A Different approach on Graceful Labeling For Some Graphs

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Abstract - Graph theory is depicted as study of graphs. Graphs are considered as mathematical structures utilized to model pair-wise relations among objects from some collections. Graph can be demonstrated as set of edges and set of vertices. Vertices are measured as any abstract data types and can be offered with points in plane. These abstract data types can also be measured as nodes. A line or line segment collecting these kinds of nodes are termed as edge. However, more specifically saying, edge is measured as an abstract data type that depicts relationship among those nodes. In this article graceful labeling of graph is discussed.

Keywords: *labeling. Graceful labeling, edge graceful labeling, strong edge graceful labeling*

I. GRACEFUL LABELING

To acquire certain intuition over how to label graph gracefully, consider a path graph. Let $V(P_n) = \{u_0, u_1, \ldots, u_{n-1}\}$ be set of vertices as $u_{k-1}u_k \in E(P_n)$ be the set of edges for $0 < k < n$. As P_n holds $m = n-1$ edges, vertices must be labeled with numbers from 0 to $n-1$, however each number in $[1, n-1]$ seems to be edge label. Edge label commences with $n-1$ to attain absolute difference that is equal to $n-1$. Therefore, consider to label u_0 with 0 and u_1 with $n-1$. Subsequently, consider a label with value $n-2$. There exists only two probable ways to attain $n-2$ as absolute difference . They are $n-2 = |(n-2)-0| = |(n-1)-1|$. As u_0 has more unlabeled adjacent vertices, edge label $n-2$ can be attained by labeling u_2 with 1. By handling this case, labeling is provided as follows:

$$f(u_k) = \begin{cases} \frac{k}{2} & \text{if k is even} \\ n - \frac{k+1}{2} & \text{if k is odd} \end{cases}$$

Here, it is shown that f is measured as graceful labeling of P_n, it determines to show that edge label 1 seems to be appear on last edge $u_{n-2}u_{n-1=\frac{n}{2}} - \frac{n-2}{2} = 1$. If $'n'$ is odd, the argument provides edge label as 1.

Theorem 1.1

Path graph P_n is graceful for all $n \geq 1$.

For some instances, this work tries to find graceful labeling for complete graph k_n. As K_1 and K_2 is also determined as path graphs, it is also graceful.

Before examining general case, consider the property of graceful labeling. Graph is given with graceful label, if every vertex label k is swapped with $m-k$, resultant label is also determined as graceful as edge labels does not shows any change: end vertices of edge with labels like a and b turns to be $m-a$ and $m-b$, where $|a-b| = |(m-a)-(m-b)|$. It is also termed as complementarity property.

Here, for k_n with $n > 4$, mention vertex with label 0 which is adjacent to vertex label m to attain edge label m. However, with this case, each vertex is adjacent to each vertex. Therefore, any vertex can be labelled as 0 and any other can be labeled as m without loss of generality. To acquire edge label $m-1$, there exists two options: $m-1 = |(m-1)-0| = |m-1|$. Moreover, complementarity property facilitates to determine 1 without any loss of generality. Selection of label vertex with 1, edge label can be attained 1 and $m-1$. Now edge label of $m-2 = |(m-2)-0| = |(m-1)-1| = |m-2|$ has to be attained. A vertex cannot be labelled with $m-1$ or 2 as it may create duplicate edge label. Therefore, only option is to label vertex with $m-2$, attaining edge labels 2, $m-2$ and $m-3$.

As $m-3$ is appeared already on edge, next edge label has to attain $m-4 = |(m-4)-0| = |(m-3)-1| = |(m-2)-2| = |(m-1)-3| = |m-4|$.

As well, there is another option devoid of computing duplicate edge labels that is to label vertex with 4 attaining edge labels with 3,4, $m-6$ and $m-4$. Here, five vertices are labelled. Moreover, for K_5 may have m -6 =4 as duplicate edge label. For $n \geq 6$, next edge label is m -5. However, for all possible ways to acquire m-5 causes duplicate edge label. Henceforth, there exist no way to attain $m-5$ on edge.

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II. Edge Graceful Labelling

2.1 Path graph

Theorem Vertex union of triangular edges of any path is edge graceful labeling.

Proof Let P_n be path with vertices $'n'$. Let $'t'$ be path of triangular edges. Let central vertex is specified by $V_{0,0}$ vertices round central vertices is denoted by $V_{i,j}$. Let $e_{i,j}$ be corresponding graceful edges where $1 \leq j \leq t$, $1 \leq i \leq n-1$.

Define

$f: V(G) \rightarrow \{0,1,2,\dots,20\}$ as $f(V_{0,0}) = 0$

$f(V_1, j) = P_j$ where $1 \leq n \leq t$

$f(V_2, j) = P_{t+j} - P_j$ where $1 \leq j \leq t$

$f(V_3, j) = P_{2t+j} - P_{t+j}$ where $1 \leq j \leq t$

Similarly,

$f(V_{20}, j) = P(i-1)_{t+j} + P(i-2)_{t+j} + P + \dots + (i-19)_{t+j};$

Where, $1 \leq i \leq n-1$; $1 \leq k \leq i$ for all i, j varies from 1 to 20

Let Graph P_4^7 is union of 7 copies of path P_4

Here, $n = 4$; $t = 7$; $j = 1,2,3,\dots,7$

Number of edges in $P_4^7 = 3 * 7 = 21$ and edges are defined by graceful labeling of end vertices.

2.2 Star graph

Theorem Graph is constructed by attaching roots of different stars to vertex is a star edge graceful labeling

Proof Let $X_{1,n1}, X_{1,n2}, X_{1,n3}, \dots, X_{1,nt}$ be't' stars attached to vertices $'V'$. Label vertex $'V'$ as 0. Label central vertices of star as $P_1, P_2, \dots, P_t$ respectively.

End vertices of first start is,

$$P_{t+1} - P_1, P_{t+2} - P_1, \dots, P_{t+n} - P_1$$

End vertices of second star is,

$$P_{t+n1+1} - P_2, P_{t+n1+2} - P_2 \dots P_{t+n1+n} - P_2$$

Similarly for last star,

$$P_{t+n1+n2} \dots . n_{t-1} + 1 - P_t, P\_(t + n_1 + n_2 + \dots + n_{t-1} + 22 - P_t \dots \dots P\_(t + n_1 + n_2 + \dots + n_{t-1} + n_t - p_t$$

Here, edge labels got sum of all labels of end vertices

2.3 Cycle graph

Theorem If graph G contains cycle by joining apex vertices of cycle to next vertex 'x' is an edge labeled triangular graph

Proof Let $G = < C_n : C_m >$. let apex vertex of C_n is denoted by V_0 and vertices adjacent to V_o of cycle C_n is denoted by $V_1, V_2, \dots, V_n$. Similarly, denote vertex of other cycle C_m by C_0 and vertices adjacent to C_0 to cycle C_m by $C_1, C_2, \dots, C_m$. Let $'u'$ be vertex adjacent to apex vertices of both cycle.

Let $f: V(G) \rightarrow \{0,1,2,\dots,P_q\}$ is possible edge labeling. As $'0'$ and $'n'$ are labels of any adjacent vertices of graph $'G'$.

(i) If $'0'$ and $'1'$ are labels of adjacent vertices $'C_n'$ or $'C'_m$, then there is triangular having two vertices labeled with 0 and 1.

(ii) If $f(u) = 0$ then one vertices of V_0, C_0 is labelled with 1. Assume $f(V_0) = 1$, without loss of generality. To get edge label $P_2 = 5$.

(iii) Assume $f(V_i) = 20$ for some $i \in \{1,2,\dots,n\}$. In case, if a triangular graph is attained whose vertex labels are 1, 20 and x. Then, $x + 20$ and $x + 1$ will be edge labels of two edges with difference 19 which is not perfect square.

(iv) Assume that $f(C_i) = 9$ for $i \in \{1,2,3,\dots m\}$. In case, if triangular graph is attained whose vertex labels are 5, 9 and x. Then, $x + 9$ and $x + 5$ are labels of two edges.

(v) Assume 3 and 11 or 2 and 12 or 6 and 8 are labels of adjacent vertices from one to two of cycles. So, there is a triangular graph whose vertex labels are either 1,2,12 or 5,2,12 and 1,3,11. In these case, edge label are not triangular number.

2.4 Fan graph

Theorem Fan graph F_n is non triangular graph.

Proof Fan graph F_n attained from cycle $C_n = C_n + k_1$ by attaching edge at each vertex of C_n. Let C_1 be vertex of cycle and $V_1, V_2, \dots, V_n$ be vertices adjacent to C_1 in clockwise direction. Let vertices adjacent to $V_1, V_2, \dots, V_n$ is denoted by $u_1, u_2, \dots, u_n$ respectively.

(i) If $f(C_1) = 0$. Since $'0'$ and $'1'$ are adjacent in triangular sum graph, assign label to exactly one of vertices $V_1, V_2, \dots, V_n$. Here, triangular vertex labels are 0 and 1.

(ii) If one of vertices from $V_1, V_2, \dots, V_n$ is labeled with 0. Without loss of generality, consider $f(V_1) = 0$.

(iii) If one of vertices from C_1, V_2, V_n is labeled with 1, then there is a triangular with vertex labels 0 and 1.

(iv) If $f(u_1) = 1$, then possibility of next edge label has to be discussed

Label $P_2 = 20$ can be obtained using vertex labels. 0,20 or 1,19. As u_1 is fan based vertex having label 1, combination 1,19 is not possible.

Therefore, one vertices V_2, V_n, C_1 must be labelled within 20.

Thus, there is a triangle with two vertices labelled as 0 and 20.

2.5 Strong edge graceful labeling

A graph G with (p, q) is depicted to have strong edge graceful labeling if there is occurrence of function f from edge to $\{1,2,\dots,\left[\frac{3q}{2}\right]\}$, therefore mapping defined over vertex set is provided by $f'(x) = \sum\left\{\frac{f(xy)}{xy} \in E(G)\right\} (\text{mod } 2p)$ are unique. Graph G is

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termed as strong edge graceful if it uses strong edge graceful labeling.

Theorem 2. 5.1 Graph $k_4^{(n)}$ is determined as strong edge graceful for all $n \geq 3$, when $'n'$ is even.

Proof Consider $\{v_1, v_2, .., v_{3n}\}$ be vertices of K_4^n and $\{e_1, e_2, \ldots, e_{3n}, e_{3n}, f_1, f_2, \ldots, f_{3n-1}, f_{3n}$ be edge of K_4^n is specified as follows:

$$f(f_i) = i \qquad 1 \leq i \leq \frac{3n}{2}$$

$$f(f_i) = 3_{n+1} + i \qquad \frac{3n}{2} + 1 \leq i \leq 3n$$

$$f(e_i) = 3_{n+1} - i \qquad 1 \leq i \leq \frac{3n}{2}$$

$$f(e_i) = 6n + 2 - i \qquad \frac{3n}{2} + 1 \leq i \leq 3n$$

Then, induced vertex labels are

$$f^+(V_0) = 0$$

$$f^+(v_i) = 6n + 2 - i \qquad 1 \leq i \leq \frac{3n}{2}$$

$$f^+(v_i) = 3n + 1 - i \qquad \frac{3n}{2} + 1 \leq i \leq 3n$$

It is obvious, vertex labels are distinct.

Therefore, graph k_4^n is strong edge graceful for all $n \geq 3$, when n is even.

Theorem 2.5.2 Graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$, when $n \equiv 1 \pmod 4$

Proof Let $\{v_1, v_2, v_3, \ldots, v_{3n}\}$ be vertices of $K_4^{(n)}$ and $\{f_1, f_2, f_3, \ldots, f_{3n-1}\}$ be edges of $K_4^{(n)}$ is specified as in Fig.

Edge label of $K_4^{(n)}$ are $f(f_i) = i$

$$f(f_{3n}) = 6n$$

$$f(e_i) = 6n - i \quad 1 \leq i \leq 3n$$

Vertex labels are

$$f^+(v_0) = \frac{3n-1}{2}$$

$$f^+(v_i) = 6n - 2 - i \qquad 1 \leq i \leq 3n - 3$$

$$f^+(v_{3n-2}) = 6n$$

$$f^+(v_i) = 6n - 2 - i \qquad 3n - 1 \leq i \leq 3n - 3$$

Vertex labels are distinct. Therefore, graph $K_4^{(n)}$ is strong edge graceful label for all $n \geq 3$ when $n \equiv 1 \pmod 4$

Theorem. 2.5.3 Graph $K_4^{(n)}$ is strong edge graceful for $n \geq 3$ when $n \equiv 3 \pmod 4$

Proof Let $\{e_1, e_2, e_3, \ldots, e_{3n-1}, e_{3n}\}$be edges of $K_4^{(n)}$ and $\{v_1, v_2, v_3, \ldots, v_{3n}\}$ be vertices of $K_4^{(n)}$.

Graph Label edges of $K_4^{(n)}$ are

$$f(f_i) = i \qquad 1 \leq i \leq 3n$$

$$f(e_i) = 6n + 1 - i \qquad 1 \leq i \leq 3n$$

The vertex labels are

$$f^+(v_0) = \frac{3n+1}{2}$$

$$f^+(v_i) = 6n - i \qquad 1 \leq i \leq 3n$$

Vertex labels are distinct. Graph $K_4^{(n)}$ is strong edge graceful for $n \geq 3$ when $n \equiv 3 \pmod 4$

III. Conclusion

In this study, graceful labelling and its corresponding connections are related to combinatorial crisis. In addition, some uncovered topics are discussed for path labeling and transversal in labeling. Henceforth, benefits of graceful labeling shows some impact towards area like graph decomposition and graph databases. Conventional approach to that problem has been constructed with graceful and graceful labeling for specific class of graphs. It seems that there is some common consensus between researchers investigate in this area that various approach is essential.

In this work, an alternative solution is explored for Graceful tree from various angles. Initially, this work concentrates on relaxed labeling which is featured by some parameters that is converted to graceful labeling when these parameter is equal to number of edges or vertices. Here, this work illustrates edge graceful labeling, strong edge graceful labeling in detail. This significant approach is to validate non-trivial lower or upper bound for factor and to enhance it view of attaining graceful labeling.

Subsequently, some enumeration for graceful labeling of path is considered with edges is demonstrated here. This specific problem is associated with numerous other crises in combination with coloring, and transversal in graph labeling.

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