

# Static Voltage Stability Analysis Using Singular Value Decomposition Technique

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**Abstract:** Voltage instability is one phenomenon that could happen in a power system due to its stressed condition and in adequate reactive power supply. The result may be the occurrence of voltage collapse which leads to total blackout to the whole system. It is important to have an analytical method to predict the voltage instability in the power system, particularly with a complex and large one to prevent the occurrence of voltage collapse. In this paper, a study based on Singular Value Decomposition (SVD) techniques has been carried out to study the voltage instability phenomena. The SVD analysis can be used effectively as a powerful analytical tool to measure the maximum loadable limit of the power system and also to determine the proximity to the singular point. Above all we can also generate useful information about the mechanism of voltage instability like identifying the weakest bus in the power system that may contribute to the voltage instability and develop appropriate control measures. A MATLAB program has been developed to carry out the SVD analysis. A typical IEEE 14 – test bus power system is used to validate these techniques, and the weak bus is identified and the test results are presented and discussed.

**Keywords –** Static voltage, SVD, Power System.

## I. INTRODUCTION

The trend in modern power system is towards greater utilization of the available generation, transmission and distribution assets. As power systems are operated under increasingly stressed conditions, the ability to maintain acceptable voltage level becomes a growing concern. System failures and blackouts have already been observed in Europe, Japan, and USA due to the voltage instability. In addition, voltage stability has become one of the biggest concerns in operating and planning electric power systems. The voltage stability of a power system refers to its ability to maintain steady, acceptable voltage levels at all buses in the network at all times, even after being subjected to a disturbance or contingency. A system enters a state of voltage instability when a disturbance, increase in load demand, or change in system condition causes a progressive and uncontrollable drop in voltage.

In many cases, both static and dynamic approaches are used to analyze the problem of voltage stability. Dynamic analysis provides the most accurate indication of the time responses of the system. Dynamic analysis is therefore extremely useful for fast voltage collapse situations, following large disturbances such as loss of generation and system faults, when specific information concerning the complex sequence of events leading to instability, is required. Dynamic simulations however, fail to provide information such as the sensitivity or degree of stability. More importantly, dynamic simulations are extremely time consuming in terms of CPU and engineering resources required for the computation and analysis of the several

differential and algebraic equations needed for quantification of the phenomenon. However, such simulations do not readily provide sensitivity information or the degree of stability. For such application, the steady state analysis approach is much more suitable and can provide much insight into the voltage and reactive power loads problem.

In most cases, the system dynamics affecting voltage stability are usually quite slow and much of the problem can be effectively analyzed using the static approaches that examine the viability of a specific operating point of the power system. Static analysis methods, in addition to providing information such as sensitivity or degree of stability, involve the computation of only algebraic equations and are much more efficient and faster than dynamic approaches. The static analysis approach is more attractive than the dynamic method and well suited to voltage stability analysis of power systems over a wide range of system conditions.

In this paper the attention is given to SVD technique which provides both, relative proximity and key contributing factors to voltage insecurity. The singular value decomposition (SVD) is an important factorization of a two-dimension matrix which has a huge number of applications. Tiranuchit and Thomas (1988) proposed in [1] has used the minimum singular value of the Jacobian matrix of an electrical power grid as a voltage security index to monitor how close a power system is operating to a voltage instability. Ekwue, Wan, and et al. (1999) have applied the SVD method of voltage stability analysis to calculate the

voltage stability index, to identify the weakest buses and areas, and ranking transmission lines. [2]. Li (2000) in [3] has utilized the SVD method to analyse electricity demand data in a power grid and have developed mathematical models and algorithms to forecast the electric demand using the SVD for the UK National Grid system. In Janik, Rezmer, and et al. (2009) presented in [4] an approach based on SVD to accurate estimate of harmonic current components in a power grid network which incorporates wind generation. Krause, Lehnhoff, and et al. (2009) studied the operational limitation problems in a power grid with widely dispersed renewable energy sources in order to avoid voltage band violations and line overloads [5]. It is also possible to identify the elements of the power system contributing the most towards incipient voltage instability (critical load buses, branches).

## II. VOLTAGE STABILITY

Voltage stability is a problem in power systems which are or due to shortage of reactive power because of any one of the reasons cited as follows : Due to heavily loaded or faulted lines. The nature of voltage stability problem is influenced by the way of generation, transmission and consumption of reactive power.

The main factors that contribute to the voltage instability in the system are

1. Stressed power system; i.e. high active power loading in the system.
2. Inadequate reactive power resources.
3. Load characteristics at low voltage magnitudes and their difference from those traditionally used in stability studies.
4. Transformers tap changer responding to decreasing voltage magnitudes at the load buses.
5. Unexpected and or unwanted relay operation may occur during conditions with decreased voltage magnitudes.

The problem of voltage stability affects the whole power system, although it usually has a large involvement in one or more critical area of the power system.

In this paper Reduced Jacobian Matrix is used in the SVD techniques along with the respective mathematical modeling to analyze the stability of the system. The IEEE 14 bus system has been used to demonstrate the application of SVD for voltage stability studies.

## III. METHODS OF VOLTAGE STABILITY ANALYSIS

Many algorithms have been proposed in the literature for voltage stability analysis. Most of the utilities have a tendency to depend regularly on conventional load flows for such analysis. Some of the proposed methods are concerned

with voltage instability analysis under small perturbations in system load parameters. The analysis of voltage stability, for planning and operation of a power system, involves the examination of two main aspects:

- (i) How close the system is to voltage instability (i.e. Proximity).
- (ii) When voltage instability occurs, the key contributing factors such as the weak buses, area involved in collapse and generators and lines participating in the collapse are of interest (i.e. Mechanism of voltage collapse).

Proximity can provide information regarding voltage security while the mechanism gives useful information for operating plans and system modifications that can be implemented to avoid the voltage collapse. Many techniques have been proposed in the literature for evaluating and predicting voltage stability using steady state analysis methods. Some of these techniques are P-V curves, Q-V curves, SVD, minimum singular value and sensitivity analysis, reactive power optimization, artificial neural networks, neuro-fuzzy networks, reduced Jacobian determinant, Energy function methods, thevenin and load impedance indicator and loading margin by multiple power-flow solutions.

### 1.1 Q-V curve

Q-V curve technique is a general method of evaluating voltage stability. It mainly presents the sensitivity and variation of bus voltages with respect to the reactive power injection. Q-V curves are used by many utilities for determining proximity to voltage collapse so that operators can make a good decision to avoid losing system stability. In other words, by using Q-V curves, it is possible for the operators and the planners to know the maximum reactive power that can be achieved or added to the weakest bus before reaching minimum voltage limit or voltage instability.

### 1.2 P-V curve

The P-V curves, active power-voltage curve, are the most widely used method of predicting voltage security. They are used to determine the MW distance from the operating point to the critical voltage.

### 1.3 Modal analysis

Modal analysis can predict voltage collapse in complex power system networks. It involves mainly the computing of the smallest eigenvalues and associated eigenvectors of the reduced Jacobian matrix obtained from the load flow solution. The eigenvalues are associated with a mode of voltage and reactive power variation, which can provide a relative measure of proximity to voltage instability. Then, the participation factor can be used effectively to find out the weakest nodes or buses in the system.

### 1.4 Singular value decomposition

The main idea of the method is to find "How close is the Jacobian matrix to being singular"? One issue with this index is that it does not indicate how far in MVARs, it is to the bifurcation point (singular Jacobian value). The more important use of the index is the relationship it provides for control. That is, if VAR compensation through capacitors, excitation control or other means is available, the index provides the answer to the problem of how to distribute the resource throughout the system for maximum benefit. A disadvantage of using the minimum singular value index is the large amount of CPU time required in performing singular value decomposition for a large matrix.

## IV. POWER FLOW SOLUTION

The power flow solution for the base case is obtained using Newton Raphson (N-R) method which has been proven to be the most reliable method to solve the power flow equation. By using N-R method:

$$\begin{bmatrix} \Delta P_{(i)} \\ \Delta Q_{(i)} \end{bmatrix} = J_{(i)} \begin{bmatrix} \Delta \theta_{(i)} \\ \Delta V_{(i)} \end{bmatrix}$$

where:

$$\begin{bmatrix} \Delta P_{(i)} \\ \Delta Q_{(i)} \end{bmatrix} = \begin{bmatrix} P - P_{(i)} \\ Q - Q_{(i)} \end{bmatrix}$$

i: is the number of iteration.

$\Delta P = P_{\text{specified}} - P_{\text{calculated}}$

$\Delta Q = Q_{\text{specified}} - Q_{\text{calculated}}$

Both  $\Delta P$  and  $\Delta Q$  specify the required tolerance and therefore called the mismatch power vectors.

$\Delta V$ : is the unknown voltage magnitude correction vector.

$\Delta \theta$ : is the unknown voltage angle correction vector.

J: is called the full Jacobian matrix.

$$J = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix}$$

where

$$J_{P\theta} = \frac{\partial P}{\partial \theta}, J_{PV} = \frac{\partial P}{\partial V}$$

$$J_{Q\theta} = \frac{\partial Q}{\partial \theta}, J_{QV} = \frac{\partial Q}{\partial V}$$

## V. REDUCED JACOBIAN MATRIX

It is practically known that when the real power P changes then both voltage magnitude V and angle  $\theta$  will change.

However, the voltage angle will change much more than the voltage magnitude and when the reactive power Q changes then also both voltage magnitude V and angle  $\theta$  will change. However, the voltage magnitude will change much more than the voltage angle. By utilizing this coupling between Q & V and P &  $\theta$  and as mentioned earlier that the shortage of the reactive power Q mainly causes the voltage instability in the power system, the reduced Jacobian can be obtained as follows:

Let  $\Delta P = 0$ , then

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

i.e.

$$\Delta \theta = -J_{P\theta}^{-1} J_{PV} \Delta V$$

$$\Delta Q = J_{Q\theta} \Delta \theta + J_{QV} \Delta V$$

Substituting Eq. (4) into Eq. (5) gives:

$$\Delta Q = (J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}) \Delta V = J_R \Delta V$$

where  $J_R$  is called the reduced Jacobian

$$J_R = (J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV})$$

From Eq. (1);

$$\Delta V = J_R^{-1} \Delta Q$$

### IV. Singular value decomposition technique

It has been applied to obtain decomposition of the Jacobian matrix. For a real square matrix A of n x n dimension, the singular value decomposition is given by:

$$A = USV^T = \sum_{i=1}^n \sigma_i u_i v_i^T$$

Where U and V are n x n orthogonal matrices,  $u_i$  and  $v_i$  are called the left and right singular vectors respectively, and S is a diagonal matrix with:

$$\Sigma(A) = \text{diag}[\sigma_i(A)] \quad i = 1, 2, 3, \dots, n$$

where  $\sigma_i \geq 0$  for all i. The diagonal elements in the matrix S are usually ordered so that  $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \dots \geq \sigma_n \geq 0$ . If the matrix A has rank r ( $r \leq n$ ), its singular values  $\sigma_1, \sigma_2, \sigma_3 \dots \sigma_r$  are the only nonzero entries in the n x n diagonal matrix S. To use the above theory for voltage stability analysis, the linear power-flow equation based on the Newton- Raphson method [6] has to be found which is established by the power-flow reduced Jacobian matrix  $J_R$ .

Now, if the singular value decomposition is applied to  $J_R$ , one has



$$J_R = USV^T = \sum_{i=1}^n \sigma_i u_i v_i^T$$

The minimum singular value  $\sigma_n$  (JR) is a measure of how close to singularity the Jacobian is. From equation (3) in the case of a small disturbance point of view, let

$$\begin{bmatrix} \Delta P = 0 \\ \Delta Q \end{bmatrix} = u_n$$

where  $u_n$  is the last column of U, and

$$\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = \sigma_n^{-1} v_n$$

where  $v_n$  is the last column of V. From the above equations it can be concluded that for the minimum singular value and corresponding left and right singular vectors, the following apply:

- The smallest singular value  $\sigma_n$  is an indicator of the proximity to the steady state stability limit.
- The right singular vector,  $v_n$  corresponding to  $\sigma_n$  indicates sensitive voltage.

The left singular vector,  $u_n$  corresponding to  $\sigma_n$  indicates the most sensitive direction for changes of active and reactive power injections.

### VI. EVALUATING SVD ANALYSIS

From the Singular value decomposition of the IEEE 14 test bus system, the following inferences are arrived at. As fore said, the voltage stability analysis is carried out to first determine the proximity to voltage instability or to determine the Voltage Stability Margin (VSM). From the SVD analysis carried out, VSM was arrived from the minimum singular value which was 2.6526.

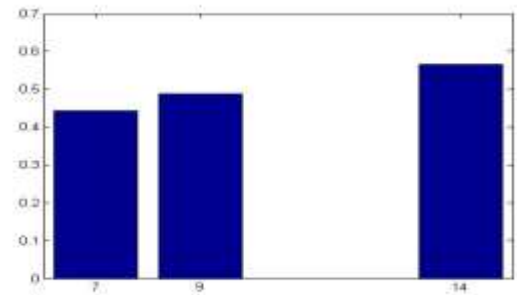
To address the second issue of identifying the weak buses, the Right vectors of the minimum singular values were used. The three least singular values are listed below in Table I .Table II and Table III, gives Singular values ,their corresponding Singular vectors and the bus numbers which are associated with the singular values.

**Table 1** Three lowest singular values

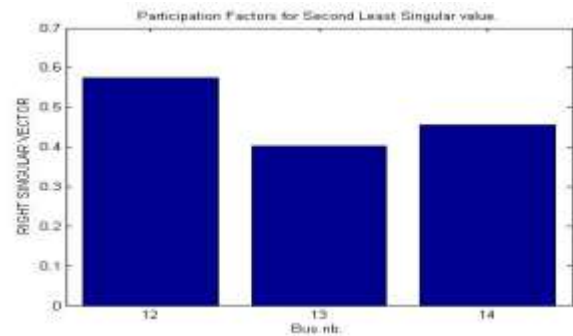
MINIMUM SINGULAR VALUES
2.6526
5.4299
7.4662

**Table 2** First least singular value

FIRST LEAST SINGULAR VALUE – 2.6526	
RIGHT VECTOR	BUS NUMBER
0.5658	14
0.4885	9
0.440	7



**Fig. 1.** Right vector and bus number for singular value – 2.6526



**Fig. 2.** Right vector and bus number for singular value – 5.4299.

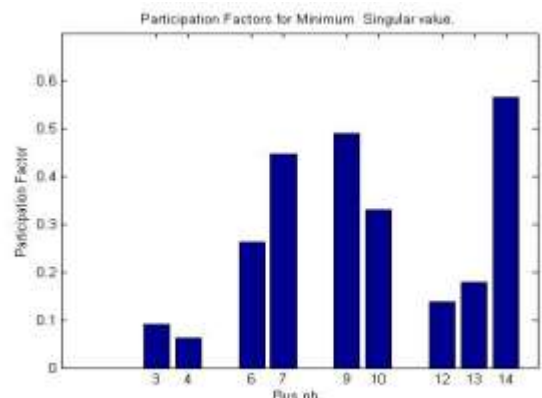
**Table 3** Second least singular value

SECOND LEAST SINGULAR VALUE – 5.4299	
RIGHT VECTOR	BUS NUMBER
0.5735	12
0.4548	14
0.4012	13

**Table 4** Participation factor of the buses at the least singular value

Bus No	Participation Factor
3	0.0893
4	0.637
6	0.2629
7	0.4440
9	0.4881
10	0.3335
12	0.1383
13	0.1878
14	0.5658

Of all the participation factors, participation factor of bus 14 is observed to be more and can be seen clearly from the following graph.



**Fig. 3.** Participation factors for IEEE 14bus system.

## VII. CONCLUSION

The paper re-examines the problem of voltage instability in power systems and the need for effective analysis techniques to detect incipient cases of instability and prevent possible escalation into wide-spread voltage collapse. Common steady-state analysis methods were reviewed including SVD. The SVD feature was tested on the IEEE 14-bus test system. The technique was demonstrated as an extremely useful method for steady state voltage stability analysis, providing a relative measure of the system to the stability limit, and correctly predicting the critical buses in the power network. The method can be incorporated into user-friendly diagnostic tools which can be applied to practical power systems and utilized by power system operators and planners to assess voltage stability at various operating conditions, as well as determine suitable remedial actions for instability and collapse.

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