International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-06, Sep 2019

Application Of Complex Intuitionistic Fuzzy Soft Graph In Decision Support System For Mobile Commerce

[1]Dr. N. Sarala, Associate Professor, Department of Mathematics, A D M College for Women(Aoutonomous), Nagapattinam, Tamilnadu, India, saralaadmc68@gmail.com

[2]R. Deepa, Associate Professor, Department of Mathematics, E.G.S. Pillay Engineering College Nagapattinam, Tamilnadu, India. srideepamuruga13@gmail.com

Abstract : **Solving, optimizing and analyzing imprecise problems involving vague and uncertain environment is one of the most interesting field in multidisciplinary research including computational intelligence, machine learning, applied mathematics, and decision analysis. It is worth mentioning that ambiguity and imprecise data arises from very different fields and cannot be modeled using conventional numerical and computational tools. Fuzzy soft graph theory enables a parameterized perspective for coping with uncertainty and vagueness. In this work, we describe a mathematical model for processing fuzzy soft information by integrating complex intuitionistic fuzzy soft sets with graph theory. This study presents some key ideas about Complex Intuitionistic Fuzzy Soft graphs (CIFS-graphs) and also explores some of their related properties. In order to demonstrate the effectiveness of CIFS-graphs, we apply this model to describe and solve a multi-criteria decision-making problem in a mobile communication network using CIFS-graphs.**

***Keywords* — fuzzy soft sets; complex intuitionistic fuzzy soft graphs; multi-criteria decision-making problems; uncertainty**

I. INTRODUCTION

Konigsberg bridge problem is the starting point of graph theory. The solution of this notable puzzle led directly to the idea of the Eulerian graph. Euler investigated the Konignberg bridge problem and developed an aptsolutionwhich is known as Eulerian graph. At present, graph theory is becoming mainstream of science and technology mainly because of its applications in different domains including networking and coding theory, data mining and image processing, optimization algorithms and computations, clustering and scheduling.

Ambiguity, fuzziness, and uncertainties are ubiquitous in real-time applications and these characteristics cannot be handled successfully by means of conventional numerical models. Recently, a number of innovative concepts including of set theory [1], fuzzy set theory [2], and intuitionistic fuzzy set theory [3] are proposed to solve these issues. In order to overcome the limitations that are intrinsic in each of these concepts, researchers have selected to integrate these concepts to design new fuzzy-based hybrid frameworks (e.g., fuzzy soft graphs [4],intuitionistic fuzzy soft graphs [4] vague soft sets [5], interval-valued fuzzy soft graphs [6], and interval-valued intuitionistic fuzzy soft graphs [7]).

The introduction of a fuzzy set by Zadeh in[2]really changed the face of engineering and science. Fuzzy sets laid the foundations for an innovative approach of philosophical thinking, "Fuzzy logic" which plays an important role in the domain of artificial intelligence. The best imperative property of a fuzzy logic is that it comprises of a class of objects that meet a definite constraint or many constraints .In 1999, Molodtsov presented the concept of soft set for dealing problems with uncertainties and vagueness in modelling real time applications in different fields such asocial and medical science, engineering, economics, and environmental science [1]. Molodtsov employed this concept in many applications like probability and measurement theory, smoothness of function, operation research, game theory, etc.

Off late, a number of research works contributed into fuzzification of soft set theory. Consequently, it captured the attention from many researchers globally. In 1975, Rosenfeld introduced the theory of fuzzy graph theory [8]. Maji et al developed fuzzy soft set-based model by integrating the notions of soft sets and fuzzy sets. Thenceforth, several stimulating applications of fuzzy soft

 DOI : 10.35291/2454-9150.2019.0432

set have been proposed by some researchers [4].Ali et al. studied many potential operations on soft set [9]. Roy and Maji discussed some potential applications of fuzzy soft sets in decision making problems [10].

Atanassov suggested an enhanced version of fuzzy set by adding a new element, called "intuitionistic fuzzy sets" (IF-sets) [3]. The IF-set can be considered as an alternate method for modelling real time system where existing data is not enough to describe the ambiguity by the traditional fuzzy set. In fuzzy group, simply the acceptance score is taken in to account, but an IF-set is pigeonholed by a membership as well as non-membership functions. The only constraint is that the sum of both membership and non-membership degree is less than or equal to one. Applications of these sets have been widely investigated in other viewpoint including pattern recognition, image processing, decision making problems, etc.

Though all of the aforesaid concepts are able to solve the uncertainty problem that arises from real time applications, they are not capable of handling the seasonality or periodicity. This led to the introduction of the complex fuzzy set frameworks[11] and, consequently the development and extension of this concept. The concept of complex sets stems from the theory of complex numbers, which is a key idea for handling problems, particularly in the domain of engineering.

The complex set has the capacity to solve large number of problems that cannot be handled by conventional numerical tools like number theory, probability theory, and original fuzzy set theory. For example, solving the improper integrals that are used to define resistance of electrical circuits and also represent the phase in two-dimensional problems. This stimulates to develop the idea of complex fuzzy sets [11], which is an upgraded and extended form of original fuzzy sets.

In 2014, Kumar & Bajaj [12] developed the idea of Complex Intuitionistic Fuzzy Soft Sets (CIFS-Set) that integrates the properties and potential benefits of complex sets, soft sets, and intuitionistic fuzzy sets in a single set. The CIFS-set is parametric in nature and pigeonholed by an amplitude term and phase term. The uniqueness of CIFS-Set is demonstrated in the additional dimension of membership, which is the phase of the grade of membership. This property offers an extra benefit to define information or data occurring recurrently over a period of time, which is often the case with problems that are two-dimensional in nature.

Even though a large number of research studies pertaining to the concepts of fuzzy set other complex fuzzy-based approaches are still in their early period. Recently, they have been progressively gaining momentum. Hitherto, a majority of the studies in this domain have revolved around the investigation of the important theoretical properties of complex fuzzy sets, complex fuzzy logic, complex fuzzy optimization and decision-making, and the application of these concepts in handling sporadic problems.

In general, complex fuzzy groups can be pigeonholed by complex-valued amplitude as well as a phase term. The amplitude term holds the conventional idea of "fuzziness". The phase term of complex fuzzy set is an important feature and can be used to model the seasonality and/or periodicity of sporadic occurrences. Nevertheless, this is not the only interpretation for the phase term. Instead, the phase term can be used to denote different features of the data, depending on the context of the scope of the problem or area that is being examined. In most of research works, the phase term has been used to represent the time factor and seasonality of the problems and has been applied to multi-criteria decision-making problems in different domains such as economics, supplier selection, engineering, artificial intelligence, pattern recognition, etc. The phase term can also be used to precisely denote the cycles in fuzzy algebraic structures. In the study of complex fuzzy algebraic theory, the fuzzy algebraic structures are defined in a complex fuzzy setting; hence, the structures consist of amplitude as well as phase terms.

The desire to utilize this unique ability of the phase term of the complex fuzzy set framework and former complex fuzzy set-based frameworks in the study of fuzzy algebra served as the main inspiration to develop the concept of CIFS-graphs in this study. By the way, the idea of CIFS-graphs and other supporting algebraic structures are introduced and developed. The lack of proper research pertaining to the algebraic theory of complex fuzzy set-based frameworks in the literature is another reason for the study done in this paper.

The main contributions of this study are as follows:

1. To present the idea ofCIFS-graphs and describe the basic operations performed on the CIFS-graphs.
2. To analyze some important properties of CIFS-graphs.
3. To demonstrate the effectiveness of CIFS-graphs by applying on the real time multi-criteria decision making environment.

II. PRELIMINARIES AND BASIC DEFINITIONS

Definition 1: A Fuzzy Set of a base set $\mathcal{V} = \{\vartheta_1, \vartheta_2, \vartheta_3 \dots \dots \vartheta_n\}$ (non-empty set) is defined by its membership function$\mathfrak{U}$; where $\mathfrak{U}: \mathcal{V} \to [0,1]$ assigning to each $\vartheta_i \in \mathcal{V}$, the grade or degree to which $\mathcal{V} \in \mathfrak{U}$.

Definition 2: A complex fuzzy soft set$\mathcal{X}$, defined on a universe of discourse *A* is an object of the form

$$\mathcal{X} = \{(a, \mathfrak{U}_{\mathcal{X}}(a) e^{i\omega_{\mathcal{X}}(a)}) \ : a \in A\}$$

 DOI : 10.35291/2454-9150.2019.0432

ISSN : 2454-9150 Vol-05, Issue-06, Sep 2019

where $i = \sqrt{-1}$, $\mathfrak{U}_{\mathcal{X}}(a) \in [0,1]$ and $0 \leq \omega_{\mathcal{X}}(a) \leq 2\pi$.

Definition 2:Let $\mathcal{X}$ be a CIFS-set defined on a universe of discourse A is an object of the form

$$\mathcal{X} = \{(a, \mathfrak{U}_{\mathcal{X}}(a)e^{i\rho_{\mathcal{X}}(a)}, \mathfrak{S}_{\mathcal{X}}(a)e^{i\sigma_{\mathcal{X}}(a)}) \ : a \in A\}$$

where $i = \sqrt{-1}$, $\mathfrak{U}_{\mathcal{X}}(a), \mathfrak{S}_{\mathcal{X}}(a) \in [0,1]$ and $\rho_{\mathcal{X}}(a), \sigma_{\mathcal{X}}(a) \in [0,2\pi]$ and $\mathfrak{U}_{\mathcal{X}}(a) + \mathfrak{S}_{\mathcal{X}}(a) \leq 1$

Definition 3: Let $\mathcal{X}$ and $\mathcal{Y}$ be two CIFS-sets in A, where

$$\mathcal{X} = \{(a, \mathfrak{U}_{\mathcal{X}}(a)e^{i\rho_{\mathcal{X}}(a)}, \mathfrak{S}_{\mathcal{X}}(a)e^{i\sigma_{\mathcal{X}}(a)}) \ : a \in A\}$$

$$\mathcal{Y} = \{(a, \mathfrak{U}_{\mathcal{Y}}(a)e^{i\rho_{\mathcal{Y}}(a)}, \mathfrak{S}_{\mathcal{Y}}(a)e^{i\sigma_{\mathcal{Y}}(a)}) \ : a \in A\}$$

Then, A∪B is given as

$$\mathcal{X} \cup \mathcal{Y} = \{(a, \mathfrak{U}_{\mathcal{X}\cup\mathcal{Y}}(a)e^{i\rho_{\mathcal{X}\cup\mathcal{Y}}(a)}, \mathfrak{S}_{\mathcal{X}\cup\mathcal{Y}}(a)e^{i\sigma_{\mathcal{X}\cup\mathcal{Y}}(a)}) \ : a \in A\}$$

where

$$\mathfrak{U}_{\mathcal{X}\cup\mathcal{Y}}(a)e^{i\rho_{\mathcal{X}\cup\mathcal{Y}}(a)} = [\mathfrak{U}_{\mathcal{X}}(a) \vee \mathfrak{U}_{\mathcal{Y}}(a)]e^{i\{\rho_{\mathcal{X}}(a) \vee \rho_{\mathcal{Y}}(a)\}},$$

$$\mathfrak{S}_{\mathcal{X}\cup\mathcal{Y}}(a)e^{i\sigma_{\mathcal{X}\cup\mathcal{Y}}(a)} = [\mathfrak{S}_{\mathcal{X}}(a) \wedge \mathfrak{S}_{\mathcal{Y}}(a)]e^{i\{\sigma_{\mathcal{X}}(a) \wedge \sigma_{\mathcal{Y}}(a)\}},$$

Definition 4: A graph is an ordered pair $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices of $\mathcal{G}^*$ and $\mathcal{E}$ is the set of edges of $\mathcal{G}^*$.

COMPLEX INTUITIONISTIC FUZZY SOFT GRAPHS

Definition 5: A CIFS with a basic set $\mathcal{V}$ is defined by a tuple $\mathcal{G} = (\mathcal{X}, \mathcal{Y})$, where $\mathcal{X}$ is a CIFS-set on $\mathcal{V}$ and $\mathcal{Y}$ is a CIFS-set on $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ such that

$$\mathfrak{U}_{\mathcal{Y}}(ab)\, e^{i\rho_{\mathcal{Y}}(ab)}\tau \leq \min\{\mathfrak{U}_{\mathcal{X}}(a), \mathfrak{U}_{\mathcal{X}}(b)\}e^{i\min\{\rho_{\mathcal{X}}(a), \rho_{\mathcal{X}}(b)\}}$$

$$\mathfrak{S}_{\mathcal{Y}}(ab)\, e^{i\sigma_{\mathcal{Y}}(ab)}\tau \leq \max\{\mathfrak{S}_{\mathcal{X}}(a), \mathfrak{S}_{\mathcal{X}}(b)\}e^{i\max\{\sigma_{\mathcal{X}}(a), \sigma_{\mathcal{X}}(b)\}}$$

for all $a, b \in \mathcal{V}$.

Definition 6: Let $\mathcal{G} = (\mathcal{X}, \mathcal{Y})$ be a CIFS-graph. The order of a CIFS-graph is given by

$$\mathcal{O}(\mathcal{G}) = \left(\sum_{a\in\mathcal{V}} \mathfrak{U}_{\mathcal{X}}(a)e^{\sum_{a\in\mathcal{V}}\rho_{\mathcal{X}}(a)}, \sum_{a\in\mathcal{V}} \mathfrak{S}_{\mathcal{X}}(a)\, e^{\sum_{a\in\mathcal{V}}\sigma_{\mathcal{X}}(a)}\right)$$

The degree of a vertex a in $\mathcal{G}$ is described by

$$\mathcal{D}(a) = \left(\sum_{ab\in\mathcal{E}} \mathfrak{U}_{y}(ab)e^{\sum_{a\in\mathcal{V}}\rho_{\mathcal{Y}}(ab)}, \sum_{ab\in\mathcal{E}} \mathfrak{S}_{\mathcal{X}}(ab)\, e^{\sum_{ab\in\mathcal{V}}\sigma_{\mathcal{Y}}(ab)}\right)$$

Definition 6: The Cartesian product $\mathcal{G}_1 \times \mathcal{G}_2$ of two CIFS-graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as $\mathcal{G}_1 \times \mathcal{G}_2 = (\mathcal{X}_1 \times \mathcal{X}_2, \mathcal{Y}_1 \times \mathcal{Y}_2$, such that:

$$\mathfrak{U}_{\mathcal{X}_1\times\mathcal{X}_2}(a_1,a_2)e^{i\rho_{\mathcal{X}_1\times\mathcal{X}_2}(a_1,a_2)} = \min\{\mathfrak{U}_{\mathcal{X}_1}(a_1), \mathfrak{U}_{\mathcal{X}_2}(a_2)\}e^{i\min\{\rho_{\mathcal{X}_1}(a_1), \rho_{\mathcal{X}_2}(a_2)\}}$$

$$\mathfrak{S}_{\mathcal{X}_1\times\mathcal{X}_2}(a_1,a_2)e^{i\sigma_{\mathcal{X}_1\times\mathcal{X}_2}(a_1,a_2)} = \max\{\mathfrak{S}_{\mathcal{X}_1}(a_1), \mathfrak{S}_{\mathcal{X}_2}(a_2)\}e^{i\max\{\sigma_{\mathcal{X}_1}(a_1), \sigma_{\mathcal{X}_2}(a_2)\}}$$

for all $a_1, a_2 \in \mathcal{V}$

$$\mathfrak{U}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a,a_2)(a,b_2))e^{i\rho_{\mathcal{Y}_1\times\mathcal{Y}_2}((a,a_2)(a,b_2))} = \min\{\mathfrak{U}_{\mathcal{X}_1}(a), \mathfrak{U}_{\mathcal{Y}_2}(a_2b_2)\}e^{i\min\{\rho_{\mathcal{X}_1}(a), \rho_{\mathcal{Y}_2}(a_2b_2)\}}$$

$$\mathfrak{S}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a,a_2)(a,b_2))e^{i\sigma_{\mathcal{Y}_1\times\mathcal{Y}_2}((a,a_2)(a,b_2))} = \max\{\mathfrak{S}_{\mathcal{X}_1}(a), \mathfrak{S}_{\mathcal{Y}_2}(a_2b_2)\}e^{i\max\{\sigma_{\mathcal{X}_1}(a), \sigma_{\mathcal{Y}_2}(a_2b_2)\}}$$

for all $a \in \mathcal{V}_1$ and $a_2b_2 \in \mathcal{E}_2$

$$\mathfrak{U}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,c)(b_1,c))e^{i\rho_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,c)(b_1,c))} = \min\{\mathfrak{U}_{\mathcal{Y}_1}(a_1b_1), \mathfrak{U}_{\mathcal{X}_2}(c)\}e^{i\min\{\rho_{\mathcal{Y}_1}(a_1b_1), \rho_{\mathcal{X}_2}(c)\}}$$

$$\mathfrak{S}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,c)(b_1,c))e^{i\sigma_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,c)(b_1,c))} = \max\{\mathfrak{S}_{\mathcal{Y}_1}(a_1b_1), \mathfrak{S}_{\mathcal{X}_2}(c)\}e^{i\max\{\rho_{\mathcal{Y}_1}(a_1b_1), \rho_{\mathcal{X}_2}(c)\}}$$

for all $a \in \mathcal{V}_2$ and $a_1b_1 \in \mathcal{E}_1$

Definition 7: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two CIFS-graphs. The degree of a vertex in $\mathcal{G}_1 \times \mathcal{G}_2$ can be defined as follows:

for any $(a_1,a_2) \in \mathcal{V}_1 \times \mathcal{V}_2$

$$\mathcal{D}_{\mathcal{G}_1\times\mathcal{G}_2}(a_1,a_2) = \begin{pmatrix} \sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}} \mathfrak{U}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))e^{\sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}}\rho_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))}, \\ \sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}} \mathfrak{S}_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))e^{\sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}}\sigma_{\mathcal{Y}_1\times\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))} \end{pmatrix}$$

Definition 8: The composition of two CIFS-graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as a pair $\mathcal{G}_1 \circ \mathcal{G}_2 = (\mathcal{X}_1 \circ \mathcal{X}_2, \mathcal{Y}_1 \circ \mathcal{Y}_2)$, such that:

$$\mathfrak{U}_{\mathcal{X}_1\circ\mathcal{X}_2}(a_1,a_2)e^{i\rho_{\mathcal{X}_1\circ\mathcal{X}_2}(a_1,a_2)} = \min\{\mathfrak{U}_{\mathcal{X}_1}(a_1), \mathfrak{U}_{\mathcal{X}_2}(a_2)\}e^{i\min\{\rho_{\mathcal{X}_1}(a_1), \rho_{\mathcal{X}_2}(a_2)\}}$$

$$\mathfrak{S}_{\mathcal{X}_1\circ\mathcal{X}_2}(a_1,a_2)e^{i\sigma_{\mathcal{X}_1\circ\mathcal{X}_2}(a_1,a_2)} = \max\{\mathfrak{S}_{\mathcal{X}_1}(a_1), \mathfrak{S}_{\mathcal{X}_2}(a_2)\}e^{i\max\{\sigma_{\mathcal{X}_1}(a_1), \sigma_{\mathcal{X}_2}(a_2)\}}$$

for all $a_1, a_2 \in \mathcal{V}$

$$\mathfrak{U}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a,a_2)(a,b_2))e^{i\rho_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a,a_2)(a,b_2))} = \min\{\mathfrak{U}_{\mathcal{X}_1}(a), \mathfrak{U}_{\mathcal{Y}_2}(a_2b_2)\}e^{i\min\{\rho_{\mathcal{X}_1}(a), \rho_{\mathcal{Y}_2}(a_2b_2)\}}$$

$$\mathfrak{S}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a,a_2)(a,b_2))e^{i\sigma_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a,a_2)(a,b_2))} = \max\{\mathfrak{S}_{\mathcal{X}_1}(a), \mathfrak{S}_{\mathcal{Y}_2}(a_2b_2)\}e^{i\max\{\sigma_{\mathcal{X}_1}(a), \sigma_{\mathcal{Y}_2}(a_2b_2)\}}$$

for all $a \in \mathcal{V}$ and $a_2b_2 \in \mathcal{E}_2$

$$\mathfrak{U}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,c)(b_1,c))e^{i\rho_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,c)(b_1,c))} = \min\{\mathfrak{U}_{\mathcal{Y}_1}(a_1b_1), \mathfrak{U}_{\mathcal{X}_2}(c)\}e^{i\min\{\rho_{\mathcal{Y}_1}(a_1b_1), \rho_{\mathcal{X}_2}(c)\}}$$

$$\mathfrak{S}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,c)(b_1,c))e^{i\rho_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,c)(b_1,c))} = \max\{\mathfrak{S}_{\mathcal{Y}_1}(a_1b_1), \mathfrak{S}_{\mathcal{X}_2}(c)\}e^{i\max\{\rho_{\mathcal{Y}_1}(a_1b_1), \rho_{\mathcal{X}_2}(c)\}}$$

for all $a_1 \in \mathcal{V}_1$ and $a_1b_1 \in \mathcal{E}_1$

$$\mathfrak{U}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))\, e^{i\rho_{\mathcal{Z}_1\circ\mathcal{Z}_2}((a_1,a_2)(b_1,b_2))} = \min\{\mathfrak{U}_{\mathcal{X}_2}(a_2), \mathfrak{U}_{\mathcal{X}_2}(b_2), \mathfrak{U}_{\mathcal{Y}_1}(a_1b_1)\}e^{i\min\{\rho_{\mathcal{X}_2}(a_2), \rho_{\mathcal{X}_2}(b_2), \rho_{\mathcal{Y}_1}(a_1b_1)\}}$$

$$\mathfrak{S}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))\, e^{i\rho_{\mathcal{Z}_1\circ\mathcal{Z}_2}((a_1,a_2)(b_1,b_2))} = \max\{\mathfrak{S}_{\mathcal{X}_2}(a_2), \mathfrak{S}_{\mathcal{X}_2}(b_2), \mathfrak{S}_{\mathcal{Y}_1}(a_1b_1)\}e^{i\max\{\rho_{\mathcal{X}_2}(a_2), \rho_{\mathcal{X}_2}(b_2), \rho_{\mathcal{Y}_1}(a_1b_1)\}}$$

for all $a_1, a_2 \in \mathcal{V}_2$, $a_2 \neq b_2$, and $a_1b_1 \epsilon \mathcal{E}_1$

Definition 9: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two CIFS-graphs. The degree of a vertex in $\mathcal{G}_1 \circ \mathcal{G}_2$ can be defined as follows:

$$\mathcal{D}_{\mathcal{G}_1\circ\mathcal{G}_2}(a_1,a_2) = \begin{pmatrix} \sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}} \mathfrak{U}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))e^{\sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}}\rho_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))}, \\ \sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}} \mathfrak{S}_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))e^{\sum_{(a_1,a_2)(b_1,b_2)\in\mathcal{E}}\sigma_{\mathcal{Y}_1\circ\mathcal{Y}_2}((a_1,a_2)(b_1,b_2))} \end{pmatrix}$$

for any $a_1, a_2 \in \mathcal{V}_1 \times \mathcal{V}_2$

Definition 10: The union $\mathcal{G}_1 \cup \mathcal{G}_2 = (\mathcal{X}_1 \cup \mathcal{X}_2, \mathcal{Y}_1 \cup \mathcal{Y}_2)$ of two CIFS-graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as follows:

$$\mathfrak{U}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \mathfrak{U}_{\mathcal{X}_1}(a)e^{i\rho_{\mathcal{X}_1}(a)}$$

$$\mathfrak{S}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\sigma_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \mathfrak{S}_{\mathcal{X}_1}(a)e^{i\sigma_{\mathcal{X}_1}(a)}$$

for $a \in \mathcal{V}_1$ and $a \notin \mathcal{V}_2$

$$\mathfrak{U}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \mathfrak{U}_{\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_2}(a)}$$

$$\mathfrak{S}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\sigma_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \mathfrak{S}_{\mathcal{X}_2}(a)e^{i\sigma_{\mathcal{X}_2}(a)}$$

for $a \in \mathcal{V}_2$ and $a \notin \mathcal{V}_1$

$$\mathfrak{U}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \max(\mathfrak{U}_{\mathcal{X}_1}(a), \mathfrak{U}_{\mathcal{X}_2}(a))e^{i\max(\rho_{\mathcal{X}_1}(a), \rho_{\mathcal{X}_2}(a))}$$

$$\mathfrak{S}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)} = \min(\mathfrak{S}_{\mathcal{X}_1}(a), \mathfrak{S}_{\mathcal{X}_2}(a))e^{i\min(\sigma_{\mathcal{X}_1}(a), \sigma_{\mathcal{X}_2}(a))}$$

for $a \in \mathcal{V}_1 \cap \mathcal{V}_2$

DOI : 10.35291/2454-9150.2019.0432

ISSN : 2454-9150 Vol-05, Issue-06, Sep 2019

$$\mathfrak{U}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \mathfrak{U}_{\mathcal{Y}_1}(ab)e^{i\rho_{\mathcal{Y}_1}(ab)}$$

$$\mathfrak{S}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \mathfrak{S}_{\mathcal{Y}_1}(ab)e^{i\sigma_{\mathcal{Y}_1}(ab)}$$

for $ab \in \mathcal{E}_1$ and $ab \notin \mathcal{E}_2$

$$\mathfrak{U}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \mathfrak{U}_{\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_2}(ab)}$$

$$\mathfrak{S}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \mathfrak{S}_{\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{Y}_2}(ab)}$$

for $ab \in \mathcal{E}_2$ and $ab \notin \mathcal{E}_1$

$$\mathfrak{U}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \max(\mathfrak{U}_{\mathcal{Y}_1}(ab), \mathfrak{U}_{\mathcal{Y}_2}(ab))e^{i\max(\rho_{\mathcal{Y}_1}(ab),\rho_{\mathcal{Y}_2}(ab))}$$

$$\mathfrak{S}_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1\cup\mathcal{Y}_2}(ab)} = \min(\mathfrak{S}_{\mathcal{Y}_1}(ab), \mathfrak{S}_{\mathcal{Y}_2}(ab))e^{i\min(\sigma_{\mathcal{Y}_1}(ab),\sigma_{\mathcal{Y}_2}(ab))}$$

for $ab \in \mathcal{E}_1 \cap \mathcal{E}_2$

Definition 11: The join $\mathcal{G}_1 + \mathcal{G}_2 = (\mathcal{X}_1 + \mathcal{X}_2, \mathcal{Y}_1 + \mathcal{Y}_2)$ of two CIFS-graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as follows

$$\mathfrak{U}_{\mathcal{X}_1+\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1+\mathcal{X}_2}(a)} = \mathfrak{U}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)}$$

$$\mathfrak{S}_{\mathcal{X}_1+\mathcal{X}_2}(a)e^{i\sigma_{\mathcal{X}_1+\mathcal{X}_2}(a)} = \mathfrak{S}_{\mathcal{X}_1\cup\mathcal{X}_2}(a)e^{i\sigma_{\mathcal{X}_1\cup\mathcal{X}_2}(a)}$$

If $a \in \mathcal{V}_1 \cup \mathcal{V}_2$

$$\mathfrak{U}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)} = \mathfrak{U}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{X}_1\cup\mathcal{X}_2}(a)}$$

$$\mathfrak{S}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)} = \mathfrak{S}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{X}_1\cup\mathcal{X}_2}(a)}$$

for $ab \in \mathcal{E}_1 \cap \mathcal{E}_2$

$$\mathfrak{U}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\rho_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)} = \min(\mathfrak{U}_{\mathcal{X}_1}(a), \mathfrak{U}_{\mathcal{X}_2}(b))e^{i\min(\rho_{\mathcal{X}_1}(a),\rho_{\mathcal{X}_2}(b))}$$

$$\mathfrak{S}_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)e^{i\sigma_{\mathcal{Y}_1+\mathcal{Y}_2}(ab)} = \max(\mathfrak{S}_{\mathcal{X}_1}(a), \mathfrak{S}_{\mathcal{X}_2}(b))e^{i\max(\sigma_{\mathcal{X}_1}(a),\sigma_{\mathcal{X}_2}(b))}$$

for $ab \in \mathcal{E}'$ here $\hat{\mathcal{E}}$ is the set of all edges linking the vertices $\mathcal{V}_1$ and $\mathcal{V}_2$.

Definition 12: The complement of an CIFS-graph $\mathcal{G} = (\mathcal{X}, \mathcal{Y})$ is denoted by $\bar{\mathcal{G}} = (\bar{\mathcal{X}}, \bar{\mathcal{Y}})$ and is defined by

$\bar{\mathcal{V}} = \mathcal{V}$

$$\mathfrak{U}_{\bar{\mathcal{X}}}(a)e^{i\rho_{\bar{\mathcal{X}}}(a)} = \mathfrak{U}_{\mathcal{X}}(a)e^{i\rho_{\mathcal{X}}(a)}$$

$$\mathfrak{S}_{\bar{\mathcal{X}}}(a)e^{i\sigma_{\bar{\mathcal{X}}}(a)} = \mathfrak{S}_{\mathcal{X}}(a)e^{i\sigma_{\mathcal{X}}(a)}$$

$$\mathfrak{U}_{\bar{\mathcal{Y}}}(ab)e^{i\rho_{\bar{\mathcal{Y}}}(ab)} = \begin{Bmatrix} 0, & \text{if } \mathfrak{U}_{\mathcal{Y}}(ab)e^{i\rho_{\mathcal{Y}}(ab)} \neq 0 \\ \min(\mathfrak{U}_{\mathcal{X}}(a), \mathfrak{U}_{\mathcal{X}}(b))e^{i\min(\rho_{\mathcal{X}}(a),\rho_{\mathcal{X}}(b))}, & \text{if } \mathfrak{U}_{\mathcal{Y}}(ab)e^{i\rho_{\mathcal{Y}}(ab)} = 0 \end{Bmatrix}$$

$$\mathfrak{S}_{\bar{\mathcal{Y}}}(ab)e^{i\sigma_{\bar{\mathcal{Y}}}(ab)} = \begin{Bmatrix} 0, & \text{if } \mathfrak{U}_{\mathcal{Y}}(ab)e^{i\rho_{\mathcal{Y}}(ab)} \neq 0 \\ \max(\mathfrak{S}_{\mathcal{X}}(a), \mathfrak{S}_{\mathcal{X}}(b))e^{i\max(\sigma_{\mathcal{X}}(a),\sigma_{\mathcal{X}}(b))}, & \text{if } \mathfrak{U}_{\mathcal{Y}}(ab)e^{i\rho_{\mathcal{Y}}(ab)} = 0 \end{Bmatrix}$$

III. APPLICATION OF CIFSG ON MULTI-CRITERIA DECISION MAKING PROBLEM

CIFS-graph have many applications including decision support system, database management system, machine learning and optimization techniques, mobile commerce, vehicular ad-hoc networks, resource management problems, etc.Decision support System (DSS) is an interactive information system that automatically supports the decision process to help experts for making vital judgments.DSS enables decision makers to exploit large dataset to make predictions and verdicts about utilization of valuable resources .In this section, we design a strategy to solve a multi-criteria decision making problem that can be implemented in real-time. Assume a wireless service provider that has a proposal to erect the minimum number of base stations in a city to serve maximum number of subscribers. For this purpose, the following factors can be considered:

- Suitability of place to construct a base station.
- Mode of conveyance
- Number of customers connected
- Connectivity with the main server
- Rural or urban area
- Ambient weather conditions around the base station
- Availability ofother competing service provider
- Availability of other resources
- Disbursements and outcomes

Consider a planning team selected 5 places where they are interested in fixing a base station, so that they can service maximum numbers of the subscribers. They observe the following two scenarios:

1. Installing a base station exactly at the most suitable place out of 5 selected places
2. Installing a base station between any 2out of 5selected places

The algorithm used to find out the optimum place to install the base station is given below

Algorithm 1:	
1:	Input : number of places, set of choice of parameters.
2:	Output: Optimum place to install base station
3:	For each place or edge
4:	Find the amplitude and phase values of membership and non-membership score of selected places (refer table 1 and table 2)
5:	Calculate the absolute value and score function of each point
6:	Select the place or edge with maximum score function
7:	If more than one place or edge is having equal and maximum score then find the accuracy
8:	A place or edge with maximum score function and maximum accuracy is selected as the optimum point for installation

Scenario 1: Let $V = \{P_1, P_2, P_3, P_3, P_4, P_5\}$ be the set of places where the planning team is interested in installing a base station as a vertex set. Assume that 70% of the members of the planning team suggest that P_1 should have a base station and 10% of the team members suggest that there is no need to install a base station at P_1 after cautiously considering the various factors mentioned earlier in this section. Hence, in this manner, the amplitude term is calculated for membership as well as non-membership functions. Then ,thev ague of the phase term that denotes

 DOI : 10.35291/2454-9150.2019.0432

the period needs to be found. Let50% of experts believe that in a particular time the base station atP_1 can attract the maximum number of customers(profit)and 30% of the team members have the opposite opinion. We model this information as

$P_1 = 0.7e^{i0.5\pi}, 0.1e^{i0.3\pi}$

Likewise, they visit other four places. After careful observation, they obtain amplitude and phase terms as shown in Table 1.

Table 1. Amplitude and phase values of membership and non-membership function for each vertexes

Place	Amplitude		Phase	
	Membership	Non-membership	Membership	Non-membership
P_1	0.7	0.1	0.5	0.3
P_2	0.8	0.2	0.7	0.2
P_3	0.4	0.6	0.4	0.6
P_4	0.6	0.2	0.8	0.2
P_5	0.4	0.6	0.4	0.5

We model the above information as follows

$$x = \begin{cases} \langle P_1 : 0.7\, e^{i0.5\pi}, 0.1\, e^{i0.3\pi} \rangle \\ \langle P_2 : 0.8\, e^{i0.7\pi}, 0.2\, e^{i0.2\pi} \rangle \\ \langle P_3 : 0.4\, e^{i0.4\pi}, 0.6\, e^{i0.6\pi} \rangle \\ \langle P_4 : 0.6 \quad 2\, e^{i0.2\pi} \rangle \\ \langle P_5 : 0.4\, e^{i0.4\pi}, 0.6\, e^{i0.5\pi} \rangle \end{cases}$$

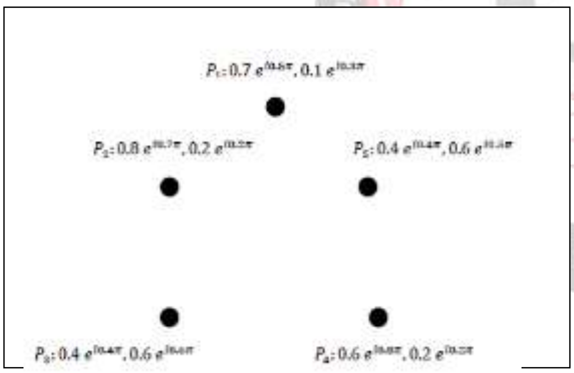

Figure 1: Places where the planning team is interested in installing a base station.

The complex membership of the vertices represents the constructive properties and complex non-membership of the vertices represents the adverse properties of a certain parameter for a certain place. Now, finding absolute values, we have

$|P_1| = (0.7, 0.1)$

$|P_2| = (0.8, 0.2)$

$|P_3| = (0.4, 0.6)$

$|P_4| = (0.6, 0.2)$

$|P_5| = (0.4, 0.6)$

In order to find the ideal choice, we find the score function (*sf*) of the absolute values of $P_1, P_2, P_3, P_3, P_4, P_5$. Thus, we have

$$sf(P_1) = (0.7 - 0.1) = 0.6$$
$$sf(P_2) = (0.8 - 0.2) = 0.6$$
$$sf(P_3) = (0.4 - 0.6) = -0.2$$
$$sf(P_4) = (0.6 - 0.2) = 0.4$$
$$sf(P_5) = (0.4 - 0.6) = -0.2$$

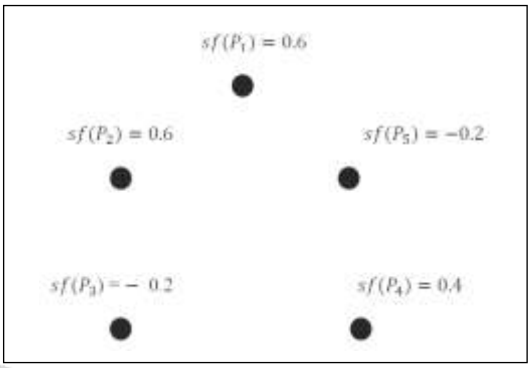

Figure 2: Score function of places where the planning team is interested in installing a base station.

Since scores for P_1 and P_2 are equal, we find the accuracies of P_1 and P_2 as follows

$acc(P_1) = (0.7 + 0.1) = 0.8$

$acc(P_2) = (0.8 + 0.2) = 1.0$

As $acc(P_2) > acc(P_1)$, then P_2 is considered as the appropriate place to fix the base station.

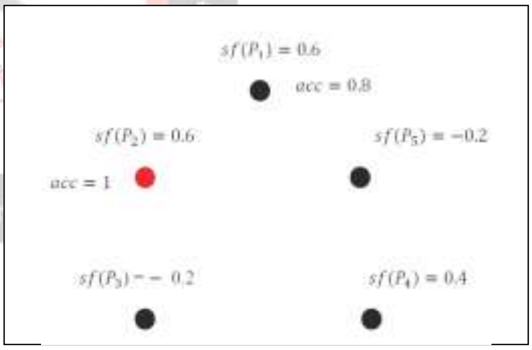

Figure 3: Place P_2 is selected to install a base station.

Scenario 2: If a base station is installed between places $P_1$and P_2, then it will characterize the edge P_1P_2. In order to model the edge P_1P_2, we find amplitude and phase terms using Definition 1. The edge is modeled as follows

$P_1P_2 : 0.7\, e^{i0.5\pi} + 0.2\, e^{i0.4\pi}$

In this case, the amplitude term indicates that 70% of the team members recommend that there should be a base station between these two places and 20% of the team members believe the opposite. The phase term indicates that 50% of the team members recommend that in a certain time the installation of a base station between these two places will give higher profit, while 40% of the team members

 DOI : 10.35291/2454-9150.2019.0432

believe the opposite. The amplitude and phase values of all other cases are obtained and given in Table 2.

Table 2. Amplitude and phase values of membership and non-membership function for each edges

Edges	Amplitude		Phase	
	Membership	Non-membership	Membership	Non-membership
P_1P_2	0.7	0.2	0.5	0.4
P_1P_3	0.4	0.6	0.4	0.4
P_1P_4	0.6	0.2	0.5	0.7
P_1P_5	0.4	0.6	0.4	0.3
P_2P_3	0.4	0.6	0.4	0.4
P_2P_4	0.6	0.2	0.7	0.7
P_2P_5	0.4	0.6	0.4	0.4
P_3P_4	0.4	0.6	0.4	0.7
P_3P_5	0.4	0.6	0.4	0.4
P_4P_5	0.4	0.6	0.4	0.7

We model the information in Table 2 as follows

$$\mathcal{Y} = \begin{cases} P_1P_2 : 0.7\, e^{i0.5\pi} + 0.2\, e^{i0.4\pi} \\ P_1P_3 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.4\pi} \\ P_1P_4 : 0.6\, e^{i0.5\pi} + 0.2\, e^{i0.7\pi} \\ P_1P_5 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.3\pi} \\ P_2P_3 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.4\pi} \\ P_2P_4 : 0.6\, e^{i0.7\pi} + 0.2\, e^{i0.7\pi} \\ P_2P_5 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.4\pi} \\ P_3P_4 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.7\pi} \\ P_3P_5 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.4\pi} \\ P_4P_5 : 0.4\, e^{i0.4\pi} + 0.6\, e^{i0.7\pi} \end{cases}$$

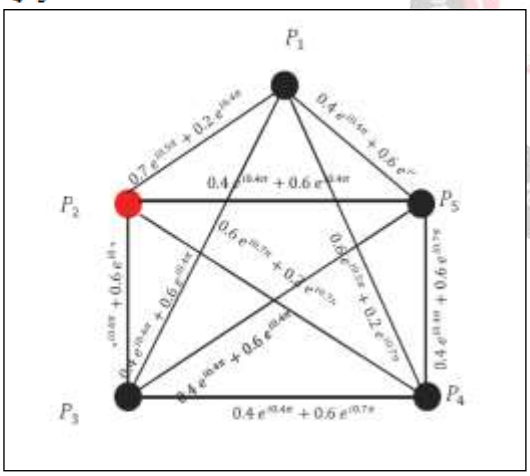

Figure 4: Edges where the planning team is interested in installing a base station.

The absolute value of edges are calculated as below

$|P_1P_2| = (0.7, 0.2)$
$|P_1P_3| = (0.4, 0.6)$
$|P_1P_4| = (06, 0.2)$
$|P_1P_5| = (0.4, 0.6)$
$|P_2P_3| = (0.4, 0.6)$
$|P_2P_4| = (0.6, 0.2)$
$|P_2P_5| = (0.4, 0.6)$
$|P_3P_4| = (0.4, 0.6)$
$|P_3P_5| = (0.4, 0.6)$
$|P_4P_5| = (0.4, 0.6)$

In order to find the optimal choice, we need to find the score function of the absolute values of the edges. Thus, we have

$sf(P_1P_2) = (0.7 - 0.2) = 0.3$
$sf(P_1P_3) = (0.4 - 0.6) = -0.2$
$sf(P_1P_4) = (0.6 - 0.2) = 0.3$
$sf(P_1P_5) = (0.4 - 0.6) = -0.2$
$sf(P_2P_3) = (0.4 - 0.6) = -0.2$
$sf(P_2P_4) = (0.6 - 0.2) = 0.4$
$sf(P_2P_5) = (0.4 - 0.6) = -0.2$
$sf(P_3P_4) = (0.4 - 0.6) = -0.2$
$sf(P_3P_5) = (0.4 - 0.6) = -0.2$
$sf(P_4P_5) = (0.4 - 0.6) = -0.2$

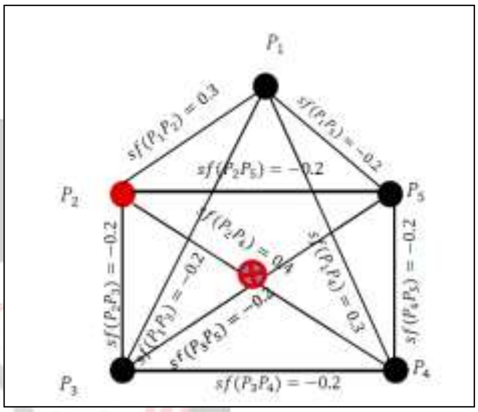

Figure 5: Vertex and Edge selected for installing base station

The score function of the edge is the maximum (i.e., $sf(P_2P_4) = 0.4$), and therefore the optimum choice to install the base station in this scenario is any point between the places P_2 and P_4.

IV. CONCLUSION

Graph models have numerous applications in various domains including mathematics, biotechnology, communication networks, information coding, operations research and many other domains of science and technology. An Intuitionistic fuzzy soft model plays an important role as a numerical tool for modeling a problem of multi-criteria decision analysis with uncertainty. A CIFS-graph is a combination of the fuzzy soft model which provides more compatibility, reliability and accuracy to a system as related to other soft graph frameworks. We have used the properties of CIFS-graph to solve a multi-criteria decision making problem in a mobile communication network to select the optimum to install a base station. In the near future, we plan to assimilate the notions of CIFS-graph with interval number theory, bipolar hyper graphs and rough-soft graphs.

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