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Matrix Application in Social Network Analysis

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Abstract: In this paper we have applied the concept of adjacency matrix and matrix operation to social network. In the first part of the paper we defined the concept of graph and various matrix operations. In the second part we considered two social network and applied concept of matrix operation on them, to find the popularity and type of bond between the members. In third part we applied the concept of permuted matrix and blocks to the same example to find block density matrix.

Keywords — Adjacency matrix, binary, blocks, connected graph, Decomposition of matrix, Inverse, Linear mapping, Ordinal relations, Permutation, supernodes.

I. INTRODUCTION

A Graph G is a pair (V, E) where V is a nonempty set and E is a set of unordered pairs of elements taken from the set V. The set V is called the vertex set and the set E is called the edge set. It is noted that there exist countless graph matrices in the mathematical and mathematico-chemical literature. Many real life situations like social network can be represented by graph but sometimes they can become so visually complicated that it is very difficult to understand them. It is always possible to represent information about social networks in the form of matrices. It becomes easy to analyze the graph and extract many important information using matrix operations, so representing the information in this way also allows the application of mathematical and computer tools to handle it. Social network analysts use matrices in a number of different ways. So, understanding a few basic things about matrices from mathematics is necessary.

Rectangular arrangement of numbers or abstract quantities in general is called matrix or matrices in plural .It represents mapping, so can be used to describe linear equations. Addition, subtraction, decomposition, transpose, inverse, matrix permutation, blocks, images, multiplication, powers of matrices are few operations defined on it. These operations are key concept in matrix theory and linear Algebra. Matrix is represented by $A_{m\times n}$ with m horizontal lines called rows and n vertical lines called columns. A matrix with *m* rows and *n* columns is called an *m*-by-*n* matrix, where *m* and *n* are called its dimensions. a_{ij} Represents an element of a matrix in i[th] row and j[th] column.

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nm} \end{bmatrix}$$

A matrix where one of the dimensions equals to one is often called a *vector*, $A_{1\times n}$ matrix is called (one row and *n* columns) a row vector, and $A_{m\times 1}$ matrix (one column and *m* rows) is called a column vector.

Adjacency matrix A = A (G) of the graph G is the square matrix of order n whose (i, j) entry is defined as

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{, and } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{if } i \neq j \text{, and } v_i \text{ and } v_j \text{ are not adjacent} \\ 0 & \text{if } i = j \end{cases}$$

which is binary in nature , That is, we enter one in a cell if a tie is present, a zero is entered in a cell, if there is no tie. This kind of a matrix is the starting point for almost all network analysis, and is called an "adjacency matrix" because it represents who is next to, or adjacent to whom in the "social space" mapped by the relations .Adjacency matrix can be classified as "symmetric" or "asymmetric." If there is a "bonded tie" and the entry in the $X_{i,j}$ cell is same as the entry in the $X_{j,i}$ cell then matrix is called Symmetric, If the entry in $X_{i,j}$ cell is different as the entry in the $X_{j,i}$ cell then matrix is called asymmetric. [7][18]

EXAMPLE 1: Let us consider directed graph of four students A, B, C, D in a group who are sending friend request to each other on some social media.

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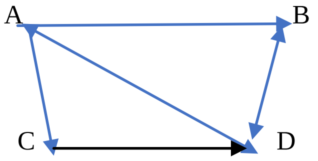
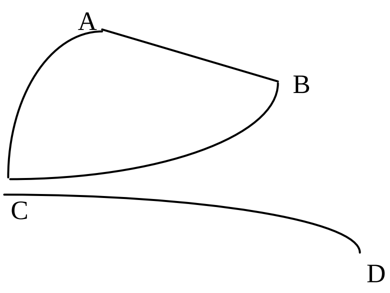


Adjacency matrix is given by

$$R = \begin{array}{c|cccc|} & A & B & C & D \\ A & - & 1 & 1 & 1 \\ B & 0 & - & 0 & 1 \\ C & 0 & 0 & - & 1 \\ D & 1 & 1 & 0 & - \end{array}$$

The sender of connection (friend request) is the row and the target is column, for i=j i.e. main diagonal entries are ignored in this case. But it plays important role when it comes to super nodes or blocks. Clearly entries at position(i,j) is not same as (j,i) as it is not necessary that if A want to be friend with B then B also wants the same.so the adjacency matrix is asymmetric. From this matrix we can find out popularity level of a person sum of row element tells how many connections that person want to make and sum of column elements tells, how many people want to be friend with it and hence tell popularity level. For example, for A, row vector is (-,1,1,1) means A want to have tie with three people on the other hand column vector for A is (-,0,0,1) means only one person want to have tie with A. From matrix it is very clear that D is very popular and everyone wants to have a tie with D. This concept helps a lot when it comes to business or politics as you can judge from relationship matrix that who is popular and influential candidate.[17]

EXAMPLE2:Now consider graph with bounded ties ,we have four business firms having partnerships



Adjacency matrix is given by

$$R = \begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 0 \\ C & 1 & 1 & 0 & 1 \\ D & 0 & 0 & 1 & 0 \end{array}$$

The above adjacency matrix is symmetric as there is a edge between A and B means there is partnership between them ,here (i.j) element will be same as (j,i)element .i.e $R=R^T$.

SIGNED GRAPH: Ordinal relations are represented by signed graphs with -1,0 and +1 to represent negative relations, neutral relation and positive relation.

II. Operations on Matrix

Let us analyze social network adjacency matrix under various matrix operation -

MATRIX ADDITION (SUBTRACTION):Network analysis is a field where Matrix addition and matrix subtraction are used to simplify or reduce the complexity of multiplex data to simpler version, Matrix with same dimensions can be added or subtracted.

TRANSPOSE: Transposing a Matrix means exchanging the rows and columns, so that i becomes j and vice versa. To indicate that we are transposing a matrix, we add a "T" to the top right-hand corner of the matrix .As discussed above for symmetric matrix A,$A=A^T$. If we observe relation matrix above in example1 and take transpose of a directed adjacency matrix then rows are the sources of ties directed to the student. Degree of similarity between adjacency matrix and the transpose is a way of summarizing degree of symmetry in the pattern of relations among students. This correlation is a measure of the degree of reciprocity of ties.

INVERSE: This is a mathematical operation that finds a matrix which when multiplied to the original matrix gives Identity matrix. [1]

PRODUCT: For any $m \times r$ matrix A and $r \times n$ matrix B product **AB** is an $m \times n$ matrix, in which the r entries across a row of **A** are multiplied with the r entries down a column of **B** and summed to produce an entry of **AB**.

Let AB=C then (i,j) entry of C, c_{ij} is given by

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{ir}b_{rj} = \sum_{k=1}^{r} a_{ik}b_{kj}$$

Being square matrix finding product is always possible for adjacency matrix, repeated multiplication gives power of matrix. In case of adjacency matrix we can conclude many important information from it. [3]

If A is any adjacency matrix of a graph .Then i,j entry of A^n gives count of n-step path from vertex i to vertex j.

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For example let us have a adjacency matrix $A = a_{ij}$ of a simple graph with 6 vertices and

C= A^2 then c_{34} (3, 4 entry of A^2) is

$$c_{34} = a_{31}a_{14} + a_{32}a_{24} + a_{33}a_{34} + a_{34}a_{44} + a_{35}a_{54}$$

Now $a_{31}a_{14}$ product of counts of number of paths from vertex 3 to 1 and the number of paths from 1 to vertex 4, this product exactly gives count of two step path from vertex 3 to vertex 4, via vertex 1.

In general terms we can say $a_{3k}a_{k4}$ counts two step path from vertex 3 to vertex 4 via vertex k.

And the sum of all terms of (1) gives all two-step path from vertex 3 to vertex 4.so if $c_{34} = n$

Means there are n two step path from vertex 3 to vertex 4.

Here we can conclude one more interesting fact, consider a matrix S_k such that

$S_k = M + M^2 + \cdots + M^k$ .The (i,j) entry in this matrix tells us the number of ways to get from vertex i to j in k steps or less, If entry is zero means you can't go .so if we find $S_1, S_2 \ldots S_k$

Then the first K for which (i ,j) entry in S_k is non-zero is the shortest number of steps between i and j. By this we can compute shortest number of steps but we can't determine what those steps are. This procedure can also help us to determine connected nature of graph .i.e. if S_k have zero till the last step for any vertex in above procedure then it is impossible to connect those vertices and the graph is not connected.

This concept can be used in many practical problems like business, social network, transportation problems etc .To understand this more let us consider example1

$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

This matrix shows, connection between friends, Like B is connected directly to D only

$$R^2 = \begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

This matrix counts number of pathways between two nodes that are of length two, see now B is connected to A also through one member, which we can't tell through matrix but graph tells us B to D and then from D to A.Now only B is not connected to C in one or two step so let's check third power of R

$$R^3 = \begin{bmatrix} 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 2 & 2 & 0 & 2 \end{bmatrix}$$

Now B is connected to C in 3 step, B to D, D to A and then A to C, Now all all members are connected but the tie of B to C is very weak. [4] [8]

Let us brief how these friends are connected

Table-1 connectivity

	A	B	C	D
A	-	Direct	Direct	Direct
B	One step	-	Two step	Direct
C	One step	One step	-	Direct
D	Direct	Direct	One step	-

III. Permuted Matrix and Blocks

When we rewrite matrix by exchanging rows to form different groups, process is called matrix permutation. EXAMPLE-3: Consider a adjacency matrix of four business people A-Abhay, B-Bharti, C-Chetan, D-Divya with network matrix

$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Let us permute it gender wise by changing position of rows and corresponding columns .

$$R = \begin{array}{c} \\ A \\ C \\ B \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{bmatrix} \end{array}$$

We have rearranged rows and columns, value of element is not changed at all, we have divided it in four section based on genders called blocks ,this process is called partitioning or blocking of matrix . In network analysis we do this type of grouping to understand how some sets of members are embedded in social roles. If we observe above example males (Abhay, Chetan) chose each other as friends but females (Divya, Bharti) don't choose each other as friends. Also males are more likely to choose females (3 out of 4 possibilities) than females are to choose males (only 2 out of 4 possible choices).We have grouped the males together to create a "partition" or "super-node" or "social role" or "block." In social network we often partition matrices in this way to identify and test ideas about how members are "embedded" in social roles or other "contexts." [6]

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BLOCK DENSITY MATRIX: By calculating proportion of all ties in a block, we can find block density matrix. We should ignore self-ties in this case. Like for above example we have, Block density matrix

	Male	Female
Male	1.00	0.75
Female	0.50	0.00

IMAGE OF BLOCKED MATRIX: Now average density in this matrix is 0.58, let it be cut-off score. Image of this block is prepared by entering "1" in a cell if density in block is greater than cutoff else we enter "0".Image matrix of gender blocked data, using overall mean density as the cut-off

	Male	Female
Male	1	1
Female	0	0

Images of blocked matrices helps to simplify the presentation of complex patterns of data.

IV. Conclusion

How members are connected to each other is most fundamental properties of a social network. Networks that have few or weak ties, or where members are connected by pathways of great length display weak solidarity, a tendency to get separated, slow response to stimuli etc. Networks that have more and stronger connections with shorter paths among members respond quickly, effectively and be in contact for long. Count of number and lengths of pathways among the members in a network help us to index important tendencies of whole networks. Also individual influence can also be evaluated ,members who have many pathways to other members may be more influential .Members with short pathways are central figures.so the number and length of pathways in a network are very important for understanding individuals personality, constraints and opportunities, which help for understanding both behavior and potentials of the network as a whole.

References

[1] DATTA, B. N. (1995): Numerical Linear Algebra and Applications. Brooks&Cole Publishing Company, Pacific Grove.

[2] DOREIAN, P. BATAGELJ, V., FERLIGOJ, A. (1994): Partitioning Networks Based on Generalized Concepts of Equivalence. Journal of Mathematical Sociology, 19, 1, 1-27.

[3] GOLUB, G. H., van LOAN, C. F. (1996): Matrix Computations. The John Hopkins University Press, Baltimore.

[4] HUMMON, N. P., DOREIAN, P. (1990): Computational Methods for Social Network Analysis. Social Networks, 12, 273-288.

[5]. C. Godsil and G.Royle, Algorithmic Graph Theory, Cambridge University Press, 2001.

[6]. J. A. Bondy and U.S.R. Murty, Graph Theory with applications, North-Holland, New York, Amsterdam, Oxford, 1982,PP. 45-65.

[7]. I.Gutman, The energy of a graph, Ber. Math Statist.

Sekt. Forschungsz. Graz 103, 1978.

[8] R. B. Bapat, Graphs and Matrices, Springer, London, UK, 2006.

[9] J. Clark and D. A. Holton, A First Look at Graph Theory, World Scientific, 1995.

[10] Horn and C. R. Johnson, Matrix Analysis, Cambridge University Press, Cambridge, UK, 1985.

[11] E.Ulrychova; Several real world applications of Linear Algebra Tools.

[12] Loet Leydesdorff and Liwen Vaughan; Co-Occurance Matrices and their Applications in Information Science; Extending ACA to web environment, JASIST.

[13] Ting Yip; Matrices in Computer Graphics, Math, 308A, 12/3/2001.

[14] Athanasios Karamalis; Introduction to applied Matrix Transformation for computer Graphics and Image Processing.

[15] Keijo Ruohonen, Graph Theory, 2010,PP. 27-35.

[16] Kenneth H. Rosen, Discrete Mathematics and its Applications, seventh edition, McGraw Hill, 2007, PP. 45-48.

[17] Jonathan L. Gross and Jay Yellen, Graph Theory and its applications, Columbia University, New York, USA, 2005

[18] D.Cvetkovi´c, P.Rowlinson, S.Simi´c, An Introduction to the Theory of Graph, Spectra, Cambridge Univ. Press, Cambridge, 2010,

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