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k and (*k*, *d*)- Odd Edge Mean Labeling of Triangular Graphs

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Abstract : A (p,q) graph G is said to have a (*k*, *d*)- odd edge mean labeling $(k, d \geq 1)$, if there exists an injection *f* from the edges of G to $\{0, 1, 2, 3, \ldots, 2k+2d(p-1)-1\}$ such that the induced map f^* defined on V by $f^*(v) = \left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1, 2k+2d-1, 2k+4d-1, \ldots, 2k+2(p-1)d-1\}$. A graph that admits a (*k*, *d*)-odd edge mean labeling is called a (*k*, *d*)-odd edge mean graph. In this paper, we have introduced (*k*, *d*)-odd edge mean labeling and we have investigated the (*k*, *d*)-odd edge mean labeling of triangular graphs like triangular Snake and Friendship graphs.

***Keywords* — k- Odd edge mean labeling, k-Odd edge mean graph, (k, d)- Odd edge mean labeling, (k, d)-Odd edge mean graph**

I. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [5]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges). Then the labeling is called a vertex labeling (or an edge labeling). Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [6]. In this paper, we have introduced the concept of (*k*, *d*) –odd edge mean labeling graphs.

For brevity, we use (*k*, *d*) – OEML for (*k*, *d*) – odd edge mean labeling and (*k*, *d*) – OEMG for (*k*, *d*) – odd edge mean graph.

II. Definitions

1. Defintion

A (p,q) graph G is said to have a (*k*, *d*)- odd edge mean labeling $(k, d \geq 1)$, if there exists an injection f from the edges of G to $\{0, 1, 2, 3, \ldots, 2k+2(p-1)d-1\}$ such that the induced map f^* defined on V by $f^*(v) = \left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1, 2k+2d-1, 2k+4d-1, \ldots, 2k+2(p-1)d-1\}$. A graph that admits a (k, d)-odd edge mean labeling is called a (k, d)-odd edge mean graph.

2. Defintion

A graph is said to be mC_n – snake if it is a connected graph with m blocks whose block cut point graph is a path and each of the m- blocks is isomorphic to C_n.

3. Defintion

Let $C_n^{(t)}$ denote the one-point union of t cycles of length n. $C_3^{(t)}$ is called friendship graph.

III. Main Results

Theorem 3.1

The Friendship graph $C_3^{(n)}$ is a k-odd edge mean graph for all k when n (n ≥3) is odd.

Proof

Let $V\left(C_3^{(n)}\right) = \{u_1, u_2, u_3, \ldots u_n, v_1, v_2, v_3, \ldots v_n\}$

$$E\left(C_3^{(n)}\right) = \{u_i u_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i, 1 \leq i \leq n\}$$

First we label the edges as follows:

Define $f : E \to \{0, 1, 2, 3, \ldots, 2k+2p-3\}$ by

$$f(u_1 v_1) = 2k-1$$

$$f(u_i v_i) = \begin{cases} 2k+4i-4, & for\ 2 \leq i \leq n/2 \\ 2k+4i-2, & for\ n/2+1 \leq i \leq n \end{cases}$$

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$$f\left(uu_1\right)=2k$$

$$f\left(uu_i\right)=\begin{cases}2k+4i-7, & for\ 2\le i\le n/2\\ 2k+4i-4, & for\ n/2+1\le i\le n\end{cases}$$

$$f\left(uv_1\right)=2k+3$$

$$f\left(uu_i\right)=\begin{cases}2k+4i-7, & for\ 2\le i\le n/2\\ 2k+4i-4, & for\ n/2+1\le i\le n\end{cases}$$

Then the induced vertex labels are

$$f^*\left(u\right)=2k+2n-1$$

$$f^*\left(u_i\right)=\begin{cases}2k+4i-5, & for\ 1\le i\le (n+1)/2\\ 2k+4i-3, & for\ (n+3)/2\le i\le n\end{cases}$$

$$f^*\left(v_i\right)=\begin{cases}2k+4i-3, & for\ 1\le i\le (n-1)/2\\ 2k+4i-1, & for\ (n+1)/2\le i\le n\end{cases}$$

Thus

$$V(C_3^{(n)})=\{2k-1,\ 2k+1,\ 2k+3,\ .\ .\ .\ .\ .\ ,\ 2k+2p-3\}$$

Hence, the graph $C_3^{(n)}$ is a k-odd edge mean graph for all k when n is odd and n ≥4.

Illustration: 3.2

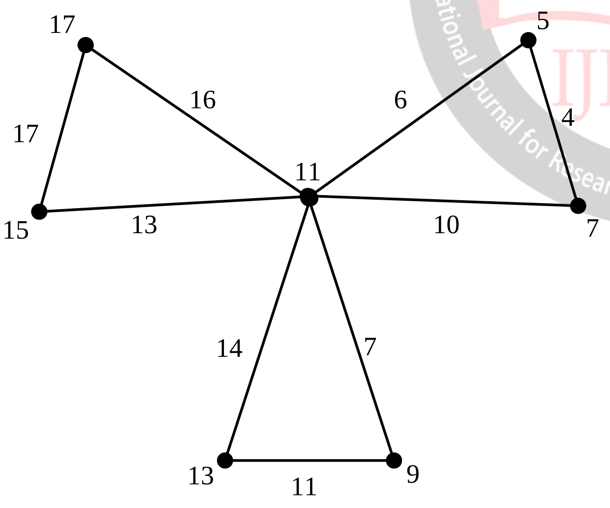

Fig 3.1: 3- OEML of $C_3^{(3)}$

Theorem 3.3

The Friendship graph $C_3^{(n)}$ is a k-odd edge mean graph for all k when n (n ≥4) is even.

Proof

Let $V\left(C_3^{(n)}\right)=\{u_1,\ u_2,u_3,\ .\ .\ .\ u_n,\ v_1,v_2,v_3,\ .\ .\ .\ v_n\}$

$$E\left(C_3^{(n)}\right)=\{u_iu_{i+1},\ 1\le i\le n-1\}\cup\{u_iv_i,\ 1\le i\le n\}$$

First we label the edges as follows:

Define $f:E\rightarrow\{0,\ 1,\ 2,\ 3,\ .\ .\ .\ .\ ,\ 2k+2p-3\}$ by

$$f\left(u_1v_1\right)=2k-2$$

$$f\left(u_iv_i\right)=\begin{cases}2k+4i-4, & for\ 2\le i\le n/2\\ 2k+4i-2, & for\ n/2+1\le i\le n\end{cases}$$

$$f\left(uu_1\right)=2k$$

$$f\left(uu_i\right)=\begin{cases}2k+4i-7, & for\ 2\le i\le n/2\\ 2k+4i-4, & for\ n/2+1\le i\le n\end{cases}$$

$$f\left(uv_1\right)=2k+3$$

$$f\left(uu_i\right)=\begin{cases}2k+4i-7, & for\ 2\le i\le n/2\\ 2k+4i-4, & for\ n/2+1\le i\le n\end{cases}$$

Then the induced vertex labels are

$$f^*\left(u\right)=2k+2n-1$$

$$f^*\left(u_i\right)=\begin{cases}2k+4i-5, & for\ 1\le i\le n/2\\ 2k+4i-3, & for\ n/2+1\le i\le n\end{cases}$$

$$f^*\left(v_i\right)=\begin{cases}2k+4i-3, & for\ 1\le i\le n/2\\ 2k+4i-1, & for\ n/2+1\le i\le n\end{cases}$$

Thus

$$V(C_3^{(n)})=\{2k-1,\ 2k+1,\ 2k+3,\ .\ .\ .\ .\ .\ ,\ 2k+2p-3\}$$

Hence, the graph $C_3^{(n)}$ is a k-odd edge mean graph for all k when n is even and n ≥4.

Illustration: 3.4

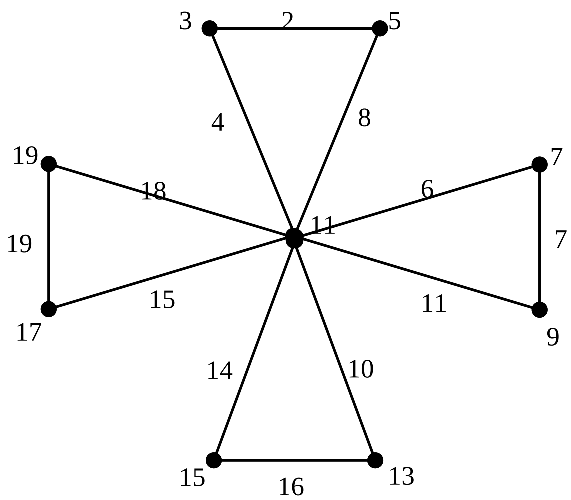

Fig 3.2: 2-OEML of $C_3^{(4)}$

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Theorem 3.5

The Triangular Snake graph mC_n (m,n≥3) is a (k, d)-odd edge mean graph for all k and d.

Proof.

Let $V(mC_n)=\{u_i : 1\le i\le n-1\}\cup\{v_i, 1\le i\le n\}$ and

$$E(mC_n)=\{u_iv_i,\ u_iv_{i+1}, v_iv_{i+1} : 1\le i\le n-1\}.$$

First we label the edges as follows:

Define $f : E\to\{0, 1, 2, 3, \dots, 2k+2(p-1)d-1\}$ by

$$f(u_1v_1)=2k-2$$

For $2\le i\le n-2$,

$$f(u_iv_i)=2k+2di-2$$

$$f(u_1v_2)=2k+4d-1$$

For $2\le i\le n-2$,

$$f(u_iv_{i+1})=2k+2d(i-1)$$

$$f(u_{n-1}v_n)=2k+4d(n-1)-1$$

$$f(v_1v_2)=2k$$

For $2\le i\le n-1$,

$$f(v_iv_{i+1})=2k+2d(2i-1)-1$$

$$f(v_{n-1}v_n)=2k+4d(n-1)-2$$

Then the induced vertex labels are

For $1\le i\le n$,

$$f^*(v_i)=2k+4d(i-1)-1$$

For $1\le i\le n-1$,

$$f^*(u_i)=2k+2d(2i-1)-1$$

Thus

$$V(mC_n)=\{2k-1, 2k+2d-1, 2k+4d-1, \dots, 2k+2d(p-1)-1\}$$

Hence, the Triangular Snake graph mC_n is a (k, d)-odd edge mean graph for all k and d.

Illustration: 3.6

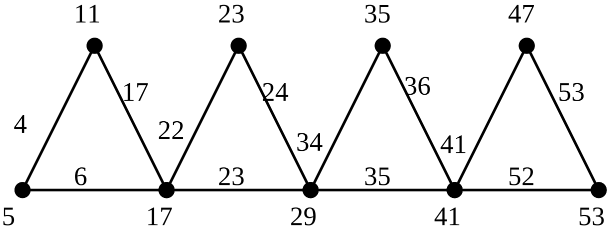

Fig 3.3: (3, 3) – OEML of $4C_3$

CONCLUSION

Since graph labeling serve as practically useful model for a wide range of applications. It is desired to have generalized the following results. This paper focused on k and (k, d) Odd edge mean graphs. We proved that Friendship graph is k- odd edge mean graphs and triangular snake graph is (k, d) odd edge mean graphs. To investigate similar results for other families and in the context of different labeling techniques is an open area of research.

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