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Radiation effects on unsteady two-dimensional flow through a porous medium over a stretching sheet with slip boundary conditions

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Abstract: The present study deals with the effects of heat transfer in the boundary layer unsteady flow of a viscous incompressible fluid through a permeable (porous) medium adjacent to a stretching sheet. In addition, the investigation of the heat effects in the consideration of radiations has been concluded. The governing time dependent boundary layer flow and thermal energy equations are reduced to a set of ordinary differential equations using similarity transformations. In order to obtain the results, these equations are elucidated numerically under the velocity slip and temperature jump conditions at the surface of the sheet. Effects of various pertinent parameters on the unsteady fluid flow and temperature fields are examined and analyzed graphically.

***Keywords**: Radiation, stretching sheet, porous medium, unsteady flow, velocity slip, Temperature jump condition*.

I. INTRODUCTION

The effort to study the transfer of heat and its flow over a stretching surface has significant importance because of its wide applications in various engineering and industrial manufacturing processes, like, in polymer industries, extrusion of plastic sheets, metal spinning, drawing films and fibers, glass blowing and paper production. In such processes, the quality of final products depends on the rate of cooling and skin-friction at the stretching sheet. Since the pioneering work by [13] of presenting an absolute solution analytically for the steady 2-dimensional flow of a viscous fluid over a stretching sheet, many researchers have considered various other aspects of this problem and obtained analytical and numerical solutions, e.g. [20], [4], [21], [11], [17], [2], [3], [23], [19], and [6].

The above researchers examined steady flow and heat transfer in a semi-infinite fluid layer over a stretching surface, however, in many cases of practical interest the flow and temperature field can be unsteady. For example, a sudden stretching or impulsively stretching of the sheet leads to time dependent flow field. Such unsteady flow and heat transfer over a stretching sheet have been investigated by a number of authors, e.g. [27], [18], [34], [24],[33] ,[35], [1], [23].

In high temperature processes, the effect of radiation is also important. These radiative effects have significant applications in physics and space Technology. Many research works have been reported concerning radiation impacts on the boundary layer fluid flow over a stretching sheet. See for example,[28] , [31], [25], [16],[30] ,[26] , and [12].

In view of industrial applications, it is required to analysed the steady and unsteady flow and transfer of heat of the fluids through permeable (porous) medium adjacent to stretching surfaces. [14] investigated flow and heat transfer through a porous medium adjacent to a stretching sheet with internal heat generation/absorption. [15] also studied such flow of an electrically conducting second-grade fluid with chemically reactive species. [8] presented numerical and approximate solution of flow in the presence of a magnetic field through the permeable (porous) medium adjacent to a stretching sheet. [9] analyzed heat transfer and entropy generation in such flow through porous medium. [10] examined slip effects on flow and heat transfer of a second-grade fluid through permeable (porous) medium adjacent to a stretching sheet with power- law surface temperature/heat flux. [7] analyzed stagnation-point flow of a polar fluid adjacent to a stretching sheet embedded in a porous medium.

In the present investigation, a viscous incompressible fluid unsteady flow through a permeable (porous) medium over a stretching sheet is considered. Heat transfer effects on such unsteady flow are investigated in the presence of radiation. Using similarity transformations and slip conditions at the surface of the sheet, the governing equations for velocity and temperature fields are solved numerically, and effects of various physical parameters on the velocity profiles, temperature profiles coefficient of skin-friction and rate of heat transfer are examined and discussed.

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II. PROBLEM FORMULATION

A viscous fluid unsteady flow in a permeable (porous) medium over a stretching sheet is considered. The sheet coincides with the plane $y = 0$, and the flow in permeable (porous) medium is considered above the sheet $y > 0$. The axis of x is taken along the stretching sheet and the axis of y is taken normal to it in the outward direction into the fluid. The sheet moves in its own plane, with a velocity,

$$U_{\omega}(x,t) = \frac{bx}{1-\lambda t}. \qquad (1)$$

where b and λ, are positive constants and measures stretching rate and unsteadiness respectively.

The temperature distribution is defined as:

$$T_{\omega}(x,t) = T_{\infty} + T_0\left(\frac{bx^2}{2\upsilon}\right)(1-\lambda t)^{-3/2}. \qquad (2)$$

where, T_0, a reference temperature; T_{∞}, the ambient temperature; and υ, the kinematic viscosity. Here, $T_{\omega}(x,t) > T_{\infty}$.

The governing equations, for unsteady flow through a permeable (porous) medium and temperature distribution, under the boundary layer approximations, are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (3)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\phi\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{k_0}u, \qquad (4)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_\rho}\frac{\partial q_r}{\partial y}.$$

(5)

The boundary conditions for the present problem are given by:

$$y = 0,\ \ u = U_{\omega}(t,x) + L_1\frac{\partial u}{\partial y},\ \ v = 0,$$

at

$$T = T_{\omega}(t,x) + L_2\frac{\partial T}{\partial y}$$

as $y \to \infty,\ \ u \to 0,\ \ \ T \to T_{\infty}$ (6)

where, u and v are the velocity components in x and y direction respectively; T, the temperature ; k_0, the permeability of the porous medium; ρ, the density; C_p, the specific heat at the constant pressure; k, the thermal conductivity ; L_1 and L_2, the velocity and temperature slip constants; $\phi = (\bar{\mu}/\mu)$, the viscosity ratio; μ, the viscosity of the clear fluid; $\bar{\mu}$ the effective viscosity of the fluid in porous medium; and $\upsilon = \mu/\rho$, the kinematic viscosity.

In this research, an optically thick fluid is considered, which is absorbing , emitting, gray but not scattering , in a highly porous medium domain . In particular, the Rosseland radiation flux model is employed in the analysis following [5] which gives excellent results for optically thick fluids. As suggest in this model, the radiative heat transfer takes the following form:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (7)$$

where, σ^*, the Stefan–Boltzman constant; k^*, the mean absorption coefficient for thermal radiation ;and the temperature function T^4 in (2.5.7) , following [30], can be expressed as a linear function of T, by expanding T^4 in a Taylor Series about T_{∞} and neglecting higher order term as,

$$T^4 = 4T_{\infty}^3 T - 3T_{\infty}^4$$

Let us introduce the following similarity transformations:

$$u = \frac{bx}{1-\alpha t}f'(\eta),$$

$$v = -(\upsilon b)^{1/2}(1-\lambda t)^{-1/2}f(\eta),$$

$$\theta(\eta) = \frac{T-T_{\infty}}{T_{\omega}-T_{\infty}}. \qquad (8)$$

where, $\eta = (b/\upsilon)^{1/2}(1-\lambda t)^{-1/2}y$

Using above similarity transformations (8), the partial differential equations (3) to (5) are reduced to, the following dimensionless system of ordinary differential equations:

$$\phi f''' - S\left[f' + \frac{\eta}{2}f''\right] - (f')^2 + ff'' - \frac{f'}{K} = 0 \quad (9)$$

$$\theta''\left(\frac{3R+4}{3R}\right) - \Pr\left[-\theta' f + 2f'\theta + S\left(\frac{3}{2}\theta + \eta\frac{1}{2}\theta'\right)\right] = 0 \quad (10)$$

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The corresponding boundary conditions are reduced to

At

$$\eta = 0\,,\ f(0) = 0,\ f'(0) - 1 = \alpha f''(0),\quad \theta(0) - 1 = \beta\theta'(0) \qquad (11)$$

as $\eta \to \infty,\quad f'(\infty) = 0,\quad \theta(\infty) = 0 \qquad (12)$

where, $S = \dfrac{\lambda}{b}$, the unsteadiness parameter;

$\Pr = \dfrac{\mu C_p}{k},$ the Prandtl number;

$K = \dfrac{k_0 b}{\upsilon(1-\lambda t)},$ the Permeability parameter;

$R = \dfrac{kk^*}{4\sigma^* T_\infty^3}$, the Radiation parameter;

$\alpha = L_1\left[\dfrac{b}{\upsilon(1-\lambda t)}\right]^{1/2}$, the velocity slip parameter; and

$\beta = L_2\left[\dfrac{b}{\upsilon(1-\lambda t)}\right]^{1/2}$, the temperature slip parameter.

The skin friction coefficient at the stretching sheet is given by

$$C_f = \frac{-2\tau_w}{\rho(U_w)^2} = \frac{-2}{(R_{ex})^{1/2}} f''(0), \qquad (13)$$

where $R_{ex} = \dfrac{U_\omega x}{\upsilon}$, is the local Reynolds number.

The non-dimensional rate of heat transfer at the stretching sheet is given by

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = -\theta'(0) \qquad (14)$$

III. NUMERICAL SOLUTION

For the numerical solution for this problem, governed by the set of non-linear ordinary differential equations (9) and (10) subjected to the boundary conditions (11) and (12), new variables are defined as follows:

$$x_1 = f,\ \ x_2 = f',\ \ x_3 = f'',\ \ x_4 = \theta,\ x_5 = \theta' \qquad (15)$$

Using above variables (15), the equations (9) and (10) are reduced into an equivalent system of five first order differential equations as follows:

$$x_1' = x_2$$
$$x_2' = x_3$$
$$x_3' = \left[-x_1x_3 + x_2^2 + S\left(x_2 + \frac{1}{2}\eta x_3\right) + \frac{x_2}{K}\right]\frac{1}{\phi}$$
$$x_4' = x_5$$
$$x_5' = \Pr\left(\frac{3R}{3R+4}\right)\left(-x_5x_1 + 2x_2x_4 + S\left(\frac{3}{2}x_4 + \eta\frac{1}{2}x_5\right)\right) \qquad (16)$$

The corresponding boundary conditions for this problem are reduced to the following:

$$\text{at}\ \ \eta = 0\ \ x_1(0) = 0,\ x_2(0) - 1 = \alpha x_3(0),\quad x_4(0) - 1 = \beta x_5(0),$$

$$\text{and as}\ \ \eta \to \infty\ \ x_2(\infty) = 0\ \ x_4(\infty) = 0 \qquad (17)$$

As explained by [32], it is convenient in solving an ODE system numerically, by describing the problem in terms of a system of first-order differential equations. The numerical solution of the problem governed by the system of equations (16) - (17) is obtained by using MATLAB BVP solver bvp4c. For numerical solution the asymptotic boundary conditions of the problem described in (12) at $\eta = \infty$ are replaced by those conditions at a large but finite value of η where no considerable variation in velocity, temperature etc. occur, as is usually the standard practice, in the boundary layer analysis. The computational procedure is repeated until; we get the results up to the desired degree of accuracy, 10^{-6}.

IV. DISCUSSIONS

Radiation effects on the unsteady flow via a permeable (porous) medium next to a stretching sheet have been investigated with, slip-velocity and temperature, boundary conditions.

The variations of velocity profiles $f'(\eta)$ with η are plotted in figures 1-3, for various different values of the parameters such as, the unsteadiness parameter (S), the permeability parameter (K), and the velocity slip parameter (α) . It is found that the velocity component $f'(\eta)$ for any fixed η decreases with the increase in the value of the parameter (S) . It is seen that increasing permeability (K) of the permeable (porous) medium, causes flow faster in the boundary layer adjacent to the stretching sheet, while, the velocity slip parameter causes flow slower for all values of η, in the boundary layer .In fact, with the increase in value (α), the magnitude of slip at the sheet increases, because as (α) increases the resistance between the fluid and the stretching surface

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decreases. Figure 4, shows the variations of skin -friction in terms of $-f''(0)$ at the stretching sheet for various values of the physical parameters. It is noticed that the unsteadiness parameter causes an increase in the magnitude of skin-friction $-f''(0)$, while the permeability (K) of the permeable (porous) medium reduces it. It is observed that the scale of the skin-friction $-f''(0)$ decreases as the value of slip parameter (α) increases, because larger velocity slip lowers the friction at the sheet.

Figure 5 to 8 show the variations of temperature distribution for various different values of the pertinent parameters. Figure 5 illustrates the effect of the unsteadiness parameter (S) and it is seen that its effect is to decrease temperature in the boundary layer. Figure 6, depicts the effect of Prandtl number $(\Pr)$ on the temperature in the boundary layer. As expected, the temperature in the boundary layer decreases with the increase in $(\Pr)$ value because the thermal conduction in the boundary layer is lowered by the increase in $(\Pr)$ value, which results in lowering the molecular movement of the fluid elements and thus causing a temperature fall. Figure 7, illustrates that as we increase the value of the radiation parameter (R) or the temperature slip parameter (β), the temperature in the boundary layer decreases at all points, while the velocity slip (α) causes an increase in temperature in the flow domain. Figure 8 depicts the effects of the permeability parameter (K) on the temperature in the flow domain. It is observed that temperature decreases in the flow domain by increasing the value of (K).

Table 1 illustrates the variations of the non-dimensional rate of heat transfer $-\theta('0)$ at the stretching sheet. It is observed that the effect of unsteadiness parameter (S), or the Prandtl number $(\Pr)$, or the viscosity ratio parameter (ϕ), or the radiation parameter (R), or the permeability parameter (K), is to increase the rate of heat transfer at the stretching sheet. However, it is seen that the effect of the velocity slip parameter (α), or the temperature slip parameter (β), is to reduce the rate of heat transfer at the sheet.

V. CONCLUSIONS

In conclusion, this study presents numerical solutions of the unsteady boundary layer flow and heat transfer through a porous medium adjacent to a stretching sheet in the presence of the radiation. The numerical solutions depict the following main results of the study:

It is found that the permeability (K) of the porous matrix enhances the flow and the rate of the heat transfer at the sheet, while it reduces the skin-friction at the sheet and the temperature in the boundary layer.

The effect of the unsteadiness parameter (S) is to decrease the flow and the temperature in the boundary layer, while it enhances the skin-friction and the rate of the heat transfer at the sheet.

It is seen that the effect of the velocity slip (α) is to decrease the flow, the skin-friction and the rate of the heat transfer at the sheet, while it enhances the temperature in the boundary layer.

The heat transfer rate at the stretching sheet enhances with increasing $\Pr$, causing therefore suppress in the thermal boundary layer thickness.

It is seen that the temperature slip (β), or the radiation parameter (R), decreases the temperature in the boundary layer, however R increases the rate of the heat transfer at the sheet, while β decreases it.

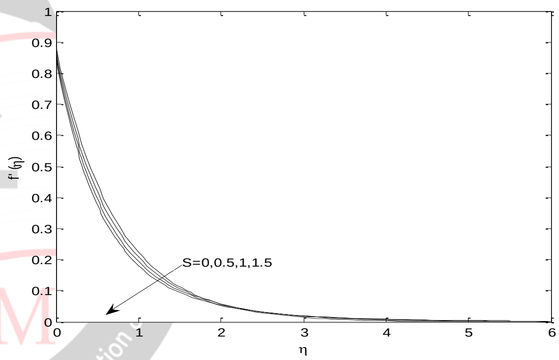

Figure.1 Velocity profiles $f'(\eta)$ versus η for $\phi=1, K=1, \alpha=0.1$

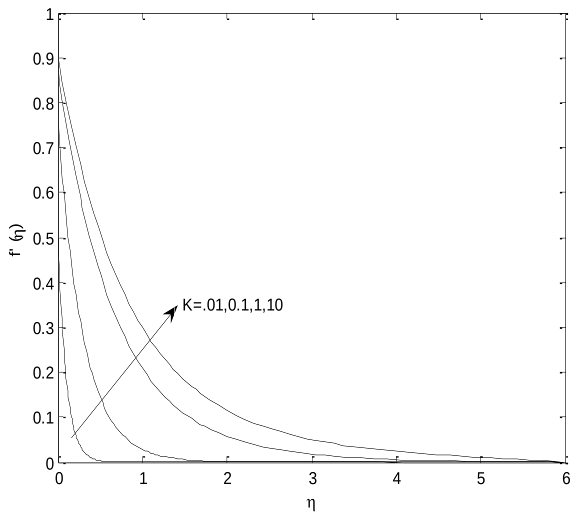

Figure2. Velocity profiles $f'(\eta)$ versus η fo $\phi=1, S=0.5, \alpha=0.1$

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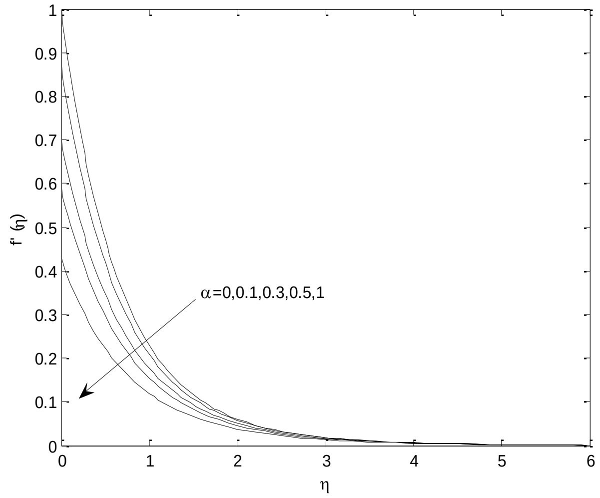

Figure. 3 Velocity profiles $f'(\eta)$ versus η for $\phi = 1, S = 0.5, K = 1$

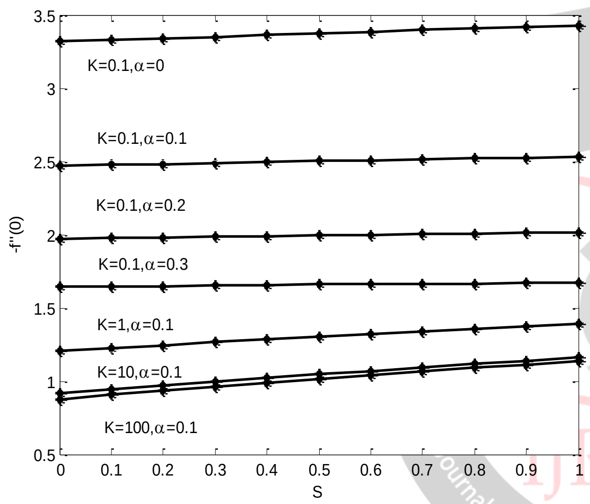

Figure. 4 Variations of $-f''(0)$ versus S

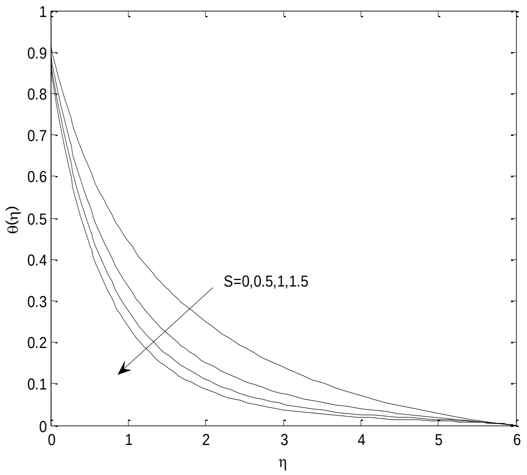

Figure.5 Temperature profiles $\theta(\eta)$ versus η for $\Pr = 1, K = 1, R = 3, \alpha = 0.1, \beta = 0.1$

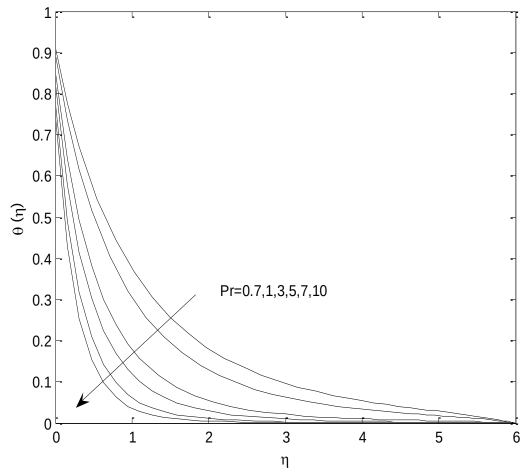

Figure.6 Temperature profiles $\theta(\eta)$ versus η for $S = 1, K = 1, R = 3, \alpha = 0.1, \beta = 0.1$

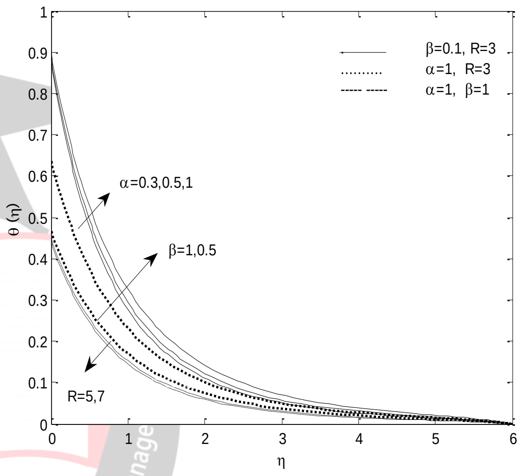

Fig.7 Temperature profiles $\theta(\eta)$ versus η for $\Pr = 1, S = 1, K = 1$

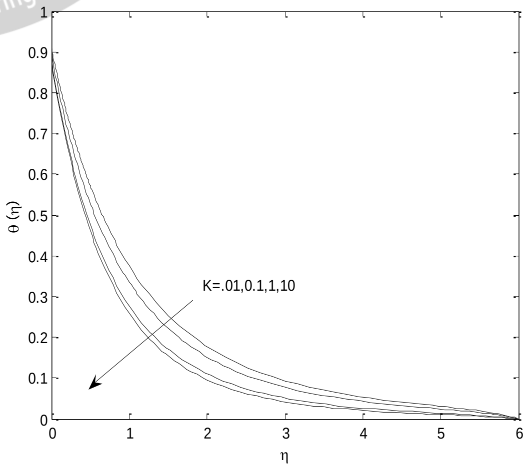

Figure.8 Temperature profiles $\theta(\eta)$ versus η for $S = 1, \Pr = 1, R = 3, \alpha = 0.1, \beta = 0.1$

Table 1 Variations of $-\theta'(0)$ with different values of the parameters

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S	Pr	φ	R	K	α	β	−θ′(0)
0.5	1	1	1	1	0.1	0.1	0.795
1							0.927
1.5							1.033
2							1.125
1	0.7	1	1	1	0.1	0.1	0.772
	1						0.927
	3						1..579
	5						1.984
	7						2.287
1	1	1	1	1	0.1	0.1	0.927
		1.25					0.941
		2					0.969
		4					1.007
1	1	1	0.5	1	0.1	0.1	0.735
			1				0.927
			5				1.255
			7				1.293
			10				1.324
1	1	1	1	0.01	0.1	0.1	0.727
				0.1			0.824
				1			0.927
				10			0.956
1	1	1	1	1	0	0.1	0.959
					0.1		0.927
					0.3		0.883
					0.5		0.854
					1		0.81
1	1	1	1	1	0.1	0	1.022
						0.1	0.927
						0.3	0.782
						0.5	0.676
						1	0.505

V. REFERENCES

[1] M. Abd-El-Aziz, "Flow and Heat Transfer over an unsteady stretching surface with Hall effect", *Acta Mechanica*, vol. **45**, pp. 97-109, 2010.

[2] P.D. Ariel, "The flow of an elastic-viscous fluid past a stretching sheet with partial slip", *Acta Mechanica*, vol. **187**, pp. 29-35, 2006.

[3] P.D. Ariel, T. Hayat and S. Asghar, "Homotopy perturbation method and axisymmetric flow over a stretching sheet", *Int. J. Nonlinear Sci.Numer.Simul.*,vol.**7,** pp. 399-406, 2006.

[4] P. Carragher, L.J. Crane, "Heat transfer on a continuous stretching sheet", *Z Angew Math Mec*, vol. **62**, pp. 564-565, 1982.

[5] R. Siegel and J. R. Howell , "Thermal Radiation Heat and Transfer", *Mc-Graw Hill Book Company, New York*, 1972.

[6] A.J. Chamkha, A.M. Aly and M.A. Mansour, "Similarity Solution for Unsteady Heat and Mass Transfer from a Stretching Surface Embedded in a Porous Medium with Suction/Injection and Chemical Reaction Effects" *Chemical Engineering Communications*, vol. **197**, pp. 846-858, 2010.

[7] A.J. Chamkha and A.M. Aly, "Heat and Mass Transfer in Stagnation-Point Flow of a Polar Fluid towards a Stretching Surface in Porous Media in the Presence of Soret, Dufour and Chemical Reaction Effects", *Chemical Engineering Communications*, vol. **198**, pp. 214-234, 2011.

[8] D.S. Chauhan and R. Agarwal, "MHD flow through a porous medium adjacent to a stretching sheet :numerical and an approximate solution" *Eur. Phys. J. Plus*, vol. **126(5).** Doi:10.1140/epjp/i2011-11047-3, 2011.

[9] D. S. Chauhan and P. Rastogi, "Heat transfer and entropy generation in MHD flow through a porous medium past a stretching sheet", *Int. J. Energy Technology*, vol. **3 (15)**, pp. 1-13, 2011.

[10] D.S. Chauhan and A. Olkha,A, "Slip flow and heat transfer effect of a second-grade fluid in a porous medium over a stretching sheet with power-law surface temperature or heat flux", *Chem.Eng.Communication*, vol. **198(9),** pp. 1129-1145,2011.

[11] C. K. Chen and M.I. Char, "Heat transfer of continuous stretching surface with suction r blowing", *J Math Anal Appl*, vol. **135**, pp. 568-580, 1988.

[12] C.H. Chen, "On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation", *Int. J. of Heat and Mass Transfer***,** vol. **53**, pp. 4264-4273, 2010.

[13] L.J. Crane, "Flow past a stretching plate", *Z Angew Math Phys*, vol. **21**, pp. 645-647,1970.

[14] R. Cortell, "Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation /absorption and suction/blowing", *Fluid Dyn.Res.*, vol. **37,** pp. 231-245, 2005.

[15] R. Cortell, "MHD flow and mass transfer of an electrically conducting fluid of a second grade in a porous medium over a stretching sheet with chemically reactive species", *Chem. Eng. Process*, vol. **46,** pp. 721-728,2007.

[16] R. Cortell, "Effects of viscous dissipation and radiation on the thermal boundary layer over a non linearly stretching sheet Physics Letters **A",** vol. **372**, pp. 631- 636, 2008.

[17] E.M.A Elbashbeshy, "Heat transfer over a stretching surface with variable surface heat flux", *J. Phys .D. : Appl. Phys.*, vol. **31**, pp. 1951-1954, 1998.

[18] E.M.A Elbashbeshy and M.A.A, Bazid, "Heat transfer over an unsteady stretching surface", *Heat Mass Transf.*, vol. **4,** pp. **1**-4, 2004.

 DOI : 10.35291/2454-9150.2019.0309

[19]` E.M.A. Elbashbeshy and D.A. Aldawody, "Heat transfer over an unsteady stretching surface with variable heat flux in the presence of heat source or sink", *Computer and Mathematics with Applications*, vol. **60(10)**, pp. 2806-2811, 2010.

[20] P.S. Gupta and A.S. Gupta, "Heat and mass transfer on a stretching sheet with suction or blowing", *Can J Chem Engg.*, vol. **55**, pp. 744-746, 1977.

[21] L.J. Grubka and K.M. Bobba, "Heat transfer characteristics of a continuous stretching surface with variable temperature", *ASME J Heat transfer*, vol. **107**, pp. 248-250, 1985.

[22] W. Ibrahim and B. Shanker, "Unsteady MHD boundary layer flow and heat transfer duet to stretching by quasi –linearization technique", *Int. J. of Appl. Math and Mech*, vol. **8 (7),** pp. 18-30, 2012.

[23] S.J. Liao, "An analytic solution of unsteady boundary- layer flows caused by an impulsively stretching plate", *Commun Nonlinear Sci Numer Simul*., vol. **11(3)**, pp. 326-339, 2006.

[24] S.J. Liao, "A new branch of solutions of boundary layer flows over a permeable stretching plate", *Int. J. Non- linear Mech*, vol.**42**, pp. 819-830, 2007.

[25] S. Mukhopadhyay and G.C. Layek, "Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface", *Int. J. Heat and Mass transfer***,** vol. **51**, pp. 2167-2178 , 2008.

[26] S. Mukhopadhyay, "Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in a porous medium", *Int. J. of Heat and Mass Transfer*, vol. **52**, pp. 3261-3265, 2009.

[27] T.Y. Na and I. Pop, "Unsteady flow past a stretching sheet", Mech Res Commun, vol. **23**, pp. 413-422, 1996.

[28] M.E.M., Quaf, "Exact solution of thermal radiation on MHD flow over a stretching porous sheet", *Appl. Mathematics and Computations*, vol. **170,** pp. 117-1125, 2005.

[29] A. Rapits, "Radiation and free Convection flow through a porous medium", *Int. Commun. Heat Mass Transfer*, vol. **25**, pp. 289-295, 1998.

[30] M. Sajid and T. Hayat, "Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet", *Int. Comm. In Heat and Mass transfer*, vol. **35**, pp. 347-356, 2008.

[31] M. A. Seddeek and M.S. Abdelmeguid, "Effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux", *Physics Letters A***,** vol. **348**, pp. 172-179, 2006.

[32] L.F. Shampine, I. Gladwell and S. Thompson, "Solving ODEs with MATLAB", *Cambridge University Press,* 2003 (Chapter 3).

[33] Y. Tan and S.J. Liao, "Series solution of three-dimensional unsteady laminar viscous flow due to a stretching surface in a rotating fluid", *ASME J. Appl. Mech*, vol. **74(5),** pp. 1011-1018, 2007.

[34] H. Xu and S.J. Liao, "Analytic solutions magneto hydrodynamic flows of non- Newtonian fluids caused by an impulsively stretching plate", *J. Non-Newtonian Fluid Mech*. vol. **129,** pp**.** 46-55, 2005.

[35] H. Xu, S.J. Liao and I. Pop, "Series solutions of unsteady three-dimensional MHD flow and heat transfer in theboundary layer flows caused by impulsively stretching plate", *commun Nonlinear Sci Numer Simul*, vol. **11(3)**, pp. 326-339, 2007.

 DOI : 10.35291/2454-9150.2019.0309