International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-05, Issue-09, Dec 2019

Pairwise S**GLC Continuous Maps in Bitopological Space

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Abstract- The purpose of this paper is to introduce the distinct concepts of pairwise SGLC – continuity and we discuss some of their consequences and the restriction maps of pairwise S**GLC – continuity.**

Keywords – pairwise S**GLC – continuous, pairwise S**GLC – irresolute.

I. Introduction

The difference of two closed subsets of an n – dimensional Euclidean space was studied by Kuratowski and Sierpinski [10] in 1921 and indirectly their work represents the idea of a locally closed subset of a topological space (X, τ). Bourbaki [1] defined a subset of a topological space as locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly [6] introduced locally closed sets in topological spaces and studied three different ideas of general continuity and Stone[12] called locally closed sets as FG sets. H. Maki, P. Sundaram and K. Balachandran[11] introduced the concept of generalized locally closed sets and obtained different notions of generalized continuities. Ganster, Arockiarani and Balachandran[7] introduced regular generalized locally closed sets and rglc-continuous functions and discussed some of their properties.

K. Chandrasekhara Rao and K.Joseph[2] introduced the concepts of semi star generalized open sets and closed sets in topological spaces.K. Chandrasekhara Rao and K. Kannan[3,4] introduced the concepts of semi star generalized locally closed sets, semi star generalized maximal spaces with the help of semi star generalized closed sets.

J.C. Kelly[9] introduced the concept of bitopological spaces. M.Jelic[8] introduced lodcally closed sets and locally closed continuity in bitopological spaces.K. Chandrasekhara Rao and K. Kannan[5] introduced the concepts of semi star generalized closed sets in bitopological spaces.

In this paper the concepts of Pairwise S**GLC – continuous, Pairwise S**GLC – irresolute are introduced and study their basic properties.

II. PRELIMINARIES

In this section (X , τ_1 , τ_2) or simply X denotes a bitopological space .By τ_i – S*GO (X , τ_1 , τ_2){ respectively, τ_i – S*GC (X , τ_1 , τ_2)}, we shall mean the collection of all τ_i – s*g open sets(respectively, τ_i -s*g closed sets) in (X , τ_1 , τ_2).

For any subset A $\subseteq$ X , τ_i int (A) and τ_i cl (A) denote the interior and closure of a set A with respect to the topology τ_i respectively . A^c denotes the complement of A in X unless explicitly stated .

Definition 2.1 A subset of a bitopological space (X , τ_1 , τ_2) is called

a) $\tau_i\tau_j$ – semi open if there exists a τ_1 – open set U such that U $\subseteq$ A $\subseteq$ τ_2– cl(U), i, j =1, 2, i $\neq$ j .
b) $\tau_i\tau_j$ – semi closed if X – A is $\tau_i\tau_j$ – semi open.
c) $\tau_i\tau_j$ – generalized closed ($\tau_i\tau_j$ − g closed) if τ_j– cl(U) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i -open in X.
d) $\tau_i\tau_j$ – generalized open ($\tau_i\tau_j$ – g open) if X – A is $\tau_i\tau_j$ – g closed.
e) $\tau_i\tau_j$ –semi star generalized closed ($\tau_i\tau_j$ − s*g closed) if τ_j– cl(U) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i – semi open in X.
f) $\tau_i\tau_j$ –semi star generalized open ($\tau_i\tau_j$ − s*g open) if X – A is $\tau_i\tau_j$ – s*g closed in X.

Definition 2.2 A subset A of a bitopological space (X , τ_1 , τ_2) is said to be a

a) $\tau_i\tau_j$ -locally closed set if A = G $\cap$ F , where G is τ_1 - open & F is τ_2 - closed in X , i,j =1, 2, i $\neq$ j .

 DOI : 10.35291/2454-9150.2019.0550

b) $\tau_i\tau_j$ -locally semi closed set if A = G ∩ F , where G is τ_1 - open & F is τ_2 - semi closed in X , i , j = 1 , 2 , i ≠ j .

c) $\tau_i\tau_j$ - semi locally closed set if A = G ∩ F , where G is τ_1 - semi open & F is τ_2 - semi closed in X , i , j = 1 , 2 , i ≠ j .

d) $\tau_i\tau_j$ - g locally closed set if A = G ∩ F , where G is τ_1 - g open & F is τ_2 - g closed in X , i , j = 1 , 2 , i ≠ j .

e) $\tau_i\tau_j$ -sg locally closed set if A = G ∩ F , where G is τ_1 - sg open & F is τ_2 - sg closed in X , i , j = 1 , 2 , i ≠ j .

f) $\tau_i\tau_j$ -s*g locally closed set if A = G ∩ F , where G is τ_1 - s*g open & F is τ_2 - s*g closed in X , i , j = 1 , 2 , i ≠ j .

g) $\tau_i\tau_j$ -sg locally closed* set if A = G ∩ F , where G is τ_1 - sg open & F is τ_2 - closed in X , i , j = 1 , 2 , i ≠ j .

h) $\tau_i\tau_j$ -sg locally closed** set if A = G ∩ F , where G is τ_1 - open & F is τ_2 - sg closed in X , i , j = 1 , 2 , i ≠ j .

i) $\tau_i\tau_j$ -gs locally closed set if A = G ∩ F , where G is τ_1 - gs open & F is τ_2 - gs closed in X , i , j = 1 , 2 , i ≠ j .

Definition 2.3 A function f : (X , τ_1 , τ_2)→ (Y , σ_1 ,σ_2) is

a) Pairwise l c - continuous if f^{-1} (U) is $\tau_i\tau_j$ - locally closed for each σ_j - open set U in Y , i , j = 1 , 2 , i ≠ j .

b) Pairwise glc - continuous if f^{-1} (U) is $\tau_i\tau_j$ - g locally closed for each σ_j - open set U in Y , i , j = 1 , 2 , i ≠ j .

c) Pairwise sglc - continuous if f^{-1} (U) is $\tau_i\tau_j$ - sg - locally closed for each σ_j - open set U in Y , i , j = 1 , 2 , i ≠ j .

d) pairwise gslc - continuous if f^{-1} (U) is $\tau_i\tau_j$ – gs - locally closed for each σ_j - open set U in Y , i , j = 1 , 2 , i ≠ j .

e) pairwise s*glc - continuous if f^{-1} (U) is $\tau_i\tau_j$ - s*g locally closed for each σ_j - open set U in Y , i , j = 1 , 2 , i ≠ j .

III. Pairwise S**GLC continuous Maps

Definition 3.1 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tau_1\tau_2$ - S**GLC - continuous {resp. $\tau_1\tau_2$ - S**GLC* - continuous, $\tau_1\tau_2$ - S**GLC** - continuous} if f^{-1}(U)is $\tau_1\tau_2$ - s**g locally closed [resp. $\tau_1\tau_2$ - s**g - locally closed*, $\tau_1\tau_2$ - s**g - locally closed**] for each σ_1- open set U in Y.

Definition 3.2 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tau_2\tau_1$ - S**GLC - continuous { resp. $\tau_2\tau_1$ - S**GLC* - continuous, $\tau_2\tau_1$ - S**GLC** - continuous } if f^{-1}(U)is $\tau_2\tau_1$ - s**g locally closed [resp. $\tau_2\tau_1$ - s**g - locally closed*, $\tau_2\tau_1$ - s**g - locally closed**] for each σ_2 - open set U in Y.

Definition 3.3 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC - continuous { resp. pairwise S**GLC* - continuous, pairwise S**GLC** - continuous } if f is both $\tau_1\tau_2$ - S**GLC - continuous { resp. $\tau_1\tau_2$ - S**GLC* - continuous, $\tau_1\tau_2$ - S**GLC** - continuous} and $\tau_2\tau_1$ - S**GLC - continuous { resp. $\tau_2\tau_1$ - S**GLC* - continuous, $\tau_2\tau_1$ - S**GLC** - continuous}.

Example 3.4 Let X = Y = {a, b, c, d}, τ_1 = {ϕ, X, {c, d}, {c}} and τ_2 = {ϕ, X, {a, b}, {a, b, d}}, σ_1 = {ϕ, Y, {c, d}}, σ_2 = {ϕ, Y, {a, b}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is both pairwise S**GLC - continuous and pairwise S**GLC** - continuous.

Example 3.5 Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {b, c}, {b}} and τ_2 = {ϕ, X, {a}}, σ_1 = {ϕ, Y, {b, c}}, σ_2 = {ϕ, Y, {c}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is pairwise S**GLC* - continuous.

Definition 3.6. A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is strongly $\tau_1\tau_2$ - S**GLC – continuous {resp. strongly $\tau_1\tau_2$ - S**GLC* - continuous, strongly $\tau_1\tau_2$ - S**GLC** - continuous} if f^{-1}(U) is τ_1 - open for each $\sigma_1\sigma_2$ - s**g - locally closed [resp.$\sigma_1\sigma_2$ - s**g - locally closed*, $\sigma_1\sigma_2$ - s**g - locally closed**] set U in Y.

Definition 3.7 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is strongly $\tau_2\tau_1$ - S**GLC - continuous {resp. strongly $\tau_2\tau_1$ - S**GLC* - continuous, strongly $\tau_2\tau_1$ - S**GLC** - continuous} if f^{-1}(U)is τ_2 - open for each $\sigma_2\sigma_1$ - s**g - locally closed [resp. $\sigma_2\sigma_1$ - s**g - locally closed*,$\sigma_2\sigma_1$ - s**g - locally closed**] set U in Y.

Example 3.8 Let X = Y = {a, b, c, d}, τ_1 = {ϕ, X, {c, d}, {c}} and τ_2 = {ϕ, X, {a, b}, {a, b, d }}, σ_1 = {ϕ, Y, {c, d}}, σ_2 = {ϕ, Y, {a, b}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is both strongly pairwise S**GLC - continuous and strongly pairwise S**GLC** - continuous.

Example 3.9 Let X = Y = {a, b, c, d}, τ_1 = {ϕ, X, {c, d}, {c}} and τ_2 = {ϕ, X, {c, d}, {a, b, d}}, σ_1 = {ϕ, Y, {c, d}}, σ_2 = {ϕ, Y, {a, b}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is both strongly pairwise S**GLC** - continuous.

Definition 3.10 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tau_1\tau_2$ - S**GLC - irresolute {resp. $\tau_1\tau_2$ - S**GLC* - irresolute, $\tau_1\tau_2$ - S**GLC** - irresolute} if f^{-1}(U) is $\tau_1\tau_2$ - S**G locally closed [resp. $\tau_1\tau_2$ - s**g - locally closed*, $\tau_1\tau_2$ - s**g - locally closed**] for each $\sigma_1\sigma_2$ - s**g - locally closed set {resp. $\sigma_1\sigma_2$ - s**g - locally closed*, $\sigma_1\sigma_2$ - s**g - locally closed**} U in Y.

Definition 3.11 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tau_2\tau_1$ - S**GLC - irresolute {resp. $\tau_2\tau_1$ - S**GLC* -

 DOI : 10.35291/2454-9150.2019.0550

irresolute, $\tau_2\tau_1$ - S**GLC** - irresolute} if $f^{-1}(U)$ is $\tau_2\tau_1$ - s**g locally closed [resp. $\tau_2\tau_1$ - s**g locally closed*, $\tau_2\tau_1$ - s**g locally closed**] for each $\sigma_2\sigma_1$ - s**g locally closed set {resp. $\sigma_2\sigma_1$ - s**g - locally closed*, $\sigma_2\sigma_1$ - s**g - locally closed**} U in Y.

Definition 3.12 A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC - irresolute {resp. pairwise S**GLC* - irresolute, pairwise S**GLC** - irresolute} if f is both $\tau_1\tau_2$ - S**GLC- irresolute {resp. $\tau_1\tau_2$ - S**GLC* - irresolute, $\tau_1\tau_2$ - S**GLC** - irresolute} and $\tau_2\tau_1$ - S**GLC - irresolute {resp. $\tau_2\tau_1$ - S**GLC* - irresolute, $\tau_2\tau_1$ - S**GLC** - irresolute}.

Example 3.13 Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}, σ_1 = {ϕ, Y, {a},{a, b}}, σ_2 = {ϕ, Y, {b, c}, {c}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is both pairwise S**GLC - irresolute & pairwise S**GLC** - irresolute.

Example 3.14 Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {c}, {b, c}} and τ_2 = {ϕ, X,{a, b}, {a}}, σ_1 = {ϕ, Y, {c}}}, σ_2 = {ϕ,Y, {a, b}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is pairwise S**GLC* - irresolute.

Theorem 3.15

a) Every pairwise lc - continuous is pairwise S**GLC - continuous function
b) Every pairwise S**GLC* - continuous function is pairwise S**GLC - continuous.
c) Every pairwise S**GLC** - continuous function is pairwise S**GLC*- continuous.
d) Every pairwise S**GLC - irresolute function is pairwise S**GLC - continuous.

Proof.(a) Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be pairwise lc-continuous. Then $f^{-1}(U)$ is $\tau_i\tau_j$ - locally closed for each σ_i - open set U in Y, i ≠ j and i, j = 1, 2. Then $f^{-1}(U)$ is $\tau_i\tau_j$ s**g - locally closed for each σ_i - open set U in Y. Therefore we have f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC - continuous.

(b) Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be pairwise S**GLC* - continuous function. Then $f^{-1}(U)$ is $\tau_i\tau_j$ - S**GLC* for each σ_i - open set U in Y, i ≠ j and i, j = 1, 2. Then $f^{-1}(U)$ is $\tau_i\tau_j$ - S**GLC for each σ_i - open set U in Y.Hence f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC – continuous.

(c) and (d) are similar to prove.

Remark 3.16 The reverse of the above theorems need not be true as seen from the following examples.

Example 3.17a) In example 3.4, f is pairwise S**GLC - continuous, but neither pairwise lc - continuous nor pairwise S**GLC* - continuous.

b) Let X = Y = {a, b, c }, τ_1 = {ϕ, X, {a, c}} and τ_2 = {ϕ, X, {b}, {a, b}}, σ_1 = {ϕ, Y, {c}}, σ_2 = {ϕ, Y, {a, b}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a function defined by f (a) = f (b) = b, f (b) = c. Then f is pairwise S**GLC - continuous.

We know that every $\tau_i\tau_j$ - s**g closed sets is $\tau_i\tau_j$ - s*g closed, $\tau_i\tau_j$ - g closed, $\tau_i\tau_j$ - sg closed, $\tau_i\tau_j$ - gs closed, i, j = 1, 2, i ≠ j and therefore we have every pairwise S**GLC - continuous function is pairwise s*glc - continuous, pairwise glc - continuous, pairwise sglc - continuous, pairwise gslc - continuous.

Anyhow none of these implications can be reversed. The following example supports the claim.

Example 3.18 Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a, b}}and τ_2 = {ϕ, X, {c}}, σ_1 = {ϕ, Y, {b, c}}, σ_2 = {ϕ, Y, {a}}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is pairwise glc - continuous and pairwise gslc - continuous but not pairwise S**GLC - continuous.

Theorem 3.19 (a) If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC** continuous and W be a subset of X. Then the restriction map f/w : (W, τ_{1w}, τ_{2w}) → (Y, σ_1, σ_2) is pairwise S**GLC** - continuous.

(b) If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise S**GLC* continuous and W be a subset of X. Then the restriction map f/w : (W, τ_{1w}, τ_{2w}) → (Y, σ_1, σ_2) is pairwise S**GLC* - continuous.

Proof. (a) Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be pairwise S**GLC** continuous. Then $f^{-1}(U)$is $\tau_i\tau_j$ - s**g locally closed** for each σ_i - open set U in Y, i≠ j and i, j = 1, 2. Therefore, $f^{-1}(U)$is the intersection of an τ_i - open set G and F is τ_j - s**g closed set in (X, τ_1, τ_2). Now $\left(f/_{W}\right)^{-1}(U)$= (G ∩ W) ∩ (F ∩ W). Since G is τ_i - open in X then G ∩ W is τ_i - open in W and since F is τ_j - s**g closed in X then F ∩ W is τ_j - s**g closed in W. Hence f/w : (W, τ_{1w}, τ_{2w}) → (Y, σ_1, σ_2) is pairwise S**GLC** continuous.

(b) Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be pairwise S**GLC* continuous. Then $f^{-1}(U)$is $\tau_i\tau_j$ - s**g locally closed* for each σ_i - open set U in Y, i≠ j and i, j = 1, 2. Therefore, $f^{-1}(U)$is the intersection of an τ_i - open set G and F is τ_j - s**g closed set in (X, τ_1, τ_2). Now $\left(f/_{W}\right)^{-1}(U)$= (G ∩ W) ∩ (F ∩ W). Since G is τ_i - open in X then G ∩ W is τ_i - open in W and since F is τ_j - s**g closed in X then F ∩ W is τ_j - s**g closed in W. Hence f/w : (W, τ_{1w}, τ_{2w}) → (Y, σ_1, σ_2) is pairwise S**GLC* continuous.

IV. CONCLUSION

In this work we have established the concepts of pairwise S**GLC – continuity and pairwise S**GLC irresolute functions which is applicable in most areas of pure

 DOI : 10.35291/2454-9150.2019.0550

mathematics. However, there is scope for further research studies of these topics that will be investigated in our future works.

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