Fuzzy Ideals in Right Almost Semigroups

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ABSTRACT: It is observed that many researchers studied on different structures of semigroups. Many fundamental results on semigroup theory have been extended to fuzzy semigroups. Various concepts of fuzzification have been investigated and fuzzy ideal theory is one of that which motivates to characterize some of the properties of fuzzy ideals in almost semigroups. The present work deals with different results obtained with various fuzzy ideals such as quasi primary, completely weakly primary in Right almost semigroups. In this paper mainly we have studied some aspects on fuzzy right (left) ideals of right (left) almost semigroups and the direct product of right almost semigroups.

Keywords: Right Almost semigroup, fuzzy primary, fuzzy quasi primary, weakly fuzzy quasi primary, fuzzy weakly completely primary.

I. INTRODUCTION

The idea of generalization of a common semigroup was first introduced by Kazim and Naseeruddin[1]. They named it as left almost semigroup (LA semigroup). It is also called Abel Grassmanns groupoid (AG semigroup). Quasier Mustaq and Madad Khan [2] studied left almost semigroups in ideals, ideals as intra regular left almost semigroups. In a similar way right almost semigroups is defined. We extend the study of right almost semigroups in fuzzy ideals and ideals as right almost semigroups.

The fundamental concept of fuzzy subsets was first introduced by Zadeh [3]. Fuzzy algebraic structures were studied by Rosenfeld [4] with the introduction of fuzzy subgroupoid of a groupoid. The theory of fuzzy bi-ideals in semigroups was introduced by Kuroki [5]. Almost semigroup is a new algebraic structure obtained by imposing a pseudo associative postulate on a groupoid . Also Left almost and right almost semigroups which come between a groupoid and a semigroup were investigated by introducing left invertive and right invertive elements by Mohd. Naseeruddin. Thus it is considered as a generalization of semigroup with vast range of usages in theory of flocks. Abdulla et al.,[6]and many others such as Musthaq and Khan[7], Naveed Yaqoob[8], Yousafzai Faizal [9], continued their work which added many results to Left Almost semigroups where in Shah and Rehman[10], Akin C[11] and Yiarayong[12] focused on different fuzzy ideals on Left Almost - Γ-semigroups.

The following are basic definitions and preliminaries.

II. FUZZY SETS-DEFINITIONS

- Fuzzy subset of a non empty set is a collection of objects with each object being assigned a value between 0 & 1 by a membership function
- Let X be a non empty set. A fuzzy subset μ of the set X is a function $\mu : X \rightarrow [0,1]$.
- Let S be a semigroup. A map μ from S to [0,1] is called a fuzzy subset in S.
- A fuzzy subset $\mu: X \rightarrow [0,1]$ is nonempty if μ is not the constant map which assumes the value 0
- The characteristic function of a subset A of S is denoted by f_A .
- If μ is a fuzzy subset of S then the image of μ denoted by $Im(\mu) = \{ \mu(m)/ m \in M\}$
- Let μ be any fuzzy subset of S. For $t \in [0,1]$, the set $\mu_t = \{x \in M/ \mu(x) \geq t\}$ is called level subset of μ.
- A fuzzy subset $A \in F(S)$ is said to be a fuzzy sub semigroup of S if

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A(xy) ≥ Min { A(x), A(y) } ∀ x,y ∈ S.

A fuzzy subset A ∈ F(S) is said to be a fuzzy left ideal of S if

A(xy) ≥ A(y) ∀ x,y ∈ S

- A fuzzy subset A ∈ F(S) is said to be a fuzzy right ideal of S if

  A(xy) ≥ A(x) ∀ x,y ∈ S

- A fuzzy subset A ∈ F(S) is said to be a fuzzy ideal of S if it is both a fuzzy left and fuzzy right ideal of S.

  A fuzzy sub semigroup A ∈ F(S) is said to be a fuzzy bi ideal of S if

  A(xyz) ≥ Min { A(x), A(z) } ∀ x,y,z∈ S

- A subset A of S is called a quasi ideal of S if AS ∩ SA ⊆ A.
- A fuzzy subset A of S is called a fuzzy quasi ideal of S if AS ∩ SA ⊆ A.
- An ideal P of S is called a completely primary ideal if for a,b $\in S$ such that $ab \in P$ implies that $a^n \in P\ or\ b \ \in P$, for some positive integer n.
- An ideal P of S is called a weakly completely primary ideal if for a,b $\in S$ such that $0 \neq ab \in P$ implies that $a^n \in P\ or\ b \ \in P$, for some positive integer n.
- **Remark**: Every completely primary ideal is weakly completely primary ideal and {0} is always weakly completely primary ideal of S. But vice versa need not be true.
- A left almost semigroup (LASG) or Abel Grassmanns groupoid is a groupoid S with left invertive law $(ab)c = (cb)a\, \forall\ a, b, c \in S$.
- A right almost semigroup (RASG) is a groupoid S satisfying the right invertive law $a(bc) = c(ba)\, \forall\ a, b, c \in S$
- A groupoid in which both left invertive and right invertive postulates hold will be called as Almost semigroup.
- The right invertive and left invertive laws are independent of each other and are neither associative nor commutative.
- A non empty subset A of a LASG 'S' is said to be a sub LASG 'S' if $AA \subseteq A$.
- A non empty subset A of a LASG 'S' is called a right(left) ideal of S if $AS \subseteq A(SA \subseteq A)$. Also A is said to be an ideal of S if it is both left and right ideal of LASG 'S'.
- A LASG (RASG) always satisfies the medial law:

  $(ab)(xy) = (ax)(by)$ for all $\forall\ a, b, x, y \in S$

- A LASG (RASG) with left(right) identity always satisfies the paramedial law:

  $(ab)(xy) = (yx)(ba)$ for all $\forall\ a, b, x, y \in S$.

- If a LASG contains left identity then we have the law:

  $a(bc) = b(ac)\, \forall\ a, b, c \in S$

- A LASG 'S' itself a fuzzy subset of S such that $\mathrm{S(x)} = 1;\ \forall\, x \in S$.
- A fuzzy subset A of S is called fuzzy sub LASG of S if,

  $A(xy) \geq A(x) \wedge A(y)\}$ for all x,y in S.

  OR

- A fuzzy subset A of S is called a fuzzy sub LASG of S if,

  $A(xy) \geq \min\{A(x), A(y^n)\}$ for all x,y in S.

- Also A is called a fuzzy left(right) ideal of S if $A(xy) \geq A(y)\ \big(A(xy) \geq A(x)\big) \forall\, x, y \in S$
- Also if A is both fuzzy right and left ideal of S then A is called a fuzzy ideal of S

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- If A is (right,left) ideal of S if and only if the function f_A of A is a fuzzy (right,left) ideal of S.

- Definition: Let S be a RA semigroup, $x \in R\ and\ t \in (0,1]$. A fuzzy point x_t of S is defined by the rule that $x_t(y) = \begin{cases} t; & x = y \\ 0; & otherwise \end{cases}$

- If f, g, h are fuzzy subsets of RASG S then $fo(goh) = ho(gof)$.
- Let S be a LASG.A fuzzy subset f of a LASG is called fuzzy quasi primary if for any two fuzzy left ideals g and h of S such that $gh \subseteq f$ implies $g \subseteq f$ or $h^n \subseteq f$ for some positive integer n.

- Let S be a LASG and f be a fuzzy left ideal of S. Then SS=S and Sf=f.

- Let S be a LASG.A fuzzy subset f of a S is called fuzzy primary of S if for any two fuzzy left ideals g and h of S such that $gh \subseteq f$ implies $g \subseteq f$ or $h^n \subseteq f$ for some positive integer n.

- Let S be a RASG.A fuzzy subset f of a S is called fuzzy primary of S if for any two fuzzy right ideals g and h of S such that $gh \subseteq f$ implies $g \subseteq f$ or $h^n \subseteq f$ for some positive integer n.

- Let S be a LASG. We can see that every fuzzy quasi primary ideal is fuzzy primary.

- Let S be a LASG. A fuzzy subset f of a LASG is called fuzzy weakly completely primary if $\max\{f(x), f(y^n)\} \geq f(xy)$ where $x, y \in S$ for some positive integer n.
- An ideal A of a LASG(RASG) is called primary if $XY \subseteq P$ implies $X \subseteq P$ or $Y^n \subseteq P$
- Let S be a LASG, , $x \in S$ and , $t \in [0,1]$. A fuzzy point x_t of S is defined as $x_t(y) = \begin{cases} t; x = y \\ 0; x \neq y \end{cases}$

- It is accepted that x_t is a mapping from S into [0,1] and a fuzzy point of S is a fuzzy subset of S. For any fuzzy subset f of S, we also denote $x_t \subseteq f$ by $x_t \in f$ in sequel.
- Let tf_A be a fuzzy subset of S defined as : $tf_A(x) = \begin{cases} t \in (0,1]; & x \in A \\ 0; & otherwise \end{cases}$

- An ideal P of a left almost semigroup S is called primary if $XY \subseteq P$ then $X \subseteq P$ or $y^n \subseteq P$

- A fuzzy subset f of a RA semigroup of S is called fuzzy completely primary if, $f(xy) \geq \max\{f(x), f(y)\}$

- Let A be a subset of LASG and f be a fuzzy set of S. Then the following are equivalent.
  (i) $tg_A \subseteq f, t \in [0,1]$

  (ii) $A \subseteq f_t, t \in [0,1]$

  (iii) A fuzzy subset f of S is said to be weakly fuzzy primary if $t\, g_A\, t\, h_B \subseteq f$

  implies $t\, g_A \subseteq f$ or

  (iv) $t\, h_B{}^n \subseteq f$, for some positive integer n, where A and B are two ideals of S and

  $t \in [0,1]$.

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(v) A fuzzy subset f of S is said to be weakly fuzzy quasi primary if $t\,g_A\,t\,h_B \subseteq f$ implies $t\,g_A \subseteq f$ or $t\,h_B{}^n \subseteq f$, for some positive integer n, where A and B are two left ideals of S and $t \in [0,1]$.

(vi) We can see that every weakly fuzzy quasi primary ideal is weakly fuzzy primary.

Lemma: Let A and B be two non empty subsets of right almost semigroup S. Then for any $t \in (0,1]$ the following exists:

1. $t\,\mu_A\,t\,\mu_B = t\mu_{AB}$
2. $t\,\mu_A \cap t\,\mu_B = t\,\mu_{A\cap B}$
3. $t\,\mu_A \cup t\,\mu_B = t\,\mu_{A\cup B}$
4. $t\,\mu_A = \bigcup_{a\in A} a_t$
5. $S\,t\,\mu_A = t\mu_{SA}, t\,\mu_A\,S = t\mu_{AS}$ and $S\,t\,\mu_A\,S = t\,\mu_{SAS}$
6. If A is a left ideal (right ideal) of S, then $t\,\mu_A$ is a fuzzy left ideal (fuzzy left or fuzzy ideal) of S.

Theorem: 1

Let A be an ideal of right almost semigroup S. Then P is a primary ideal of S if and only if the fuzzy subset μ_A is a fuzzy weakly completely primary ideal of S.

Proof: Suppose that A is a primary ideal of right almost semigroup S.

Let μ_A be a fuzzy subset of S.

Let $x, y \in S$ and if $xy \in A$ then

$\mu_A(xy) = 0 \leq \max\{\mu_A(x), \mu_A(y^n)\}$ for some positive integer n.

Let $xy \in A$ then $\mu_A(xy) = 1$

Since A is a primary ideal of S we have $x \in A$ or $y^n \in A$ for some positive integer n

Thus $\mu_A(x) = 1$, or $\mu_A(y) = 1$, or $\mu_A(xy) = 1$.

Conversely,

Since μ_A is a fuzzy weakly completely primary ideal of S, we get

$1 = \mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y^n)\}$ and so $x \in A$ or $y^n \in A$ for some positive integer n.

Hence A is a primary ideal of S.

Theorem: 2

Let μ be a fuzzy subset of of right almost semigroup S. Then μ is a fuzzy weakly completely primary ideal of S if and only if the level subset μ_t, $[\,t \in Im(\mu)]$ of μ is weakly completely primary ideal of S, for every $t \in [0,1]$.

Proof: Suppose μ is a fuzzy weakly completely primary ideal of S

Let $x, y \in S$ such that $xy \in \mu_t$ then $\mu(xy) \geq t$.

Since μ is a fuzzy weakly completely primary ideal of S,

We have $\mu(xy) \leq \max\{\mu(x), \mu(y^n)\}$, for some positive integer n

If $\mu(x) \leq \mu(y^n)$, then

$t \leq \max\{\mu(x), \mu(y^n)\} = \mu(x)$ and $\mu(x) \geq t$

So $x \in \mu_t$

If $\mu(x) > \mu(y^n)$, then

$t \leq \max\{\mu(x), \mu(y^n)\} = \mu(y^n)$ and $\mu(y^n) \geq t$

So $y^n \in \mu_t$

Therefore the level subset μ_t, is weakly completely primary ideal of S, for every $t \in [0,1]$.

Conversely assume that μ_t, is weakly completely primary ideal of S, for every $t \in [0,1]$.

Let $x, y \in S$ then $\mu(xy) \geq 0$.

Since $xy \in \mu_{\mu(xy)}$, by hypothesis we have $x \in \mu_{\mu(xy)}$ or $y^n \in \mu_{\mu(xy)}$ for some

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positive integer n

Thus $\mu(x) \geq \mu(xy)$ or $\mu(y^n) \geq \mu(xy)$

Hence $\max\{ \mu(x) , \mu(y^n)\} \geq \mu(xy)$

Theorem : 3

Let L be a fuzzy left ideal of right almost semigroup with left identity S. Then the following are equivalent:

(a) L is weakly fuzzy quasi primary of S.

(b) For any $x, y \in S$ and $t \in (0,1]$, if $x_t(Sy_t) \subseteq L$ then $x_t \in L$ or $y^n{}_t \in L$.

(c) For any $x, y \in S$ and $t \in [0,1]$, if $t\mu_x t\mu_y \subseteq L$ then $x_t \in L$ or $y^n{}_t \in L$ for some positive integer n.

(d) If A and B are left ideals of S such that $t\mu_A t\mu_B \subseteq L$ then $t\,\mu_A \subseteq L$ or $t\,\mu_B{}^n \subseteq L$ for some positive integer n.

Proof: **(i)** Let L be a fuzzy left ideal of right almost semigroup with left identity.

(i)We prove $(a) \Rightarrow (b)$

We assume that L is a weakly fuzzy quasi primary of S.

Now For any $x, y \in S$ and $t \in (0,1]$, if $x_t(Sy_t) \subseteq L$ then

$$t\,\mu_{xes}\,t\,\mu_{yes} = (t\,\mu_{xe}\,S)\,(t\,\mu_{ye}\,S)$$

$$= (t\,\mu_{xe}\;t\,\mu_{ye})\,(S\,S)$$

$$= (t\,\mu_x\;t\,\mu_e)\,(t\,\mu_y\,t\,\mu_e)\,(S\,S)$$

$$= (t\,\mu_x\;t\,\mu_y)\,(t\,\mu_e\,t\,\mu_e)(S\,S)$$

$$= (t\,\mu_e\,t\,\mu_e)\,(t\,\mu_y\,t\,\mu_x)\,(S\,S)$$

$$= \left(t\,\mu_{ee}\,(t\,\mu_y\,t\,\mu_x)\right)(S\,S)$$

$$= \left(t\,\mu_e\,(t\,\mu_y\,t\,\mu_x)\right)(S\,S)$$

$$= \left(t\,\mu_y\,(t\,\mu_e\,t\,\mu_x)\right)(S\,S)$$

$$= \left(t\,\mu_y\,(t\,\mu_{ex})\right)(S\,S)$$

$$= (S\,S)(t\,\mu_x\;t\,\mu_y)$$

$$= S(t\,\mu_x\;t\,\mu_y)$$

$$= t\,\mu_x(S\;t\,\mu_y)$$

$$= x_t(Sy_t) \subseteq L$$

Since μ is weakly fuzzy quasi primary ideal, we have

$$x_t = t\,\mu_x = t\,\mu_{(xe)e} \subseteq t\,\mu_{(xe)s} \subseteq L \qquad \text{(or)}$$

$$y^n{}_t = t\,\mu\,y^n = t\,\mu_{((ee)y)}{}^n$$

$$= t\,\mu_{((ye)e)}{}^n$$

$$= t\,\mu_{((ye)s)}{}^n$$

$$\subseteq L$$

Hence $x_t \in L$ or $y^n{}_t \in L$, for some positive integer n.

i.e., $(a) \Rightarrow (b)$ is proved.

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(ii) To prove $(b) \Rightarrow (c)$

Let $x, y \in S$ and $t \in (0,1]$ and $t\mu_x t\mu_y \subseteq L$ then

$$x_t\, S\, y_t \subseteq t\,\mu_x\, S\, t\,\mu_y$$
$$= S(t\,\mu_x\ t\,\mu_y)$$
$$= S\,L$$
$$\subseteq L$$

Thus by hypothesis, $x_t \in L$ or ${y^n}_t \in L$ for some positive integer n.

Hence $(b) \Rightarrow (c)$ is proved.

(iii) To prove $(c) \Rightarrow (d)$

Let A and B be left ideals of S.

Then by the above lemma , we have $t\,\mu_A$ and $t\,\mu_B$ are two fuzzy left ideals of S.

Suppose that $t\mu_A t\mu_B \subseteq L$ and $t\,\mu_{B^m} \not\subseteq L$, then there exists $y \in B$ such that

${y_t}^n \notin L$ for all positive integer n.

For any $x \in A$ and by lemma, we write

$$t\,\mu_x\ t\,\mu_y = t\,\mu_{xy}$$
$$\subseteq t\,\mu_{AB}$$
$$= t\,\mu_A\,\mu_B$$
$$\subseteq L$$

Since ${y_t}^n \notin L,\ t\,\mu_{y^n} \not\subset L$ which implies $t\,\mu_x \subseteq L$ and so $x_t \in L$

By lemma, it follows that $t\,\mu_A = \cup_{x \in A} x_t$

(iv)Finally we prove that $(d) \Rightarrow (a)$

Let A and B be left ideals of S such that $t\,\mu_A\ t\,\mu_B \subseteq L$ then by hypothesis $t\,\mu_A \subseteq L$ or $t\,{\mu_B}^n \subseteq L$ for some positive integer n.

By definition of weakly fuzzy quasi primary, we get

L is a weakly fuzzy quasi primary of S.

Theorem:4

Let S be a right almost semigroup with left identity. If μ is a fuzzy quasi primary ideal of S, then $\inf\{\mu(a^2\, S\, b^2)\} \leq \max\{\mu(a^2), \mu(b^2)^n\}$ for some positive integer n where $a, b \in S$

Proof:

Let μ is a fuzzy quasi primary ideal of S.

We assume that $\inf\{\mu(a^2 S\, b^2) > \max\{\mu(a^2), \mu(b^2)^n\}$ for some positive integer n.

Let $\inf\{\mu(a^2 S\, b^2)\} = m$, where m is a positive integer.

We define two fuzzy subsets g and h of S as follows:

$$g(x) = m;\ x \in a^2 S \qquad\qquad h(x) = m;\ x \in b^2 S$$
$$\quad = 0;\ x \notin a^2 S \quad \text{and} \quad\quad = 0;\ x \notin b^2 S$$

Then g and h are fuzzy left ideals of S.

$$(goh)(x) = \underbrace{sup}_{x=yz}(\min\{g(y), h(z)\}) = m$$

Then there exists an $u \in a^2 S, v \in b^2 S$ such that $uv = x.$

Put $u = a^2 t$ and $v = b^2 k$ for some $t, k \in S$

Then $\mu(x) = \mu(uv)$

$$= \mu(a^2 t)(b^2 k)$$
$$= \mu((a^2\, b^2)(tk))$$

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$= \mu((kt)(a^2 b^2))$
$\geq \mu(b^2 a^2)$
$= \mu(a^2 b^2)$
$= \mu(a^2 (eb^2))$
$\geq \inf\{\mu(a^2 S b^2)\}$
$= m$
$\Rightarrow goh \subseteq \mu$

Since μ is a fuzzy quasi primary ideal, we get $g \subseteq \mu$ or $h^n \subseteq \mu$, for some positive n.

Thus $h^n(x) = \cup_{x=a_1 b_1} \min\{h^{n-1}(a_1), h(b_1)\}$

$= \cup_{x=a_1 b_1} \min\{\cup_{a_1=a_2 b_2} \min\{h^{n-2}(a_2), h(b_2)\}, h(b_1)\}$

= ………………

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$= \cup_{x=a_n b_n} \min\{\min\{h(a_n), h(b_n)\}, \ldots.. h(b_1)\}$

And $(b^2)^n = (b^2 b^2)\, b^2 \ldots \ldots \ldots \ldots \ldots b^2$.

Then $g(a^2) = g(a^2 e) = m$

Or $h^n((b^2)^n) = \cup_{x=(b^2 b^2)\, b^2 \ldots\ldots b^2} \min\{\min\{h(b^2), h(b^2)\}, \ldots.. h(b^2)\}$

$= \cup_{x=(b^2 b^2)\, b^2 \ldots\ldots b^2} \min\{\min\{h((ee)b^2), h((ee)b^2)\}, \ldots.. h((ee)b^2)\}$

$= \cup_{x=(b^2 b^2)\, b^2 \ldots\ldots b^2} \min\{\min\{h(b^2 e), h(b^2 e)\}, \ldots.. h(b^2 e)\}$

$= \cup_{x=(b^2 b^2)\, b^2 \ldots\ldots b^2} \min\{\min\{m, m\} \ldots \ldots m\}$

$= m$

But from $m = \max\{\mu(a^2), \mu(b^2)^n\} < \inf\{\mu(a^2 S b^2)\}$, we have a contradiction.

So we get $\inf\{\mu(a^2 S b^2)\} \leq \max\{\mu(a^2), \mu(b^2)^n\}$.

Theorem:5

Let S be a right almost semigroup with left identity and μ is a fuzzy ideal of S. If $\mu(xy) = \max\{\mu(x), \mu(y^n)\}$ then μ is a weakly fuzzy quasi primary ideal of S for some positive integer n, where $x, y \in S$.

Proof: Let S be a right almost semigroup.

Let x_t, y_t $(t \in (0,1])$ are the fuzzy points of S such that $x_t\, S\, y_t \subseteq \mu$

Since $S(xy)_t = S(x_t\, y_t) \subseteq x_t\, S\, y_t$
$\subseteq \mu$

And $\mu(xy) = \max\{\mu(x), \mu(y^n)\}$, we have $\mu(xy) \geq t$

Which implies that $\mu(x) \geq t$ or $\mu(y^n) \geq t$, for some positive integer n then,

$x_t \in P$ or $y_t{}^n \in \mu$ then μ is a weakly fuzzy quasi primary ideal of S.

Corollary:

Let S be a right almost semigroup with left identity . If μ is a fuzzy weakly completely primary, then μ is a weakly fuzzy quasi primary ideal of S.

Proof:

The proof of this follows from the above.

Theorem: 6

Let S be a right almost semigroup with left identity. A fuzzy subset μ of a right almost semigroup S is weakly fuzzy quasi primary if and only if $\mu(xy) \leq \max\{\mu(x), \mu(y^n)\}$ for some positive integer n, where $x, y \in S$.

Proof:

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Suppose that μ is a weakly fuzzy quasi primary ideal of S.

if $\mu(xy) > \max\{\mu(x), \mu(y^n)\}$ then there exists $t \in (0,1)$ such that

$\mu(xy) > t > \max\{\mu(x), \mu(y^n)\}$

Then $x_t(Sy_t) = S(x_t\ y_t)$

$= S(xy)$

$= S\,\mu$

$\subseteq \mu.$

For all $x, y \in S$, since μ is a weakly fuzzy quasi primary of S,

we get $x_t \in \mu$ *or* $y_t{}^n \in \mu$, for some positive integer n.

But $x_t \notin \mu$ *and* $y_t{}^n \notin \mu$, which is impossible.

Therefore $\mu(xy) \leq \max\{\mu(x), \mu(y^n)\}$, for some positive integer n, where $x, y \in S$.

Definition:

Let S_1 and S_2 be two right almost semigroups. Then,

$S_1 X\, S_2 = \{(x,y) \in S_1\, X\, S_2 \,/\, x \in S_1, y \in S_2\}$ and for any $(a,b),(c,d) \in S_1\, X\, S_2$, we define $(a,b)(c,d) = (ac, bd)$. Then $S_1\, X\, S_2$ is a right almost semigroup as well.

Let $f: S_1 \to [0,1]$ and $g: S_2 \to [0,1]$ be two fuzzy subsets of right almost semigroups S_1 and S_2 respectively. Then the product of fuzzy subsets is denoted by fXg and defined as $fXg : S_1\, X\, S_2 \to [0,1]$ where $(fXg)(x,y) = \min\{f(x), g(y)\}$.

Theorem: 7

If f_1 and f_2 are fuzzy sub right almost semigroups of S_1 and S_2 respectively, then $f_1 X\, f_2$ is a fuzzy sub right almost semigroup of $S_1\, X\, S_2$.

Proof:

Since f_1 and f_2 are fuzzy sub right almost semigroups of S_1 and S_2 respectively, then

$f_1(x_1, x_2) \geq \min\{f_1(x_1), f_1(x_2)\} \forall\, x_1, x_2 \in S_1$ and

$f_2(y_1, y_2) \geq \min\{f_2(y_1), f_2(y_2)\} \forall\, y_1, y_2 \in S_2$

To show that $f_1 X\, f_2$ is a fuzzy sub right almost semigroup of $S_1\, X\, S_2$

i.e., $(f_1 X f_2)\{(x_1, x_2,\, y_1, y_2)\} \geq \min\{ (f_1 X f_2)(x_1, y_1), (f_1 X f_2)\,(x_2, y_2) \}$

$$\forall\, (x_1, y_1)(x_2, y_2) \in S_1\, X\, S_2 \text{ and } x_1, x_2 \in S_1;\; y_1, y_2 \in S_2$$

Consider $(f_1 X f_2)\{(x_1, y_1)(x_2, y_2)\} = (f_1 X f_2)\{(x_1, x_2,\, y_1, y_2)\}$

Since $(f_1 X f_2)(x,y) = \min\{f_1(x), f_2(y)\}$

We have $(f_1 X f_2)\{(x_1, x_2,\, y_1, y_2)\} = \min\{f_1(x_1, x_2), f_2(y_1, y_2)\}$

$\geq \min\{\min\big(f_1(x_1), f_1(x_2)\big), \min(f_2(y_1), f_2(y_2))\}$

$= \min\{\min\big(f_1(x_1), f_2(y_1)\big), \min(f_1(x_2), f_2(y_2))\}$

$= \min\{(f_1 X\, f_2)\,(x_1, y_1), (f_1 X\, f_2)\,(x_2, y_2)\}$

Therefore $(f_1 X f_2)\{(x_1, x_2,\, y_1, y_2)\} \geq \min\{ (f_1 X\, f_2)\,(x_1, y_1), (f_1 X\, f_2)\,(x_2, y_2) \}$

Thus $f_1 X\, f_2$ is a fuzzy sub right almost semigroup of $S_1\, X\, S_2$.

Theorem: 8

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Let f_1 and f_2 be two weakly completely primary (fuzzy primary, quasi primary) ideals of right almost semigroups S_1 and S_2 respectively. Then $f_1 X f_2$ is a fuzzy weakly completely primary (fuzzy primary, quasi primary) ideal of $S_1\ X\ S_2$

Proof:

Let $(a,b),(c,d) \in S_1\ X\ S_2$. Since f_1 and f_2 are weakly completely primary ideals of S, we get

$$(f_1 X f_2)\big((a,b)(c,d)\big) = (f_1 X f_2)(ac,bd)$$

$$= \min\{f_1(ac), f_2(bd)\}$$

$$\leq \min\{\max\{f_1(a), f_1(c^n)\}, \max\{f_2(b), f_2(d^n)\}\}$$

$$= \max\{\min\{f_1(a), f_2(b^n)\}, \min\{f_1(c^n), f_2(d)\}\}$$

$$\leq \max\{\min\{f_1(a), f_2(b)\}, \min\{f_1(c), f_2(d)\}\}$$

$$= \max\{(f_1 X f_2)(a,b), (f_1 X f_2)(c,d)\}$$

For some positive integer n. Hence $(f_1 X f_2)$ is a fuzzy weakly completely primary of $S_1\ X\ S_2$.

III. CONCLUSION

In this paper we discussed a non associative algebraic structure which is a generalization of semigroup termed as Almost semigroup where in left invertive law leads to Left Almost Semigroups and right invertive law leads to Right Almost Semigroups. Different relationships between fuzzy primary, fuzzy weakly completely primary, weakly fuzzy primary and weakly fuzzy quasi primary ideals on Right Almost Semigroup were discussed. Also the direct product of Right almost semigroups was studied.

The fuzzy set theory is applicable to fuzzy technology in information processing. In the present scenario information processing is in boom and will rapidly increase in the future. Fuzzy algebra is one such attempt to analyse the fuzzy set theory in an understandable way.

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