International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-03, June 2019

Application of Matrix operation in Transportation

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Abstract - Matrix operations can give very interesting results when it comes to Adjacency matrix of a Graph .If M is the Adjacency matrix for a graph representing a route map, then (i,j) entry of M^n gives count of n-step path from vertex "i" to vertex " j". Also If we consider a matrix $S_k = M + M^2 + \cdots + M^k$ then (i,j) entry in this matrix tells us the number of ways to get from vertex i to j in k steps or less, If entry is zero means you can't go (graph is not connected),Here We have applied concept of powers of matrix to airline route map. Paper is divided into three parts In part one matrix operations are discussed ,In part two we applied concepts to Adjacency Matrix and in part three we have applied it to route map of airline.

Keywords — Adjacency matrix, Boolean Product ,connected graph, Inverse, Linear mapping, Network analysis ,Transpose

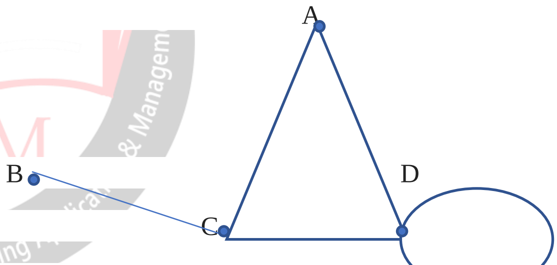
I. INTRODUCTION

Matrix(matrices plural) is a rectangular arrangement of numbers or abstract quantities in general. It represents mapping, so can be used to describe linear equations, to keep track of the coefficients of linear transformations and to record data, that depend on two parameters. Addition ,subtraction, transpose, inverse and multiplication are few operations defined on it. Matrices can be added, multiplied, and decomposed in different ways, which make them a key concept in matrix theory and linear Algebra. Matrix is represented by $A_{m\times n}$ where m represents rows(horizontal lines) and n represents columns(vertical line). A matrix with *m* rows and *n* columns is called an *m*-by-*n* matrix (written *m*×*n*) and *m* and *n* are called its dimensions. a_{ij} represents an element of a matrix in ith row and jth column.

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nm} \end{bmatrix}$$

A matrix where one of the dimensions equals to one is often called a *vector*, $A_{1\times n}$ matrix (one row and *n* columns) is called a row vector, and $A_{m\times 1}$ matrix (one column and *m* rows) is called a column vector. A Graph G(V,E) is a set of points called vertices or nodes and a set of lines connecting pairs of vertices are called edges. Adjacency matrix for a graph is a square matrix whose a_{ij} element is 1 if the ith vertex and jth vertex have edge joining them and it is 0 if they are not connected. Given a graph G of order n, with V(G)={$v_1, v_2 \ldots v_n$}, we define adjacency matrix A as

$$a_{ij} = \begin{cases} 1 & if \quad v_i v_j \in E(G) \\ 0 & if \quad v_i v_j \notin E(G) \end{cases}$$



Adjacency matrix for this graph is

$$\begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

II. OPERATIONS ON MATRIX

MATRIX ADDITION OR SUBTRACTION: We can add(subtract) matrices together as long as their dimensions are the same, i.e. both matrices have the same number of rows and columns. To add(subtract) two matrices, we add(subtract) the numbers of each matrix that are in the same element position.

TRANSPOSE: Transposing a Matrix To transpose a matrix, we swap the rows for the columns. To indicate that we are transposing a matrix, we add a "T" to the top right-hand corner of the matrix.

 DOI : 10.35291/2454-9150.2019.0200

INVERSE: This is a mathematical operation that finds a matrix which when multiplied to the original matrix gives Identity matrix.

PRODUCT: The matrix product is basically represents composition of linear maps , that are represented by matrices. Matrix multiplication Plays a important role in Linear Algebra and as such has many applications in mathematics, statistics, physics, economics and engineering. In more detail, if A is an $m \times r$ matrix and B is an $r \times$ n matrix, their matrix product AB is an $m \times$ n matrix, in which the r entries across a row of A are multiplied with the r entries down a column of B and summed to produce an entry of AB. When two linear maps are represented by matrices, then the matrix product represents the composition of the two maps.

Let AB=C then (i,j) entry of C, c_{ij} is given by

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{ir}b_{rj} = \sum_{k=1}^{r} a_{ik}b_{kj}$$

POWERS: Repeated multiplication of matrix gives powers of matrix. If C= A^2 then c_{34} (3,4 entry of A^2) is

$$c_{34} = a_{31}a_{14} + a_{32}a_{24} + a_{33}a_{34} + a_{34}a_{44} + a_{35}a_{54} \ldots$$

Matrix addition ,subtraction, transpose, inverse multiplication, product ,eigenvalues and eigen vectors are the mathematical operations which becomes very interesting when applied to adjacency matrix and plays a important role in network analysis. In this paper we are going to apply same to airlines.(social network).

III. Matrix Operations on Adjacency Matrix

MATRIX ADDITION (SUBTRACTION): Network analysis is a field where Matrix addition and matrix subtraction are used to simplify or reduce the complexity of multiplex data to simpler version. For example let A and B represents asymmetric matrix representing tie “exchanges money” and the relation “exchanges goods” respectively, then A+B represents intensity of exchange relationship like, pair with score of zero would have no relationship , with “1” would be involved in either barter or commodity exchange, and those with a “2” would have both barter and commodity exchange relations. on the other hand B-A ie subtraction of good exchange matrix from money exchange matrix, a score of -1 shows pairs with barter relationship,0 indicate no relationship or barter and commodity tie, a score of +1 express pairs with only a codified exchange relationship.

If A and B are adjacency matrix expressing

A=Exchange money

B=Exchange goods

Then $A+B=\begin{cases} 0 & \text{no relationship} \\ 1 & \text{barter or exchange} \\ 2 & \text{barter and exchange} \end{cases}$ And

$$B\text{-}A=\begin{cases} -1 & \text{barter relationship} \\ 0 & \text{no relationship or commodity tie} \\ 1 & \text{codified exchange relationship} \end{cases}$$

PRODUCT AND POWER

Adjacency matrix are square matrix so finding product or power is always possible . In many cases Boolean matrix multiplication is applied , It is similar to normal multiplication only difference is enter zero if product is zero and 1 if it is not zero.

Raising matrix to a power simply means repeated multiplication ,In case of adjacency matrix we can conclude many important information from it.

If A is any adjacency matrix of a graph .Then (i,j) entry of A^n gives count of n-step path from vertex i to vertex j.

For example let us have a adjacency matrix $A = a_{ij}$ of a simple graph with 6 vertices

Let C= A^2 then c_{34} (3,4 entry of A^2) is

$$c_{34} = a_{31}a_{14} + a_{32}a_{24} + a_{33}a_{34} + a_{34}a_{44} + a_{35}a_{54} \quad \ldots(1)$$

Now $a_{31}a_{14}$ counts product of ,count of number of paths from vertex 3 to 1 and the number of paths from 1 to vertex 4,this product exactly gives count of two step path from vertex 3 to vertex 4,via vertex 1.

In general terms we can say $a_{3k}a_{k4}$ counts two step path from vertex 3 to vertex 4 via vertex k.

And the sum of all terms of (1) gives all two-step path from vertex 3 to vertex 4.so if $c_{34} = n$

Means there are n two step path from vertex 3 to vertex 4.

Here we can conclude one more interesting fact ,consider a matrix S_k such that

$$S_k = M + M^2 + \cdots + M^k \quad \ldots(2)$$

The (i,j) entry in this matrix tells us the number of ways to get from vertex i to j in k steps or less, If entry is zero means you can’t go .so if we find $S_1, S_2 \ldots S_k$

Then the first K for which (i,j) entry in S_k is non-zero is the shortest number of steps between i and j. By this we can compute shortest number of steps but we can’t determine what those steps are.This procedure can also help us to determine connected nature of graph .i.e if S_k have zero till the last step for any vertex in above procedure then it is

 DOI : 10.35291/2454-9150.2019.0200

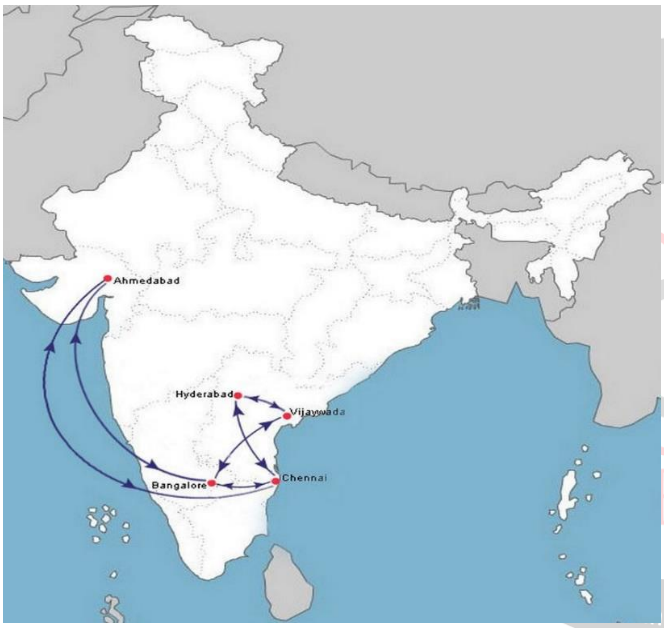
impossible to connect those vertices and the graph is not connected.

BOOLEAN PRODUCT: In case of Boolean product ,the adjacency matrix squared would tell us whether there was a path of length two between two vertices but not the number of paths .same way cube will tell that there is path of two steps between vertices but number of path it won't tell. It plays important role in social networks.

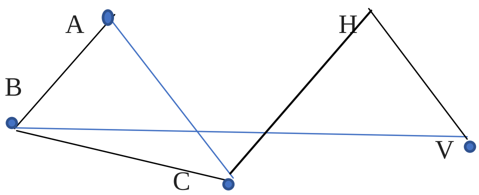
IV. Application to route Map

The above concept of raising power can be used in many practical problems like business ,social network, transportation problems etc .In this paper we will apply it on airline network.

Example:1Consider a Imaginary air route map of one private airline



Let A,B,C,V and H represents the cities Ahmedabad, Bangalore, Chennai, Vijayawada and Hyderabad respectively then the graph for this is



$$m_{ij} = \begin{cases} 1 & connected \\ 0 & otherwise \end{cases}$$

Adjacency matrix for above graph is given by

$$M= \begin{array}{c} \\ A \\ B \\ C \\ V \\ H \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & V & H \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

It is assumed that service between two cities is two ways i.e for connectivity matrix M , $M = M^T$

$M_{23} = 1$ means there is direct flight from Bangalore to Chennai and since $M_{43} = 0$,there is no direct flight from Vijayawada to Chennai.

Now

$\sum C_A = 2$,means from Ahmedabad two flights take off(land).

$\sum C_B = 3$,means from Bangalore three flights take off(land).

$\sum C_C = 3$,means from Chennai three flights take off(land).

$\sum C_V = 2$,means from Vijayawada two flights take off(land).

$\sum C_H = 2$,means from Hyderabad two flights take off(land).

In terms of connectivity Chennai and Bangalore are better than other three cities.

$$M^2 = \begin{array}{c} \\ A \\ B \\ C \\ V \\ H \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & V & H \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 0 & 2 \\ 1 & 1 & 3 & 2 & 0 \\ 1 & 0 & 2 & 2 & 0 \\ 1 & 2 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

From the above M^2 we can tell, how many two step or with one stoppage flights are there.

For example there was no direct flight from Vijayawada to Chennai but with one stoppage ,we can travel between these two cities in two ways.(from matrix we can tell number of ways but we can't tell the ways)

If you observe map two ways are

Vijayawada ⟶ Hyderabad ⟶ Chennai

Vijayawada ⟶ Bangalore⟶ Chennai

Similarly we can talk about any cities.

Let us find out shortest number of ways to travel from one city to other in this graph using $S_k(2)$

$$S_1 = M = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

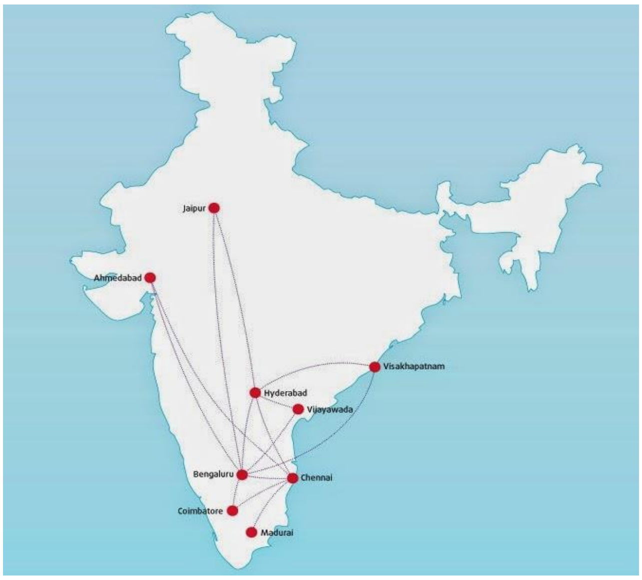
$$S_2 = M + M^2 = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 \\ 2 & 3 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{bmatrix}$$

Since in S_2 all entries are non -zero means our base graph was connected and in maximum one stoppage we can reach from any one city to any other city.

Example2. Let us analyze a bigger network, Let M be the adjacency matrix for below route map

 DOI : 10.35291/2454-9150.2019.0200

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-03, June 2019



$$M=\begin{array}{c} \\ A\\B\\C\\D\\E\\F\\G\\H\\I \end{array}\begin{array}{c} \begin{array}{ccccccccc} A&B&C&D&E&F&G&H&I \end{array}\\ \begin{bmatrix} 0&1&0&1&0&0&0&0&0\\ 1&0&1&1&0&1&1&1&1\\ 0&1&0&1&0&0&0&0&0\\ 1&1&1&0&1&0&1&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&1&0&0\\ 0&1&0&1&0&1&0&1&1\\ 0&1&0&0&0&0&1&0&0\\ 0&1&0&0&0&0&1&0&0 \end{bmatrix}\end{array}$$

Let M represents Adjacency matrix of network ,

Where,A-Ahmdabad,B-Bangalore,C-Coimbatore,D-chennai,E-Madhurai,F-Vijayawada,G-Hyderabad,

H-Vishakhapatnam-Jaipur

If we see sum of column we can conclude that Bangalore is connected to seven cities directly , Chennai and Hyderabad are connected to 5 cities ,so as per connectivity these two cities are good .Madhuri is at the end on basis of connectivity as it is connected to only Chennai, Same information can be concluded from diagonal elements of M^2.

Let us see M^2 and M^3

$$M^2=\begin{array}{c} \\ A\\B\\C\\D\\E\\F\\G\\H\\I \end{array}\begin{array}{c} \begin{array}{ccccccccc} A&B&C&D&E&F&G&H&I \end{array}\\ \begin{bmatrix} 2&1&2&1&1&1&2&1&1\\ 1&7&1&3&1&1&4&1&1\\ 2&1&2&1&1&1&2&1&1\\ 1&3&1&5&0&2&1&2&2\\ 1&1&1&0&1&0&1&0&0\\ 1&1&1&2&0&2&1&2&2\\ 2&4&2&1&1&1&5&1&1\\ 1&1&1&2&0&2&1&2&2\\ 1&1&1&2&0&2&1&2&2 \end{bmatrix}\end{array}$$

And

$$M^3=\begin{array}{c} \\ A\\B\\C\\D\\E\\F\\G\\H\\I \end{array}\begin{array}{c} \begin{array}{ccccccccc} A&B&C&D&E&F&G&H&I \end{array}\\ \begin{bmatrix} 2&10&2&8&1&3&5&3&3\\ 10&12&10&14&3&11&3&11&11\\ 2&10&2&8&1&3&5&3&3\\ 8&14&8&6&5&4&14&4&4\\ 1&3&1&5&0&2&1&2&2\\ 3&11&3&4&2&2&9&2&2\\ 5&3&5&14&1&9&8&9&9\\ 3&11&3&4&2&2&9&2&2\\ 3&11&3&4&2&2&9&2&2 \end{bmatrix}\end{array}$$

M^2 tells us two step connectivity ,like position(7,2) =4 means there are four different ways to go from Hyderabad to Bangalore with one stoppage. Also the diagonal elements tell us the connectivity of city

Like Bangalore is connected to 7 cities ,Ahmedabad to two cities, Coimbatore to two , Chennai to 5 and so on.

M^3 tells us three step connectivity or travel with two stoppage from one city to other, like position (5,3)=1 means there is only one way to travel from Coimbatore to Madurai with two stoppage and we can see in map that it is Coimbatore to Bangalore then to Chennai and then to Madurai. We must observe that we can reach from Coimbatore to Madurai with one stoppage also through Chennai as per position (5,3) in M^2.

We can get shortest number of path using S_k from (2)

If we observe value of S_2,

$S_2 = M + M^2$

$$=\begin{array}{c} \\ A\\B\\C\\D\\E\\F\\G\\H\\I \end{array}\begin{array}{c} \begin{array}{ccccccccc} A&B&C&D&E&F&G&H&I \end{array}\\ \begin{bmatrix} 2&2&2&2&1&1&2&1&1\\ 3&7&2&4&1&2&5&2&2\\ 2&2&2&2&1&1&2&1&1\\ 2&4&2&5&1&2&2&2&2\\ 1&1&1&1&1&0&1&0&0\\ 1&2&1&2&0&2&2&2&2\\ 2&5&2&2&1&2&5&2&2\\ 1&2&1&2&0&2&2&2&2\\ 1&2&1&2&0&2&2&2&2 \end{bmatrix}\end{array}$$

Now the entry at the position(5,6),(5,8),(5,9),(6,5)(8,5)(9,5) is zero which shows that ,there is no connectivity between Madurai and Vijayawada ,Madurai and Vishakhapatnam ,Madurai and Jaipur in two steps also.

Now let us see $S_3 = M + M^2 + M^3$

$$=\begin{array}{c} \\ A\\B\\C\\D\\E\\F\\G\\H\\I \end{array}\begin{array}{c} \begin{array}{ccccccccc} A&B&C&D&E&F&G&H&I \end{array}\\ \begin{bmatrix} 4&12&4&10&2&4&7&4&4\\ 12&19&12&18&4&13&18&13&13\\ 4&12&4&10&2&4&7&4&4\\ 10&18&10&11&6&6&16&6&6\\ 2&4&2&6&1&2&2&2&2\\ 4&13&4&6&2&4&11&4&4\\ 7&18&7&16&2&11&13&11&11\\ 4&13&4&6&2&4&11&4&4\\ 4&13&4&6&2&4&11&4&4 \end{bmatrix}\end{array}$$

DOI : 10.35291/2454-9150.2019.0200

ISSN : 2454-9150 Vol-05, Issue-03, June 2019

Clearly all entries are non- zero which shows that with maximum 2 stoppage we can travel from any city to other city.

V. Conclusion

Any network can be expressed as graph and for a graph we can write adjacency matrix as we shown in paper.From above two example it is very clear that matrix operations can be utilized to understand network in efficient way and help us to withdraw many important information's. same concept can be applied to very big network also and can be solved using MATLAB. We can apply same concept to transportation business and social network also. We can also extend results to directed matrix .Concept of trace can also be interpreted here.

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