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Pasting Lemma for s**g Continuous Functions

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Abstract- One of the most important concepts in topological space is the pasting lemma for continuous functions. It plays an important role in algebraic topology. In the recent years pasting lemmas for some stronger and weaker forms of continuous functions such as g - continuous functions, gp - continuous functions, gpr- continuous functions, g^*b - continuous functions have been introduced by several mathematicians.In this consequence, the pasting lemmas for sg-continuous functions and s**g irresolute functions have been introduced in this paper.**

Keywords – s**g - continuos functions, s**g - irresolute functions

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I. INTRODUCTION

Moreover, the pasting lemmas for continuous functions are established over last two decades. Anitha et al.[1] established the pasting lemma for rg-continuous, gc-irresolute and gp-continuous functions. In this sequence, the pasting lemmas for s**g-continuous functions and s**g irresolute functions have been introduced in this paper.

Throughout the paper, (X, τ) or X denote the topological space on which no separation axiom is presumed unless explicitly stated. Let A be a subset of X.Then the closure and interior of A are the intersection of all closed sets containing A and union of all open sets contained in A respectively and they are denoted by cl(A) and int(A). A set A is called semi open.

The complements of semi open, regular open, pre open, b-open sets are called semi closed, regular closed,pre closed sets, b-closed respectively. The regular closure, regular interior, preclosure, preinterior, b-closure and b-interior of a set A are defined in similar way of closure and interior of a set A. Moreover, A is called s*g – closed[3]{respectively rg-closed[5], rg*- closed[4], g*r – closed[4], gp-closed[16], gpr-closed[6] and g*b-closed[19]}if cl(A) $\subseteq U$ {respectively, cl(A) $\subseteq U$, rcl(A) $\subseteq U$, rcl(A) $\subseteq U$, pcl(A) $\subseteq U$, pcl(A) $\subseteq U$ and bcl(A) $\subseteq U$} whenever A$\subseteq U$ and U is semi open{respectively, regular open and g open}.

The complements of semi open, {respectively, regular open, preopen, s*g-closed, rg- closed, rg*-closed, g*r – closed, gp-closed, gpr-closed, g*b-closed} are called semi

closed,{respectively,regular closed, preclosed, s*g -open, rg-open, rg*-open, g*r- open, gp-open, gpr-open and g*b-open}.

A function f:(X, τ)→(Y, μ) is called continuous,{respectively, g-continuous[2], rg-continuous [15], gp continuous, gpr- continuous, s*g-continuous, rg*- continuous[15], g*r-continuous[15], g*b-continuous[20]}if f^{-1}(V) is closed{respectively, g-closed, rg-closed, gp-closed, gpr-closed, s*g-closd, rg*-closed, g*r -closed, g*b-closed} for every closed set V in Y.

A function f :(X, τ) → (Y, μ) is called irresolute,{respectively, gc-irresolute[2], rg- irresolute[15], gp- irresolute[17], gpr-irresolute[7], s*g- irresolute[3], rg*-irresolute[15], g*r- irresolute[15], g*b-irresolute[20]}if f^{-1}(V) is semi closed {resp.g- closed, rg-closed, gp-closed, gpr-closed, s*g-closed, rg*-closed, g*r-closed, g*b-closed[19]}for every semi closed{resp. g- closed, rg-closed, gp-closed[16], gpr-closed[19], s*g-closed[3], rg*-closed[4], g*r-closed[4], g*b-closed}set V in Y.A collection $\{A_\alpha : \alpha \in I\}$of subsets of a space X is locally finite if every point of X has a neighbourhood that intersects only finitely many members of $\{A_\alpha : \alpha \in I\}$.

II. MAIN RESULTS

Definition 2.1 A function f : (X, τ_1, τ_2) → (Y, μ_1, μ_2) is **s**g – continuous** if the inverse image of each closed set in Y is s**g closed set in X.

Example 2.1 Let X = Y ={a, b, c}, τ = {ϕ, X, {a,c}} and μ ={ϕ, X, {a,c},{c}, {a}}. Let f : (X, τ) → (Y, μ) be an identity mapping. Then f is s**g-continuous.

Theorem2.1 Every continuous function is s**g-continuous.

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Proof Let f :(X, τ) → (Y, µ) be continuous function and Let U be a closed set in (Y, µ) then f^{-1}(U) is closed in (X, τ).

As every closed set is s**g closed set so that we have f^{-1}(U) is s **g- closed . Hence f is s**g-continuous.

The converse need not be true as seen from the following example.

Example 2.2 Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}} and µ ={ϕ, Y, {a}}. Let f : (X, τ) → (Y, µ) be a function defined by f(a) = {b}, f(c) = {c}. Then f is s**g-continuous but not continuous.

Theorem2.2 The following are equivalent for a function f : (X, τ) → (Y, µ). **a)** f is s**g-continuous. **b)** f^{-1}(U) is s**g open for each open set U in Y.

Proof a) =>b) Let f be s**g-continuous. Let A be open in Y.Then A^c is closed in Y. Since f is s**g- continuous we have f^{-1}(A^c) is s**g closed in X. Thus f^{-1}(A) is s**g open in X.**b) => a)** Let f^{-1}(U) be s**g – open for each open set U in Y. Let V be closed in Y.Then V^c is open in Y. Therefore, by our assumption, f^{-1}(V^c) is s**g – closed in X. Thus f^{-1}(V) is s**g -open in X. Hence the proof.

Definition 2.2 A function f : (X, τ) → (Y, µ) is **s**g – irresolute** if the inverse image of each s**g- closed set in Y is s**g-closed set in X.

Example 2.2 Let X = Y = {a, b, c}, τ = { ϕ, X, {a}} and µ = { ϕ,Y, {a}, {a, b}}. Let f : (X, τ) → (Y, µ) be an identity mapping. Then f is s**g – irresolute.

We know that arbitrary union of cl(A_i), i ∈I is contained in closure of arbitrary union of subsets of A_i in any topological space. The equality holds if the collection { A_i , i ∈I} is locally finite.

Theorem 3.1[7] The arbitrary union of s**g-closed sets A_i , i ∈I in a topological space (X, τ) is s**g- closed if the family { A_i , i ∈I} is locally finite.

Theorem 3.2 Let Y be a topological spaces, V ⊆ Y and f : X →Y. Then f is s**g – continuous if and only if V is closed in Y implies that f^{-1}(V) is s**g – closed in X.

Proof Case1) Let f : X →Y be s**g- continuous. Let V ⊆ Y be closed. Then V^c ⊆ Y is open in Y. Since f is s**g – continuous, f^{-1}(V^c) is s**g open in X. f^{-1}(V^c) = [f^{-1}(V)]^c is s**g open in X. Therefore we have f^{-1}(V) is s**g – closed in X.

Case2) Let V be closed in Y then f^{-1}(V) is s**g – closed in X. Let A be open in Y. Then A^c is closed in Y. By our assumption f^{-1}(A^c) =[f^{-1}(A)]^c is s**g – closed in X. Thus, f^{-1}(A) is s**g-open in X.Hence the proof.

Theorem 3.3 Let X = A ∪ B where A and B are both open and s**g- closed sets in X. Let f : A → Y and g : B → Y be s**g – continuous(s**g – irresolute). If f(x) = g(x) for every x ∈ A ∩ B . Then f and g combine to give a s**g-continuous (s**g – irresolute) function h : X → Y defined by setting h(x) = f(x) if x ∈A and h(x) = g(x) if x ∈ B.

Proof Let U be closed (s**g – closed) in Y. Then h^{-1}(U) = f^{-1}(U) ∪ g^{-1}(U). Since f : A → Y is s**g – continuous(s**g – irresolute) and g : B → Y be s**g – continuous(s**g – irresolute), g^{-1}(U) is s**g – closed sets in B. Since A and B are both open and s**g – closed sets in X, f^{-1}(U) and g^{-1}(U) are s**g – closed sets in X. Since the union of two s**g – closed sets is s**g – closed sets. Therefore we have h^{-1}(U) = f^{-1}(U) ∪ g^{-1}(U). is s**g – closed in X.Hence proved.

Importance of pasting lemmas are recognized in the formation of continuous functions. In this regard, the pasting lemmas for s**g - continuous and s** girresolute functions are introduced herein as discussed above.

III. CONCLUSION

In this work, we have introduced the pasting lemmas for s**g – continuous and s**g – irresolute functions. This is a required development in the line of pasting lemmas for g - continuous functions, gp - continuous functions, gpr-continuous functions, g^* b - continuous functions. However, there is scope for further development of these topics that will be explored in our future works.

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