

$N\delta\hat{g}$ Closed Sets in Nano Topological Spaces

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Abstract: In this paper a new class of sets, namely $N\delta\hat{g}$ -closed sets is introduced in nano topological spaces. We prove that this class lies between the class of $N\delta$ -closed sets and the class of $N\delta g$ -closed sets. Also we find some basic properties and characterisation of $N\delta\hat{g}$ -closed sets. Applying this sets to introduce an new space namely $NT_{3/4}$ -space and $N\hat{T}_{3/4}$ -space.

Keywords: $N\delta g$ -closed sets, $N\delta$ -closure, $N\hat{g}$ -open sets, $N\delta\hat{g}$ -closed sets, $NT_{3/4}$ -space and $N\hat{T}_{3/4}$ -space.

I. INTRODUCTION

M. Lellis Thivagar and Carmel Richard [6] introduced nano topological space (or simply NTS) with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X . He has also defined nano closed sets (briefly N-CS), nano interior and nano closure of a set. In 2013, M. Lellis Thivagar and Carmel Richard [6] introduced nano semi-open, nano regular-open, nano pre open, nano α -open. R. Lalitha and Dr. A. Francina shalini [5] introduced $N\hat{g}$ -closed set in nano topological spaces. The purpose of this present paper is to define a new class of nano closed sets called $N\delta\hat{g}$ -closed sets and also we obtain some basic properties of $N\delta\hat{g}$ -closed sets in nano topological space. Applying these sets, we obtain a new space which is called $NT_{3/4}$ -space, $N\hat{T}_{3/4}$ -space.

II. PRELIMINARIES

Throughout this paper, $(U, \tau_R(X))$ (or simply U) represent Nano Topological Spaces on which no separation axioms are assumed unless otherwise mentioned. For a set A in a NTS $(U, \tau_R(X))$, $Ncl(A)$, $Nint(A)$ and A^c denote the nano closure of A , the nano interior of A and the nano complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is called the approximation space.

Remark 2.2. [8] Let (U, R) be an approximation space and $X \subseteq U$. Then

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup \{R(x) : R(x) \subseteq X, x \in U\}$, where $R(x)$ denotes the equivalence class determined by x .

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup \{R(x) : R(x) \cap X \neq \phi, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.3. [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets (briefly N-OS).

Definition 2.4. [6] A subset A of a NTS $(U, \tau_R(X))$ is called a

(i) nano semi-open set (briefly N_s -OS) if $A \subseteq Ncl(Nint(A))$. (ii) nano pre-open set (briefly N_p -OS) if $A \subseteq Nint(Ncl(A))$. (iii) nano α -open set (briefly N_α -OS) if $A \subseteq Nint(Ncl(Nint(A)))$.

(iv) nano regular open set (briefly N_r -OS) if $A = Nint(Ncl(A))$.

The complement of a nano semi-open (resp. nano pre-open, nano α -open, nano regular open) set is called nano semi-closed (resp. nano semi-closed, nano α -closed, nano-regular closed).

Definition 2.5. [1] The $N\delta$ -interior of a subset A of U is the union of all nano regular open set of U contained in A and is denoted by $NInt_{\delta}(A)$. The subset A is called $N\delta$ -open if $A = NInt_{\delta}(A)$, i.e. a set is $N\delta$ -open if it is the union of nano regular open sets. The complement of a $N\delta$ -open is called $N\delta$ -closed. Alternatively, a set $A \subseteq (U, \tau_R(X))$ is called $N\delta$ -closed if $A = Ncl_{\delta}(A)$, where $NInt_{\delta}(A) = \{x \in U: NInt(Ncl(M)) \cap A \neq \emptyset, M \in \tau_R(X) \text{ and } x \in M\}$.

Definition 2.6. A subset A of a NTS $(U, \tau_R(X))$ is called

(i) nano generalized closed set (briefly $Ng - CS$) [3] if $Ncl(A) \subseteq M$ and M is a $N - OS$ in $(U, \tau_R(X))$.

(ii) nano $sg - closed$ set (briefly $Nsg - CS$) [2] if $Nscl(A) \subseteq M$ whenever $A \subseteq M$ and M is a $Ns - OS$ in $(U, \tau_R(X))$.

(iii) nano $gs - closed$ set (briefly $Ngs - CS$) [2] if $Nscl(A) \subseteq M$ whenever $A \subseteq M$ and M is a $N - OS$ in $(U, \tau_R(X))$.

(iv) nano $ag - closed$ set (briefly $Nag - CS$) [10] if $Nacl(A) \subseteq M$ whenever $A \subseteq M$ and M is a $N - OS$ in $(U, \tau_R(X))$.

(v) nano $ga - closed$ set (briefly $Nga - CS$) [10] if $Nacl(A) \subseteq M$ whenever $A \subseteq M$ and M is a $N\alpha - OS$ in $(U, \tau_R(X))$.

(vi) nano \hat{g} -closed set (briefly $N\hat{g} - CS$) [5] if $Ncl(A) \subseteq M$ whenever $A \subseteq M$ and M is a nano semi open set in $(U, \tau_R(X))$.

(vii) nano $\delta g - closed$ set (briefly $N\delta g - CS$) [1] if $Ncl_{\delta}(A) \subseteq M$ whenever $A \subseteq M$ and M is a nano open set in $(U, \tau_R(X))$.

The complement of a Ng -closed (resp. Nsg -closed, Ngs -closed, Nag -closed, Nga -closed, $N\hat{g}$ -closed and $N\delta g$ -closed) set is called Ng -open (resp. Nsg -open, Ngs -open, Nag -open, Nga -open, $N\hat{g}$ -open and $N\delta g$ -open).

Theorem 2.7. [5] Every nano open set is $N\hat{g} - open$

Proof. Let A be a nano open set in U . Then A^c is nano closed. Therefore, $Ncl(A^c) = A^c \subseteq U$ whenever $A^c \subseteq U$ and U is nano semi-open. This implies A^c is $N\hat{g} - closed$. Hence A is $N\hat{g} - open$.

Definition 2.8. [7] A subset A of a space (X, τ) is called $\delta\hat{g}$ -closed if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is a \hat{g} -open set in (X, τ) .

Definition 2.9. A space (X, τ) is called a

(i) $T_{3/4} - space$ [4] if every $\delta g - closed$ set in it is $\delta - closed$.

(ii) $\hat{T}_{3/4} - space$ [7] if every $\delta\hat{g} - closed$ set in it is $\delta - closed$.

Definition 2.10. A NTS $(U, \tau_R(X))$ is called a $N T_{1/2} - space$ [9] if every $Ng - closed$ set in it is $N - closed$.

III. $N\delta\hat{g}$ - CLOSED SETS

In this section we define the following definition.

Definition 3.1. A subset A of a space $(U, \tau_R(X))$ is called $N\delta\hat{g}$ -closed if $Ncl_{\delta}(A) \subseteq V$ whenever $A \subseteq V$ and V is a $N\hat{g}$ -open set in $(U, \tau_R(X))$.

Proposition 3.2. Every $N\delta$ -closed set is $N\delta\hat{g}$ -closed set.

Proof. Let A be an $N\delta$ -closed set and V be any $N\hat{g}$ -open set containing A . Since A is $N\delta$ -closed, $Ncl_{\delta}(A) = A$ for every subset A of U . Therefore $Ncl_{\delta}(A) \subseteq V$ and hence A is $N\delta\hat{g}$ -closed set.

Remark 3.3. The converse of the above theorem is not true in general as shown in the following example.

Example 3.4. Let $U = \{a, b, c\}$ with $U/R = \{a, b\}$ and $X = \{a\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$, $N\delta - closed = \{U, \emptyset\}$, $N\delta\hat{g} - closed = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$. Here $\{a, c\}$ is $N\delta\hat{g}$ -closed but not $N\delta$ -closed in $(U, \tau_R(X))$.

Proposition 3.5. Every $N\delta\hat{g}$ -closed set is Ng -closed.

Proof. Let A be an $N\delta\hat{g}$ -closed set and V be an any nano open set containing A in $(U, \tau_R(X))$. Since every nano open set is $N\hat{g}$ -open and A is $N\delta\hat{g}$ -closed, $Ncl_{\delta}(A) \subseteq V$ for every subset A of U . Since $Ncl(A) \subseteq Ncl_{\delta}(A) \subseteq V$, $Ncl(A) \subseteq V$ and hence A is Ng -closed.

Remark 3.6. An Ng -closed set is not $N\delta\hat{g}$ -closed set in general as shown in the following example.

Example 3.7. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{c\}, \{a, b\}\}$ and $X = \{a, c\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a, c\}, \{b\}\}$, $Ng - closed = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $N\delta\hat{g} - closed = \{U, \emptyset, \{b\}, \{a, c\}\}$. Then the set $\{a\}$ is Ng -closed but not $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$.

Proposition 3.8. Every $N\delta\hat{g}$ closed set is Ngs -closed.

Proof. Let A be an $N\delta\hat{g}$ -closed and V be any nano open set containing A in $(U, \tau_R(X))$. Since every nano open set is $N\hat{g}$ -open, $Ncl_{\delta}(A) \subseteq V$ for every subset A of U . Since $Nscl(A) \subseteq Ncl_{\delta}(A) \subseteq V$, $Nscl(A) \subseteq V$ and hence A is Ngs -closed.

Remark 3.9. A Ngs -closed set is not $N\delta\hat{g}$ -closed in general as shown in the following example.

Example 3.10. Let $U = \{a, b, c\}$ with $U/R = \{\{c\}, \{a, b\}\}$ and $X = \{a, c\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}\}$. Then the set $\{b\}$ is Ngs -closed but not $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$.

Proposition 3.11. Every $N\delta\hat{g}$ -closed set is Nag -closed.

Proof. It is true that $Nacl(A) \subseteq Ncl_{\delta}(A)$ for every subset A of U .

Remark 3.12. A Nag -closed set is not $N\delta\hat{g}$ -closed in general as shown in the following example.

Example 3.13. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}, \{b\}\}$ and $X = \{a, b\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$.

$a, b, \{c\}$. Then the set $\{a, c\}$ is $N\alpha g$ -closed but not $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$.

Proposition 3.14. Every $N\delta\hat{g}$ -closed set is $N\delta g$ -closed.

Proof. Let A be an $N\delta\hat{g}$ -closed set and V be any nano open set containing A . Since every nano open set is $N\hat{g}$ -open, $Ncl_\delta(A) \subseteq V$, whenever $A \subseteq V$ and V is $N\hat{g}$ -open. Therefore $Ncl_\delta(A) \subseteq V$ and V is nano open. Hence A is $N\delta g$ -closed.

Remark 3.15. A $N\delta g$ -closed set is not $N\delta\hat{g}$ -closed in general as shown in the following example.

Example 3.16. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, c\}$ with nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}\}$. Then the set $\{c\}$ is $N\delta g$ -closed but not $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$.

Remark 3.17. The class of $N\delta\hat{g}$ -closed sets is properly placed between the classes of $N\delta$ -closed and $N\delta g$ -closed sets.

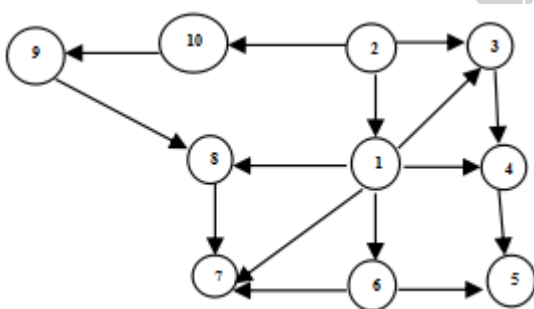
Remark 3.18. The following examples show that $N\delta\hat{g}$ -closeness is independent from $N\hat{g}$ -closeness, Nsg -closeness and $N\alpha g$ -closeness.

Example 3.19. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$, with nano topology $\tau_R(X) = \{U, \phi, \{a\}\}$. Then the set $\{a, b\}$ is $N\delta\hat{g}$ -closed but neither $N\hat{g}$ -closed nor Nsg -closed and the set $\{a, c\}$ is $N\delta\hat{g}$ -closed but not $N\alpha g$ -closed.

Also the another example, Let $U = \{a, b, c\}$ with $U/R = \{\{a, b\}, \{a\}, \{c\}\}$ and $X = \{a, c\}$ with nano topology

$\tau_R(X) = \{U, \phi, \{a, c\}, \{b\}\}$. Then the set $\{c\}$ is $N\hat{g}$ -closed, Nsg -closed, $N\alpha g$ -closed but not $N\delta\hat{g}$ -closed.

Remark 3.20. The following diagram shows the relationships of $N\delta\hat{g}$ -closed sets with other known existing sets. $A \rightarrow B$ represents A implies B but not conversely.



1. $N\delta\hat{g}$ -closed, 2. $N\delta$ -closed, 3. $N\delta g$ -closed, 4. Nsg -closed,
5. Ngs -closed, 6. $N\hat{g}$ -closed, 7. $N\alpha g$ -closed, 8. $N\alpha g$ -closed,
9. $N\alpha$ -closed, 10. N -closed.

IV. CHARACTERISATION

Theorem 4.1. The finite union of $N\delta\hat{g}$ -closed sets is $N\delta\hat{g}$ -closed.

Proof. Let $\{A_i / i = 1, 2, \dots, n\}$ be a finite class of $N\delta\hat{g}$ -closed subsets of a space $(U, \tau_R(X))$. Then for each $N\hat{g}$ -open set V_i in U containing A_i , $Ncl_\delta(A_i) \subseteq V_i$, $i \in \{1, 2, \dots, n\}$. Hence $\cup_i A_i \subseteq \cup_i V_i = W$. Since arbitrary union of $N\hat{g}$ -open sets in $(U,$

$\tau_R(X))$ is also $N\hat{g}$ -open set in $(U, \tau_R(X))$, W is $N\hat{g}$ -open in $(U, \tau_R(X))$. Also $\cup_i Ncl_\delta(A_i) = Ncl_\delta(\cup_i A_i) \subseteq W$. Therefore $\cup_i A_i$ is $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$. \square

Theorem 4.2. The arbitrary intersection of $N\delta\hat{g}$ -closed sets is $N\delta\hat{g}$ -closed.

Proof. Let $\{A_i / i = 1, 2, \dots\}$ be a arbitrary class of $N\delta\hat{g}$ -closed subsets of a space $(U, \tau_R(X))$. Then for each $N\hat{g}$ -open set V_i in U containing A_i , $Ncl_\delta(A_i) \subseteq V_i$, $i \in \{1, 2, \dots\}$. Hence $\cap_i A_i \subseteq \cap_i V_i = W$. Since arbitrary intersection of $N\hat{g}$ -open sets in $(U, \tau_R(X))$ is also $N\hat{g}$ -open set in $(U, \tau_R(X))$, W is $N\hat{g}$ -open in $(U, \tau_R(X))$. Also $\cap_i Ncl_\delta(A_i) = Ncl_\delta(\cap_i A_i) \subseteq W$. Therefore $\cap_i A_i$ is $N\delta\hat{g}$ -closed in $(U, \tau_R(X))$. \square

Proposition 4.3. Let A be a $N\delta\hat{g}$ -closed set of $(U, \tau_R(X))$. Then $Ncl_\delta(A) - A$ does not contain a non-empty $N\hat{g}$ -closed set.

Proof. Suppose that A is $N\delta\hat{g}$ -closed, let F be a $N\hat{g}$ -closed set contained in $Ncl_\delta(A) - A$. Now F^c is $N\hat{g}$ -open set of $(U, \tau_R(X))$ such that $A \subseteq F^c$. Since A is $N\delta\hat{g}$ -closed set of $(U, \tau_R(X))$, then $Ncl_\delta(A) \subseteq F^c$. Thus $F \subseteq (Ncl_\delta(A))^c$. Also $F \subseteq Ncl_\delta(A) - A$. Therefore $F \subseteq (Ncl_\delta(A))^c \cap (Ncl_\delta(A)) = \phi$. Hence $F = \phi$.

Proposition 4.4. If A is $N\hat{g}$ -open and $N\delta\hat{g}$ -closed subsets of $(U, \tau_R(X))$, then A is an $N\delta$ -closed subset of $(U, \tau_R(X))$.

Proof. Since A is $N\hat{g}$ -open and $N\delta\hat{g}$ -closed, $Ncl_\delta(A) \subseteq A$. Hence A is $N\delta$ -closed. \square

Proposition 4.5. If A is a $N\delta\hat{g}$ -closed set in a space $(U, \tau_R(X))$ and $A \subseteq B \subseteq Ncl_\delta(A)$, then B is also a $N\delta\hat{g}$ -closed set.

Proof. Let V be a $N\hat{g}$ -open set of $(U, \tau_R(X))$ such that $B \subseteq V$. Then $A \subseteq V$. Since A is $N\delta\hat{g}$ -closed set, $Ncl_\delta(A) \subseteq V$. Also since $B \subseteq Ncl_\delta(A)$, $Ncl_\delta(B) \subseteq Ncl_\delta(Ncl_\delta(A)) = Ncl_\delta(A)$. Hence $Ncl_\delta(B) \subseteq V$. Therefore B is also a $N\delta\hat{g}$ -closed set.

Theorem 4.6. Let A be $N\delta\hat{g}$ -closed of $(U, \tau_R(X))$. Then A is $N\delta$ -closed iff $Ncl_\delta(A) - A$ is $N\hat{g}$ -closed.

Proof. **Necessity:** Let A be a $N\delta$ -closed subset of U . Then $Ncl_\delta(A) = A$ and so $Ncl_\delta(A) - A = \phi$ which is $N\hat{g}$ -closed.

Sufficiency: Since A is $N\delta\hat{g}$ -closed, by proposition 4.3, $Ncl_\delta(A) - A$ does not contain a non-empty $N\hat{g}$ -closed set. But $Ncl_\delta(A) - A = \phi$. That is $Ncl_\delta(A) = A$. Hence A is $N\delta$ -closed. \square

V. APPLICATIONS

In this section we define the following definition.

Definition 5.1. A space $(U, \tau_R(X))$ is called a

(i) $NT_{3/4}$ -space if every $N\delta g$ -closed set in it is $N\delta$ -closed.

(ii) $N\hat{T}_{3/4}$ -space if every $N\delta\hat{g}$ -closed set in it is an $N\delta$ -closed.

Theorem 5.2. For a nano topological space $(U, \tau_R(X))$, the following conditions are equivalent.

(i) $(U, \tau_R(X))$ is a $N\hat{T}_{3/4}$ -space.

(ii) Every singleton $\{x\}$ is either $N\hat{g}$ -closed or $N\delta$ -open.

Proof. (i) \Rightarrow (ii) Let $x \in U$. Suppose $\{x\}$ is not a $N\hat{g}$ -closed set of $(U, \tau_R(X))$. Then $U - \{x\}$ is not a $N\hat{g}$ -open set. Thus $U - \{x\}$ is an $N\delta\hat{g}$ -closed set of $(U, \tau_R(X))$. Since $(U, \tau_R(X))$ is $N\hat{T}_{3/4}$ -space, $U - \{x\}$ is an $N\delta$ -closed set of $(U, \tau_R(X))$, i.e. $\{x\}$ is $N\delta$ -open set of $(U, \tau_R(X))$.

(ii) \Rightarrow (i) Let A be an $N\delta\hat{g}$ -closed set of $(U, \tau_R(X))$. Let $x \in Ncl_\delta(A)$. By (ii), $\{x\}$ is either $N\hat{g}$ -closed or $N\delta$ -open.

Case(i): Let $\{x\}$ be $N\hat{g}$ -closed. If we assume that $x \notin A$, then we would have $x \in Ncl_\delta(A) - A$, which cannot happen according to proposition 4.3. Hence $x \in A$.

case(ii): Let $\{x\}$ be $N\delta$ -open. Since $x \in Ncl_\delta(A)$, then $\{x\} \cap A \neq \emptyset$. This shows that $x \in A$.

So in both cases we have $Ncl_\delta(A) \subseteq A$. Trivially $A \subseteq Ncl_\delta(A)$. Therefore $A = Ncl_\delta(A)$ or equivalently A is $N\delta$ -closed. Hence $(U, \tau_R(X))$ is a $N\hat{T}_{3/4}$ -space.

Theorem 5.3. Every $NT_{3/4}$ -space is a $N\hat{T}_{3/4}$ -space.

Proof. The proof is straight forward since every $N\delta\hat{g}$ -closed set is $N\delta\hat{g}$ -closed set. \square

Remark 5.4. The converse of the above theorem is not true in general as it can be seen from the following example.

Example 5.5. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, c\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$. $(U, \tau_R(X))$ is a $N\hat{T}_{3/4}$ -space but not a $NT_{3/4}$ -space.

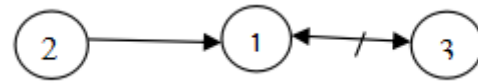
Remark 5.6. $N\hat{T}_{3/4}$ -space and $NT_{1/2}$ -space are independent of one another as the following example show.

Example 5.7. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, c\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$. $(U, \tau_R(X))$ is a $N\hat{T}_{3/4}$ -space but is not a $NT_{1/2}$ -space.

Example 5.8. Let $U = \{a, b, c\}$ with $U/R = \{\{a, c\}, \{b\}, \{a\}\}$ and $X = \{a\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, c\}, \{c\}\}$. $(U, \tau_R(X))$ is $NT_{1/2}$ -space but not a $N\hat{T}_{3/4}$ -space.

Remark 5.9. The following diagram shows the relationships

$N\hat{T}_{3/4}$ -space with other known existing spaces. $A \rightarrow B$ and $A \leftrightarrow B$ represent A implies B but not conversely and A & B are independent respectively.



1. $N\hat{T}_{3/4}$ -space, 2. $NT_{3/4}$ -space, 3. $NT_{1/2}$ -space.

VI. CONCLUSION

In this paper we introduced $N\delta\hat{g}$ -closed sets, properties and characterisation. Applying this sets to introduce a new space namely $NT_{3/4}$ -space and $N\hat{T}_{3/4}$ -space. Further using this sets we introduce $N\delta\hat{g}$ -continuous functions and $N\delta\hat{g}$ -homeomorphism. Also this type of $N\delta\hat{g}$ -continuous functions has a wide variety of applications in real life.

REFERENCES

- [1] M. Y. Bakeir, "Nano delta generalized closed sets via nano topology", 17th International Conference on Differential Geometry and Application: 1CDGA 2015, 2016.
- [2] K. Bhuvaneswari. and A. Ezhilarsi, "On nano semi-generalized and nano generalized, semi closed sets in nano topological spaces", International Journal of Mathematics and Computer Applications Research, 4(3), pp: 117-124, 2014.
- [3] K. Bhuvaneswari. and K. Mythili Gnanapriya, "Nano generalized closed sets in nano topological spaces", International Journal of Scientific and Research Publication, 4(5), pp: 1-3, 2014.
- [4] J. Dontchev and M. Ganster, "On δ -generalized closed sets and $T_{3/4}$ -spaces", Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17, pp: 15-31, 1996.
- [5] R. Lalitha and Dr. A. Francina shalini, "On nano \hat{g} -closed and open in nano topological spaces", International Journal of Applied research. 3(5): 368-371, 2017.
- [6] M. Lellis Thivagar and Carmel Richard, "On nano forms of weakly open sets", International Journal of Mathematics and Statistics Invention, 1(1), pp: 31-37, 2013.
- [7] M. Lellis Thivagar and B. Meera Devi., " $\delta\hat{g}$ -closed sets in Topological spaces", Gen. Math. Notes, vol. 1, No. 2, pp: 17-25, 2010.
- [8] Z. Pawlak, "Rough Sets", International Journal of Information and Computer Science, 11, pp: 341-356, 1982.
- [9] Qays Hatem Imran, "On Nano Generalized Semi Generalized Closed Sets", Iraqi Journal of Science, Vol. 57, No. 2C, pp: 1521-1527, 2016.
- [10] R. Thanga Nachiyar and K. Bhuvaneswari., "On nano generalized α -closed sets and nano α -generalized closed sets in nano topological spaces", International Journal of Engineering Trends and Technology, 13(6), pp: 257-260, 2014.