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Intuitionistic fuzzy quasi λ- continuous mappings

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Abstract **The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy quasi λ- continuous mappings in intuitionistic fuzzy topological space. Some of their properties are explored**.

Keywords— Intuitionistic fuzzy topology, intuitionistic fuzzy λ- closed set, intuitionistic fuzzy λ- open set and intuitionistic fuzzy quasi λ- continuous mappings.

I. INTRODUCTION

Fuzzy set (FS) as proposed by Zadeh [19] in 1965 is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

This paper aspires to overtly enunciate the notion of intuitionistic fuzzy quasi λ- continuous mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy quasi λ- continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic mappings.

II. PRELIMINARIES

Definition 2.1 ([1]) Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form A = $\{<x, \mu_A(x), \upsilon_A(x)> : x\in X\}$, where the function

$\mu_A : X \to [0,1]$ and $\upsilon_A : X\to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\upsilon_A(x)$ of each element x∈ X to the set A respectively and

$0\leq \mu_A(x)+ \upsilon_A(x) \leq 1$ for each x∈ X.

Definition 2.2 ([1]) Let A and B be intuitionistic fuzzy sets of the form

A = $\{<x, \mu_A(x), \upsilon_A(x)>: x \in X\}$, and form B= $\{<x, \mu_B(x), \upsilon_B(x)>: x\in X\}$.Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \upsilon_A(x), \mu_A(x) \rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \upsilon_A(x) \vee \upsilon_B(x) \rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \upsilon_A(x) \wedge \upsilon_B(x) \rangle / x \in X\}$.

The intuitionistic fuzzy sets $\underset{\sim}{0} = \{ <x, 0 ,1> ; x \in X \}$ and $\underset{\sim}{1} =\{ <x, 1, 0 >; x \in X \}$ are respectively the empty set and whole set of X.

Definition 2.3 [5]) An intuitionistic fuzzy topology (IFT) on X is a family of IFS which satisfying the following axioms.

(i) $\underset{\sim}{0}, \underset{\sim}{1} \in\tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1 , G_2 \in\tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in I\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS) and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS)in X.

The complement A of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in (X, τ).

Definition 2.4([3]) Let (X, τ) be an intuitionistic fuzzy topology and

A = $\{<x, \mu_A(x), \upsilon_B(x) >: x \in X\}$, be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

int (A) = ∪ {G/ G is an intuitionistic fuzzy open set in X and $G \subseteq A$}

cl (A) =∩ { K/ K in an intuitionistic fuzzy closed set in X and $A\subseteq K$ } **Definition 2.5(** [11]) Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be an

(i) intuitionistic fuzzy open mapping (IF open mapping) if f(A) is an IFOS in Y for every IFOS A in X.

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(ii) intuitionistic fuzzy closed mapping (IF closed mapping) if f(A) is an IFCS in Y for every IFCS A in X.

Remark 2.6 ([3]) For any intuitionistic fuzzy set A in (X, τ) ,we have

(i)cl (A^C) = [int (A)]^C ,

(ii) int (A^C) = [cl (A)]^C ,

(iii) A is an intuitionistic fuzzy closed set in

X ⇔ Cl (A) = A

(iv) A is an intuitionistic fuzzy open set in

X ⇔ int (A) =A

Definition 2.7 ([6]) An intuitionistic fuzzy set

A = {<x, μ_A (x), υ_B (x) >: x ∈ X} in an intuitionistic fuzzy topology space (X, τ) is said to be

(i) intuitionistic fuzzy semi closed if int (cl (A)) ⊆ A.

(ii) intuitionistic fuzzy pre closed if cl (int (A))⊆ A.

Definition 2.8 ([5]) Let X and Y are nonempty sets and

f :X→Y is a function

(a) If B = {< y, μ_B (y) ,υ_B (y) > : y ∈ Y } is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by

f^{-1}(B) , is defined by

f^{-1} (B) =={<x ,f^{-1}(μ_B(x)),f^{-1}(υ_B(x)) > : x ∈ X}

(b) If A= {<{x, μ_A (x) , υ_B (x),)> / x∈ X} is an intuitionistic fuzzy set in X, the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by

f(A) = {<y , f (μ_A (y)), f (υ_A(y)) > : y ∈ Y} where f (υ_A) = 1-f(1-(υ_A)).

Definition 2.9 ([9]) An intuitionistic fuzzy set A of an intuitionistic topology space (X , τ) is called an

(i) intuitionistic fuzzy λ-closed set (IF λ-CS) if A ⊇ cl(U) whenever A ⊇ U and U is intuitionistic fuzzy open set in X. (ii) intuitionistic fuzzy λ-open set (IF λ-OS) if the complement A^c is an intuitionistic fuzzy λ-closed set A.

Definition 2.10: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) called

(i)intuitionistic fuzzy generalized closed set [15]

(intuitionistic fuzzy g – closed) if cl(A) ⊆ U whenever

A ⊆ U and U is intuitionistic fuzzy semi open)

(ii)intuitionistic fuzzy g – open set[14], if the complement

of an intuitionistic fuzzy g – closed set is called

intuitionistic fuzzy g - open set.

(iii)intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed)[6] if there exists an intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) such that

U⊆A ⊆ Cl(U) (resp. int(U) ⊆ A ⊆ U).

Definition 2.11 : An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called

(i) an intuitionistic fuzzy w-closed **[14]** if cl(A) ⊆ O whenever A ⊆O and O is intuitionistic fuzzy semi open.

(X,τ)

(ii) an intuitionistic fuzzy generalized α-closed set [8] (IFGαCS if αcl(A) ⊆ O whenever A ⊆O and O is IFαOS in(X,τ)).

(iii) an intuitionistic fuzzy α-generalized closed set [12] (IFαGCS if αcl(A) ⊆ O whenever A ⊆O and O is IFOS in

(X,τ)

(iv) inuitionistic fuzzy regular closed set [4] (IFRCS in short) if A = cl(int(A)).

Inuitionistic fuzzy regular open set [4](IFROS in short) if A = int(cl(A)).

Definition 2.12 : A mapping f : (X,τ) → (Y, σ) is said to be

(i) an intuitionistic fuzzy w-closed [15]if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy w-closed set in Y

(ii) an intuitionistic fuzzy regular closed [17] if if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy regular closed set in Y.

(iii)) an intuitionistic fuzzy generalized α-closed [8] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy generalized α-closed set in Y

(iv) an intuitionistic fuzzy α-generalized closed [12] if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy α-generalized closed set in Y

Definition 2.13: Let f be a mapping from an IFTS (X,τ) into an IFTS (y,σ). Then f is said to be

(i) intuionisnistic fuzzy continuous [4] (IF continuous in short) if f^{-1}(B) is an IFOS in X for every IFOS B in Y

(ii) Intuitionistic fuzzy α continuous [5] (IFα continuous in short) if f^{-1}(B) is an IFαOS in X for every IFOS B in Y .

(iii) An intuitionistic fuzzy pre continuous [15] (IFP continuous in short) if f^{-1}(B) is an IFPOS in X for every IFOS B in Y .

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(iv) intuitionistic fuzzy generalized continuous [14] (IFG continuous in short) if $f^{-1}(B)$ is an IFGOS in X for every IFOS B in Y .

(v) Intuitionistic fuzzy α generalized continuous [11] (IFαG continuous in short) if $f^{-1}(B)$

is an IFαGOS in X for every IFOS B in Y .

(vi) Intuitionistic fuzzy almost continuous [18] (IFA continuous in short) if f−1(B) is an IFOS in X for every IFROS B in Y .

Definition 2.14 ([10]) A mapping f: (X,τ) → (Y, σ) is said to be intuitionistic fuzzy λ –continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy λ -closed in X.

Definition 2.15 ([11]) A topological space (X, τ) is called intuitionistic fuzzy λ - $T_{1/2}$space

(IF λ - $T_{1/2}$ space in short) if every intuitionistic fuzzy λ - closed set is intuitionistic fuzzy closed in X.

Definition 2.16 ([10]) Let *A* be an IFS in an IFTS (*X*, τ). Then the Intuitionistic fuzzy λ- interior and intuitionistic fuzzy λ- closure of A are defined as follows.

λ-int(*A*) = ∪{*G* |*G* is an IFλ-OS in *X* and *G* ⊆ *A*}

λ-cl(*A*) = ∩{*K* | *K* is an IFλ-CS in *X* and *A*⊆ *K* }

III. INTUITIONISTIC FUZZY QUASI Λ-CONTINUOUS MAPPINGS

In this section, we introduce intuitionistic fuzzy quasi λ-continuous mappings and study some of their properties.

Definition 3.1. A mapping f : (X,τ) → (Y,σ) is said to be an intuitionistic fuzzy almost λ - continuous (IFAλ - continuous in short) if $f^{-1}(B)$ is an IFλ -OS in X for every IFROS B in Y.

Theorem 3.1. Every intuitionistic fuzzy regular closed set is intuitionistic fuzzy λ- closed but not conversely.

Proof : let Abe a regular closed set in X then we have A = cl (int(A)). Let A⊒ U where U is any open set in (X, τ).Then . int A⊒ int U = U, cl(int (A)) ⊒ cl U, since A is regular closed set we have A⊒ cl(U) Hence A is an IFλ-CS in X.

Example 3.2. Let X = {a,b},and τ = { $0_\sim$, U, $1_\sim$ } where

U={ < a, 0.6,0.4 >, < b, 0.8, 0.2 > },

A= { <a, 0.2,0.7 >, <v, 0.2,0.8 > }. Is an IFλ-CS but not an IF regular closed set.

Definition 3.3. A mapping f : (X,τ) → (Y,σ) is said to be an intuitionistic fuzzy quasi λ- continuous mapping if $f^{-1}(B)$ is an IFCS in (X,τ) for every IF λ- CS B of (Y,σ).

Theorem 3.4. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFCS in Y . Since every IFCS is an IF λ-CS[8], A is an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy continuous mapping.

Example 3.5. Let X = {a,b}, Y = {u,v} and U={ < a, 0.3,0.4 >, < b, 0.4,0.5 > },

V = { <u, 0.3,0.4 >, <v, 0.4,0.5 > }. Then τ = { $0_\sim$, U, $1_\sim$ }

and σ = { $0_\sim$, V, $1_\sim$ } are IFTs on X and Y respectively.

Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = { <u, 0.6,0.2 >, <v, 0.7,0.1 > }. is an IF λ- CS in Y, but $f^{-1}(B)$= { < a, 0.6,0.2 >, < b, 0.7,0.1 > } is not an IFCS in X.

Theorem 3.6. Let f : (X,τ) → (Y,σ) be a mapping from an IFTS (X,τ) into an IFTS (Y,σ) and (Y,σ) an IF λ-$T_{1/2}$ space. Then the following statements are equivalent.

(i) f is an intuitionistic fuzzy quasi λ- continuous mapping.
(ii) f is an intuitionistic fuzzy continuous mapping.

Proof. (i)⇒(ii) Is obviously true from the Theorem 3.1.
(ii)⇒(i) Let A be an IF λ-CS in Y . Since (Y,σ) is an IF λ-$T_{1/2}$ space, A is an IFCS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy quasi λ-continuous mappings.

Theorem 3.7. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy α continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFCS in Y . Since every IFCS is an IF λ-CS [8], A is an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαCS [5], $f^{-1}(A)$ is an IFαCS in X. Hence f is an intuitionistic fuzzy α- continuous mapping.

Example 3.8 Let X = {a,b}, Y = {u,v} and U= { < a, 0.2,0.4 >, < b, 0.4,0.6 > },

V={ <u, 0.2,0.4 >, <v, 0.4,0.6 > }. Then τ = { $0_\sim$, U, $1_\sim$ }

and σ = { $0_\sim$, V, $1_\sim$ } are IFTs on X and Y respectively.

Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy α continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = { <u, 0.5,0.2 >, <v, 0.6,0.1 > }is

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an IF λ- CS in Y but f−1(B)= { < a, 0.5,0.2 >, < b, 0.6,0.1 > } is not an IFCS in X

Theorem 3.9. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy pre continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFCS in Y . Since every IFCS is an IF λ-CS[8], A is an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFPCS[15], $f^{-1}(A)$ is an IFPCS in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

Example 3.10. Let X = {a, b}, Y = {u, v} and U = { < a, 0.4,0.5 >, < b, 0.3,0.6 > }.

V={ < a, 0.2,0.3 >, < b, 0.3,0.6 > }. Then τ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and σ = $\{\underset{\sim}{0}, V, \underset{\sim}{1}\}$ are IFTs on X and Y respectively. Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy pre continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping , since the IFS B = { <u, 0.6,0.2 >, <v, 0.4,0.1 > } is an IF λ- CS in Y but $f^{-1}(B)$= { < a, 0.6,0.2 >, < b, 0.4,0.1 > } is not an IFCS in X.

Theorem 3.11. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy generalized continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ-continuous mapping. Let A be an IFCS in Y Since every IFCS is an IF λ- CS, A is an IF λ- CS in Y . By hypothesis, f−1(A) is an IFCS in X. Since every IFCS is an IFGCS[11], $f^{-1}(A)$ is an IFGCS in X. Hence f is an intuitionistic fuzzy generalized continuous mapping.

Example 3.12. Let X = {a,b}, Y = {u,v} and U=={ < a, 0.3,0.4 >, < b, 0.4,0.5 > }.

V={ < a, 0.2,0.3 >, < b, 0.2,0.4 > }. Then τ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and σ = $\{\underset{\sim}{0}, V, \underset{\sim}{1}\}$ are IFTs on X and Y respectively. Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy generalized continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = { <u, 0.4,0.2 >, <v, 0.5,0 > } is an IF λ- CS in Y but $f^{-1}(B)$= { < a, 0.4,0.2 >, < b, 0.5,0 > } is not an IFCS in X.

Theorem 3.13. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy α generalized continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFCS in Y . Since every IFCS is an IF λ- CS, A is an IF λ- CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαGCS[11], f−1(A) is an IFαGCS in X. Hence f is an intuitionistic fuzzy α generalized continuous mapping.

Example 3.14. Let X = {a,b}, Y = {u,v} and U={ < a, 0.2,0.4 >, < b, 0.3,0.5 > }.

V= { <u, 0.2,0.4 >, <v, 0.3,0.5 > }. Then τ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and σ =$\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ are IFTs on X and Y respectively. Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy α generalized continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = {<u, 0.3,0.2 >, <v, 0.4, 0> } is an IF λ- CS in Y but $f^{-1}(B)$= {< a, 0.3,0.2 >, < b, 0.4, 0> } is not an IFCS in X.

Theorem 3.15. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy almost λ- continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFRCS in Y . Since every IFRCS is an IF λ- CS, A is an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF λ-CS, $f^{-1}(A)$ is an IF λ- CS in X. Hence f is an intuitionistic fuzzy almost λ- continuous mapping.

Example 3.16. Let X = {a,b}, Y = {u,v} and U ={ < a, 0.3,0.4 >, < b, 0.4,0.5 > },

V= { <u, 0.4,0.5 >, <u, 0.5,0.5 > }. Then τ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and σ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ are IFTs on X and Y respectively. Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy almost λ- continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping , since the IFS B ={ <u, 0.6,0.3 >, <u, 0.7,0.2 > }. is an IF λ- CS in Y but $f^{-1}(B)$= {<a, 0.6,0.3 >, <b, 0.7,0.2 > }. is not an IFCS in X.

Theorem 3.17. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy almost continuous mapping but not conversely.

Proof. Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFRCS in Y . Since every IFRCS is an IF λ- CS, A is an IF λ- CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy almost continuous mapping.

Example 3.18. Let X = {a,b}, Y = {u,v} and U= { < a, 0.3,0.4 >, < b, 0.4,0.5 > }.

V= { < a, 0.3,0.4 >, < b, 0.4,0.5 > }. Then τ = $\{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and σ =$\{\underset{\sim}{0}, V, \underset{\sim}{1}\}$ are IFTs on X and Y respectively. Consider a mapping f : (X,τ) → (Y,σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy almost continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = { <u, 0.5,0.4 >, <v, 0.6,0.2 > } is an IF λ- CS in Y but $f^{-1}(B)$= { <a, 0.5,0.4 >, <b, 0.6,0.2 > } is not an IFCS in X.

Theorem 3.19. Every intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy λ- continuous mapping but not conversely.

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Proof. Let $f : (X,\tau) \to (Y,\sigma)$ be an intuitionistic fuzzy quasi λ- continuous mapping. Let A be an IFCS in Y . Since every IFCS is an IF λ- CS[8], A is an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF λ-CS, $f^{-1(}A)$ is an IF λ-CS in X. Hence f is an intuitionistic fuzzy λ- continuous mapping.

Example 3.20. Let X = {a,b}, Y = {u,v} and U = { <u, 0.2,0.4 >, <v, 0.1,0.5 > } ,
V = { < a, 0.2,0.4 >, < b, 0.1,0.5 > }. Then $\tau = \{0_\sim, U, 1_\sim\}$ and $\sigma = \{0_\sim, V, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X,\tau) \to (Y,\sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy λ- continuous mapping but not an intuitionistic fuzzy quasi λ- continuous mapping, since the IFS B = { <u, 0.6,0.2 >, <v, 0.7,0.2 > } is an IF λ-CS in Y but
$f^{-1}(B)$= { < a, 0.6,0.2 >, < b, 0.7,0.2 > } is not an IFCS in X.

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram.

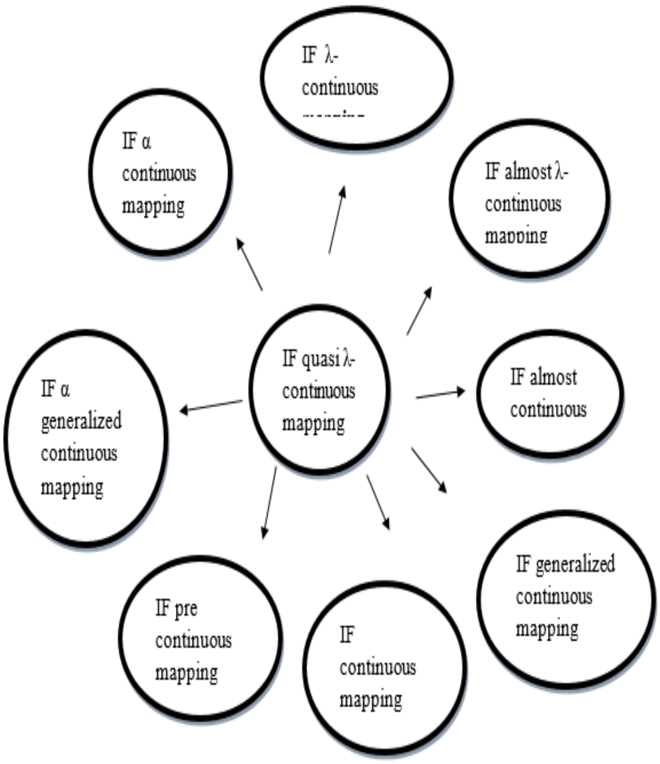

Fig 1. In this diagram by "A ⟶ B" we mean A implies B but not conversely. None of them is reversible.

Theorem 3.21. Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then the following statements are equivalent:

(i) f is an intuitionistic fuzzy quasi λ- continuous mapping.

(ii) $f^{-1}(B)$ is an IFOS in X for every IF λ-OS B in Y .

Proof. (i)⇒(ii) Let B be an IF λ- OS in Y . Then B^c is an IF λ- CS in Y . By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IFCS in X. Hence $f^{-1}(B)$ is an IFOS in X.

(ii)⇒(i) Let B be an IF λ- CS in Y . Then B^c is an IF λ-OS in Y . By (ii), $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IFOS in X. Hence $f^{-1}(B)$ is an IFCS in X. Therefore f is an intuitionistic fuzzy quasi λ- continuous mapping.

Theorem 3.22. Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IFTS (X,τ) into an IFTS (Y,σ) and let $f^{-1}(A)$ be an IFRCS in X for every IF λ-CS in Y . Then f is an intuitionistic fuzzy quasi λ- continuous mapping.

Proof. Let A be an IF λ-CS in Y . By hypothesis, $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFCS[ ], $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy quasi λ- continuous mapping.

Theorem 3.23. Let $f : (X,\tau) \to (Y,\sigma)$ be an intuitionistic fuzzy quasi λ- continuous mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then $f(cl(A)) \subseteq$ λ- cl(f(A)) for every IFS A in X.

Proof. Let A be an IFS in X. Then λ- cl(f(A)) is an IF λ-CS in Y . Since f is an intuitionistic fuzzy quasi weakly generalized continuous mapping, f−1(λ-cl(f(A))) is an IFCS in X.

Clearly $A \subseteq f^{-1}$(λ- cl(f(A))). Therefore $cl(A) \subseteq cl(f^{-1}$(λ-cl(f(A)))) = f^{-1}(λ- cl(f(A))). Hence $f(cl(A)) \subseteq$ λ- cl(f(A)) for every IFS A in X.

Theorem 3.24. Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then the following statements are equivalent:

(i) f is an intuitionistic fuzzy quasi λ- continuous mapping.

(ii) $f^{-1}(B)$ is an IFOS in X for every IF λ-OS B in Y .

(iii) f^{-1}(λ-int(B)) $\subseteq int(f^{-1}(B))$ for every IFS B in Y .

(iv) $cl(f^{-1}(B)) \subseteq f^{-1}$(λ-cl(B)) for every IFS B in Y .

Proof. (i)⇒(ii) Is obviously true from the Theorem 3.10.
(ii)⇒(iii) Let B be an IFS in Y . Then λ-int(B) is an IF λ-OS in Y . By (ii), f^{-1}(λ-int(B)) is an IFOS in X. Therefore f^{-1}(λ-int(B)) = $int(f^{-1}$(λ-int(B))). Clearly λ-int(B) $\subseteq$ B. This implies f−1(λ-int(B)) $\subseteq f^{-1}(B)$. Therefore f^{-1}(λ-int(B)) = $int(f^{-1}$(λ-int(B))) $\subseteq int(f^{-1}(B))$.
Hence f^{-1}(λ-int(B)) $\subseteq$ int(f−1(B)) for every IFS B in Y .

(iii)⇒(iv) It can be proved by taking the complement.
(iv)⇒(i) Let B be an IF λ- CS in Y . Then λ-cl(B) = B. Therefore $f^{-1}(B) = f^{-1}$(λ-cl(B)) $\supseteq cl(f^{-1}(B))$. Hence $cl(f^{-1}(B)) = f^{-1}(B)$. This implies f−1(B) is an IFCS in Y . Hence f is an intuitionistic fuzzy quasi λ- continuous mapping.

Theorem 3.25 : The composition of two intuitionistic fuzzy quasi λ- continuous mapping is an intuitionistic fuzzy quasi λ- continuous mapping.

Proof. Let A be an IF λ-CS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y . Since every IFCS is an IF λ-CS, $g^{-1}(A)$ is an IF λ-CS in Y . Then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFCS in X, by hypothesis. Hence gof is an intuitionistic fuzzy quasi λ- continuous mapping.

Theorem 3.26. Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\delta)$ be any two mappings. Then the following statements hold

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(i) Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy continuous mapping and g : (Y,σ) → (Z,δ) an intuitionistic fuzzy quasi λ- continuous mapping. Then their composition gof : (X,τ) → (Z,δ) is an intuitionistic fuzzy quasi λ- continuous mapping.
(ii) Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ-continuous mapping and
g : (Y,σ) → (Z,δ) an intuitionistic fuzzy continuous mapping [respectively intuitionistic fuzzy α continuous mapping, intuitionistic fuzzy pre continuous mapping, intuitionistic fuzzy α generalized continuous mapping and intuitionistic fuzzy generalized continuous mapping]. Then their composition gof : (X,τ) → (Z,δ) is an intuitionistic fuzzy continuous mapping.
(iii Let f : (X,τ) → (Y,σ) be an intuitionistic fuzzy quasi λ-continuous mapping and
g : (Y,σ) → (Z,δ) an intuitionistic fuzzy λ- continuous mapping. Then their composition
gof : (X,τ) → (Z,δ) is an intuitionistic fuzzy continuous mapping.

IV. PROOF

(i) Let A be an IF λ- CS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y . Since f is an intuitionistic fuzzy continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFCS in X. Hence gof is an intuitionistic fuzzy quasi λ- continuous mapping.
(ii) A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] in Y . Since every IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] is an IF λ- CS, $g^{-1}(A)$ is an IF λ- CS in Y . Then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFCS in X, by hypothesis. Hence gof is an intuitionistic fuzzy continuous mapping.
(iii)Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IF λ-CS *Z*. Since *f* is an intuitionistic fuzzy quasi λ- continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an *IFCS* in *X*.
gof is an intuitionistic fuzzy continuous mapping.

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