i\le n$

Also, we define a neighbourhood sum as follows

$N(u_i)=f(v_{i-1})+f(v_i)=f(u_i),\ 2\le i\le n$

$N(u_1)=f(v_n)+f(v_1)=f(u_1)$

$N(v_i)=f(v_{i-1})+f(u_i)+f(u_{i+1})+f(v_{i+1})$
$=f(v_{i+2}),\ 2\le i\le n-2$

From the above it is clear that neighbourhood sum of three vertices has been not assigned to any vertices of $M(C_n)$, therefore at least three isolated vertices required to label them as follows.

$N(v_{n-1})=f(v_{n-2})+f(u_{n-1})+f(u_n)+f(v_n)$
$=f(u)$

$N(v_n)=f(v_{n-1})+f(v_1)+f(u_n)+f(u_1)=f(v)$

$N(v_1)=f(u_1)+f(u_2)+f(v_n)+f(v_2)=f(w)$

and hence $on\left(M(C_n)\right)\le 3$.

REFERENCES

[1] Buckely F and Harary F., Distance in Graphs, Addison-wesley. (1990).

[2] Frank Harary, Sum Graph and Difference Graphs, Congr. Numer., 72, (1990), 101-108.

[3] Frank Harary, Some Graph over all the integers, Disc. Math., 124, (1994), 99-105

[4] B Sooryanarayana, Manjula K and Vishu Kumar M, Outer Sum Labeling og a Graph, International Journal of Combinatorial Graph Theory and Applications, Vol 4, No. I, pp. 37-60.

[5] Z Chen, Harary's Conjectures on the Integral Sum Graphs, Disc, Math., 160, (1996), 241-244.

[6] Martin Sutton, Anna Draganova and Mirka Miller, Mod Sum Number of Wheels, Ars Combinatoria, 63, (2002), 273-287.

[7] W. He, Y. Shen, L. Wang, Y. Chang, Q. Kang and X. Yu, The integral sum number of complete bipartite graphs Kr,s, Discrete Math., 239 (2001), 137-146.

[8] Y. Wang and B. Liu, The sum number and integral sum number of complete bipartite graph, Discrete Math., 239 (2001), 69-82.

 DOI : 10.35291/2454-9150.2019.0116