International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-04, July 2019

Synchronization of Chaotic Sprott-I System using Sliding Mode Control

[1]Dibyani Mili, [2]Dr. Mrinal Buragohain
[1]Post Graduate Student, [2]Associate Professor, Jorhat Engineering College, Jorhat-India,
[1]dibyani.ee16@gmail.com, [2]mrinalburagohain@gmail.com

Abstract: Chaotic Synchronization of a chaotic Sprott-I System has been presented in this paper. The Sliding Mode Control method has been utilized and also Lyapunov Stability Method has been used for validating the stability of the system. Although many methods were considered but Sliding Mode method was found to be more efficient and the required results were obtained using Sliding Mode Controller. Numerical Simulations has been done by MATLAB to validate the results and Sliding Mode Controller has been designed for the chaos synchronization of Identical Sprott-I Chaotic systems.

Keywords — Chaos, Chaos Synchronization, Sprott Systems, Sliding Mode Control, Lyapunov Stability Theory, MATLAB.

I. INTRODUCTION

A system whose state varies with time and follows differential equations where time derivatives are involved. These type of systems are very highly sensitive to initial conditions and are termed as Chaotic systems. These systems showcases high sensitive behaviour and this nature is the butterfly effect [1]. Pecora and Carroll [2] were the pioneers in the field of chaos synchronisation. They introduced methods of synchronisation with different initial condition of two identical systems. After their breakthrough their study in the field of chaos synchronization has inspired more research in these studies.

Many methods have been proposed for controlling of chaos and chaos synchronization. Some methods which have been successfully applied, are the PC method [3], the feedback approach [4], the adaptive method [5], OGY method [6]; active control method [7]; back stepping design method [8], etc. The above mentioned methods are able to synchronize only two similar identical chaotic systems.

In this paper, sliding mode control (SMC) method has been used for deriving two identical Sprott-I chaotic systems. In control systems, the most preferred method is sliding mode control (SMC) because it allows for easy realization, provides faster response, good transient behaviour and also immunity to disturbances and parameter uncertainties.

The paper has been presented in the following order. In Section 2, the problem statement has been discussed. In Section 3, a new result for the synchronization of chaos of an identical Sprott-I system is derived by sliding mode control method. In Section 4, numerical results have been presented as for the illustration of synchronization technique that has been proposed. In Section 5, The main results are concluded which we obtained in this paper.

II. PROBLEM STATEMENT

Professor J C Sprott [9] has defined some simple 3-Dimensional Ordinary Differential Equations with non-linearities which are chaotic in nature and this chaotic system are called as Sprott systems by researchers.

The Sprott I system is given by

$$\begin{aligned} \dot{x} &= -cy \\ \dot{y} &= x + y \\ \dot{z} &= x + y^2 - z \end{aligned} \quad (1)$$

which is chaotic and its chaotic attractor is shown in Fig. (1) when the constants selected as c = 0.2 with initial conditions x (0) = 0.1, y (0) = -0.1, z (0) = 0.5 [10].

Fig. (2) shows the time response of states of the system (1).

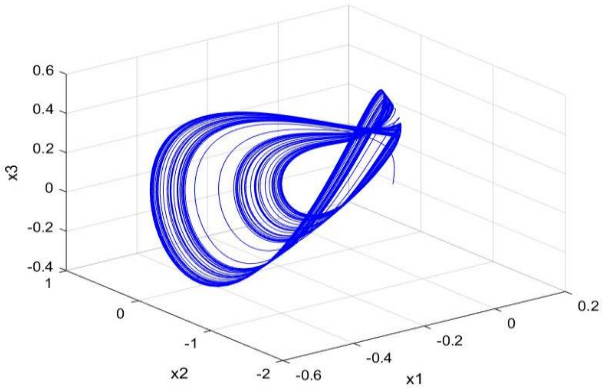

Fig1. Sprott-I Chaotic Attractor.

 DOI : 10.35291/2454-9150.2019.0307

ISSN : 2454-9150 Vol-05, Issue-04, July 2019

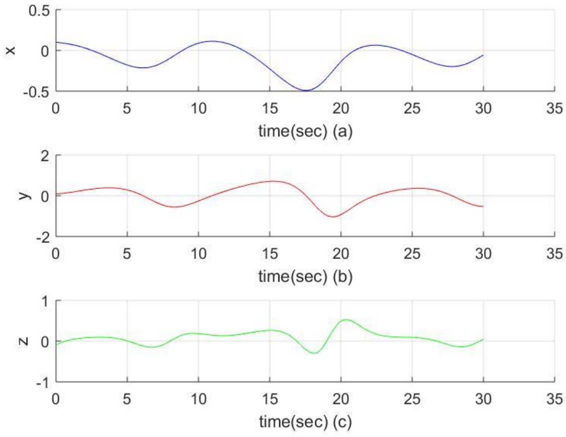

Fig2. Time Response of the systems.

III. CHAOS SYNCHRONIZATION OF IDENTICAL SPROTT-I SYSTEMS

In this section, an identical Sprott-I system is synchronized by using Sliding Mode Control Method.

Thus, the master system is given as

$$\begin{aligned} \dot{x}_1 &= -cy_1 \\ \dot{y}_1 &= x_1 + z_1 \\ \dot{z}_1 &= x_1 + {y_1}^2 - z_1 \end{aligned} \qquad (2)$$

And the slave system is given as

$$\begin{aligned} \dot{x}_2 &= -cy_2 + \mathrm{u(t)} + d(t) \\ \dot{y}_2 &= x_2 + z_2 \\ \dot{z}_2 &= x_2 + y_2^2 - z_2 \end{aligned} \qquad (3)$$

Where x_1, y_1, z_1 are the state variables, d(t) = 0.005sin(t) is nonlinear perturbation input and u(t) is nonlinear controller to be designed.

The synchronization error is defined as

$e_1(t) = x_1(t) - x_2(t)$, $e_2(t) = y_1(t) - y_2(t)$ and $e_3(t) = z_1(t) - z_2(t)$.

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \dot{x}_2 - \dot{x}_1 = -ce_2 + u_1 + d(t) \\ \dot{e}_2 &= \dot{y}_2 - \dot{y}_1 = e_1 + e_3 \\ \dot{e}_3 &= \dot{z}_2 - \dot{z}_1 = e_1 - e_3 + (y_2 + y_1)e_2 \end{aligned} \qquad (4)$$

The designing of the SMC should be very proper and, in a way, so that it is able to satisfy the resulting error vector

$$\lim_{t\to\infty}\|E(t)\| = \lim_{t\to\infty}\|e_1(t)e_2(t)e_3(t)\| \to 0 \qquad (5)$$

The error dynamics equations (4) has to be stabilized. Then synchronization has to be attained. For carrying out the above processes two proper steps have been used: first, a suitable sliding mode selection is considered because the sliding motion on the sliding manifold has to be stable and also ensure that $\lim_{t\to\infty}\|e(t)\| = 0$; second, establishment of a SMC law which will confirm the existence of the sliding mode s(t) =0. The asymptotic stability of the sliding mode has to be confirmed. This is done by the switching surface s(t) which is defined as:

$$s = c_1e_1 + c_2e_2 + c_3e_3 \qquad (6)$$

where c_1, c_2, c_3 are constants.

The proper switching surface equations has been established (6), now the next move is to design an SMC scheme so that the system trajectories can be drive onto the sliding mode, s(t) =0.

This paper has presented the following SMC:

$$u = u_{eq} + u_{sw} \qquad (7)$$

Since the system operates in the sliding mode, the following equations [11], [12] are satisfied

$s(t) = 0$ and $\dot{s}(t) = 0$

Now, the equivalent control $u_{eq}(t)$ is obtained. This is done in the sliding manifold by differentiating (6) with respect to time. The synchronization error (5) is also substituted.

$$\begin{aligned} \dot{s} &= c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 \\ &= c_1(-ce_2 + u_1 + d(t)) \\ &\quad + c_2(e_1 + e_3) \\ &\quad + c_3(e_1 - e_3 + (y_2 + y_1)e_2) = 0 \end{aligned} \qquad (8)$$

From (8) the equivalent control $u_{eq}(t)$ is established by applying the sliding mode process. It is obtained as

$$\begin{aligned} u_{eq}(t) = [&-c_1(-ce_2 + d(t)) \\ &+ c_2(e_1 + e_3) \\ &+ c_3(e_1 - e_3 + (y_2 + y_1)e_2)]/\, c_1 \end{aligned} \qquad (9)$$

Theorem: Consider the dynamic system (4) with the noise perturbation input. The condition $(s^t(t)\dot{s}(t) < 0)$ of sliding mode is satisfied, if the controller *u(t)* is

$$\begin{aligned} u &= u_{eq} + u_{sw} \\ &= [-c_1(-ce_2 + d(t)) \\ &\quad + c_2(e_1 + e_3) \\ &\quad + c_3(e_1 - e_3 + (y_2 + y_1)e_2)]/\, c_1 \\ &\quad - w.sign(s) \end{aligned}$$

where $u_{sw} = -w.sign(s)$ and

$$sign(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases}$$

DOI : 10.35291/2454-9150.2019.0307

ISSN : 2454-9150 Vol-05, Issue-04, July 2019

Proof. Consider the following Lyapunov function defined by the equation

$$V = \frac{1}{2}s^2$$

By differentiating the above equation by time, we get,

$$\dot{V} = s\dot{s}$$

$$= s[c_1\dot{e_1} + c_2\dot{e_2} + c_3\dot{e_3}]$$

$$= s[c_1(-ce_2 + u_1 + d(t)) + c_2(e_1 + e_3) + c_3(e_1 - e_3 + (y_2 + y_1)e_2)]$$

$$= s\{c_1[-ce_2 + (-c_1(-ce_2 + d(t)) + c_2(e_1 + e_3) + c_3(e_1 - e_3 + (y_2 + y_1)e_2)]/c_1 - w.sign(s) + d(t))] + c_2(e_1 + e_3) + c_3(e_1 - e_3 + (y_2 + y_1)e_2)\}$$

$$= s[c_1 d(t) - c_1 w.sign(s)]$$

$$= s[d(t) - w.sign(s)]$$

$$= s[d(t)] - w.|s|$$

If *d(t)* is bounded function and $|d(t)| \leq \gamma$, we get the following result:

$$\dot{V} \leq s[d(t)] - w.|s|$$

$$\leq |s|.[|d(t)|] - w.|s|$$

$$\leq |s|.(\gamma - w) \qquad (12)$$

where $w > \gamma$. Thus, the reaching condition $(s^t(t)\dot{s}(t) < 0)$ is satisfied. Thus, by Lyapunov stability theory, s(t) always converges to the switching surface s = 0. Hence, the proof is achieved completely.

IV. NUMERICAL RESULTS

The numerical validation is also established, the fourth-order Runge-Kutta method is used. The time-step h=0.001 is used to reach the solution of the systems of differential equations of (2) and (3) with the sliding mode controller u given by (10) using MATLAB.

The initial conditions of the master (2) and slave (3) systems are taken as $x_1(0) = 0.1, y_1(0) = -0.14,\ z_1(0) = -0.07$ and $x_2(0) = 0.12,\ y_2(0) = -0.07,\ z_2(0) = -0.1$. Fig.3 illustrates the complete synchronization of the identical Sprott-I chaotic systems (2) and (3).

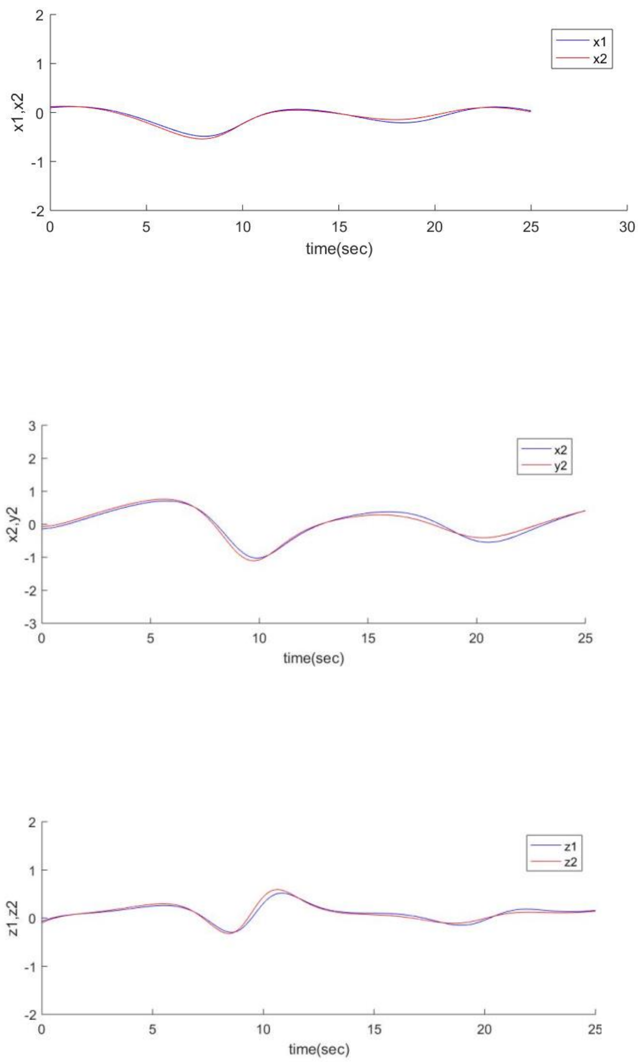

Fig. Synchronization plot of the Identical Sprott-I Systems.

V. CONCLUSIONS

The above discussed results have been realized using the Sliding Mode Controller. This controller has been designed using the theorem of Lyapunov Stability. The chaos synchronization of an Identical Sprott I systems has been examined using the controller. Numerical Solution has been carried out using MATLAB. This is done to illustrate and prove the efficiency of the Identical Sprott I systems. Thus, it can be concluded that the above discussed method in the paper is the most convenient and effective for achieving chaos Synchronization of Identical Sprott I systems.

REFERENCES

[1] K.T. Alligood, T.Sauer and J.A. Yorke, Chaos: An Introduction to Dynamical Systems, New York: Springer,1997.

[2] Pecora, Carroll,: Synchronization in Chaotic Systems. Phys.Rev. Lett, 821-823 64(1990).

 DOI : 10.35291/2454-9150.2019.0307

ISSN : 2454-9150 Vol-05, Issue-04, July 2019

[3] Pecora L M, Carroll T L,: Driving systems with chaotic signals. Phys. Rev, A44 2374-2383 (1991).

[4] Park J H, Kwon O M,: A novel criterion of delayed feedback control of time delay chaotic systems. Chaos, Solitons and Fractals, 495-501 23(2005).

[5] Chen M, Han Z,: Controlling and synchronizing chaotic genetic system via Non linear feedback control. Chaos, Solitons and Fractals,709-716 17(2003).

[6] Ott E, Grebogi C, Yorke J A,: Controlling chaos. Phys. Rev.Lett., 1196-1199 64(1990).

[7] M.C. Ho and Y.C. Hung,: Synchronization of two different chaotic systems using generalized active network, Physics Letters A,vol.301,pp.424-428 (2002).

[8] Lü Jin-hu, Zhang Suo-chun, Controlling Chen's chaotic attractor using back stepping design based on parameters identification. Physics Letters A, 286(2001) 1387-1396.

[9] Sprott J C, Some simple chaotic flows, Phys. Rev. E,R647-R650(1994).

[10] Dengguo Xu,: Chaos Synchronization between Two Different Sprott Systems. Adv. Theory. Appl. Mech., Vol. 3,no.4,195-201(2010).

[11] V.I Utkin,: Sliding Mode and their applications in Variable Structure Systems. Mir Editors, Moscow (1978).

[12] Yi-You Hou, Cheng-Shun Fang,: Sliding Mode Controller Design for Chaos Synchronization of Rititake Chaotic Systems. 978-1-5090-0832-2(2016).

 DOI : 10.35291/2454-9150.2019.0307