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Non – Markovian Queue with Multi vacation Policies

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Abstract: **This article characterizes a lining framework wherein the administration is given in a solitary stage. Clients show up in bunches follows the FCFS discipline. Customers appearance follows a Poisson dispersion. Administration follows a general conveyance. After the culmination of the administration, if there are no clients in the framework, server goes for a long get-away with a likelihood p in any case server remains in the framework with likelihood 1-p.Here long excursion is given in two phases. After the fulfillment of the long excursion of stage2, it gets into a discretionary short get-away. Next, administration proceeds once more. This covering issue is analyzed through a birth passing system of Queuing speculation and it's clarified by one among the coating issue procedure known to be gainful variable technique. The model is all around explained very well by techniques for sensible application. The model is all around supported by methods for numerical depiction and graphical technique.**

Keywords – Non – Markovian, Queue, Policies.

I. INTRODUCTION

Queuing models have been studied by many authors.Madan, K.C. [1] inferred the exhibition measures on a M[x]/G/1 line with third stage discretionary help and deterministic server.2.Madan, K.C. what's more, Abu-Dayyeh, A.Z[2] gave a nitty gritty work on a solitary server line with discretionary stage type server excursions dependent on thorough deterministic assistance and a solitary get-away .Maragathasundari. S and Srinivasan. .S[3] decided the discretionary administrations in a Non-Markovian Queue.Maragathasundari. S and Miriam cathy joy[4] examined the Queueing model of discretionary sort Of administrations with administration stoppage and redo process in web facilitating. Non-Markovian clump appearance queueing model with multi phases of administration of confined tolerability was very much broke down by Maragathasundari. S and Srinivasan.S[5]. Maragathasundari. S [6] recommended a Queuing Scheme with Feedback Service and Discretionary Administration .Maragathasundari. S [7] examined a Queuing Analysis on Multiple Vacation Policies and Reneging.Rajadurai, P. et.al [8] made a Cost advancement investigation of retrial line with K discretionary Sathiya, K. what's more, Ayyappan, G. [9] considered a Non-Markovian Batch Arrival Queue with administration interference and broadened server vacation.S. Vignesh and S. Maragathasundari [10] made an examination report on non markovian single server clump appearance lining arrangement of mandatory three phases of administrations with fourth discretionary stage administration.

II. ASSUMPTIONS AND NOTATIONS OF THE REPRESENTATION

Here we consider the arrival rate $\lambda > 0$ to follow a Poisson distribution .All the other parameters follows a general distribution.

Service

Let $F_1^*(x)$and $f_1^*(x)$ be the distribution function and the density function of phase service.

Let $\mu_q(x)$ be the conditional probability of a completion of service and it is given by,

$$\mu_q(x) = \frac{f(x)}{1-F_1^*(x)} \quad , \; f_1^*(x) = \mu_q(x)e^{-\int_0^x \mu_q(t)dt}$$

stage 1 and stage 2 of Long vacation, short vacation

Similarly, the process is repeated for the other parameters stage 1, stage 2 of long vacation and short vacation hence we have the following:

Major maintenance work of the server happens during the stage of Long vacation and short vacation.

In a case of stage 1 long vacation,we have, $\mu_{l_1}(x) = \frac{f_2^*(x)}{1-F_2^*(x)}$ and $f_2^*(x) = \mu_{l_1}(x)e^{-\int_0^x \mu_{l_1}(t)dt}$

For the stage 2 of long vacation we have, $\mu_{l_2}(x) = \frac{f_3^*(x)}{1-F_3^*(x)}$ and $f_3^*(x) = \mu_{l_2}(x)e^{-\int_0^x \mu_{l_2}(t)dt}$

For the Short Vacation $\mu_s(x) = \frac{f_4^*(x)}{1-F_4^*(x)}$ $f_4^*(x) = \mu_s(x)e^{-\int_0^x \mu_s(t)dt}$

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III. GOVERNING EQUATIONS OF THE MODEL

According to the assumptions mentioned in the previous section,the following set of equations represent the queuing system we study in this article

$$\frac{\partial}{\partial x}Q_n(x) + \left(\lambda_w + \mu_q(x)\right)Q_n(x) = \lambda_w \sum_{p=1}^{n} v_p\, Q_{n-p}(x) \tag{1}$$

$$\frac{\partial}{\partial x}Q_0(x) + \left(\lambda_w + \mu_q(x)\right)Q_0(x) = 0 \tag{2}$$

$$\frac{\partial}{\partial x}Q_{l_n}{}^{(1)}(x) + \left(\lambda_w + \mu_{l_1}(x)\right)Q_{l_n}{}^{(1)}(x) = \lambda_w \sum_{p=1}^{n} v_p\, Q_{l_{n-p}}{}^{(1)}(x) \tag{3}$$

$$\frac{\partial}{\partial x}Q_{l_0}{}^{(1)}(x) + \left(\lambda_w + \mu_{l_1}(x)\right)Q_{l_0}{}^{(1)}(x) = 0 \tag{4}$$

$$\frac{\partial}{\partial x}Q_{l_n}{}^{(2)}(x) + \left(\lambda_w + \mu_{l_2}(x)\right)Q^{(2)}{}_{l_n}(x) = \lambda_w \sum_{p=1}^{n} v_p\, Q_{l_{n-p}}{}^{(2)}(x) \tag{5}$$

$$\frac{\partial}{\partial x}Q_{l_0}{}^{(2)}(x) + \left(\lambda_w + \mu_{l_2}(x)\right)Q_{l_0}{}^{(2)}(x) = 0 \tag{6}$$

$$\frac{\partial}{\partial x}Q_{s_n}(x) + (\lambda_w + \mu_s(x) + \delta)Q_{s_n}(x) = \lambda_w \sum_{p=1}^{n} v_p\, Q_{s_{n-p}}(x) + \delta Q_{s_{n+1}}(x) \tag{7}$$

$$\frac{\partial}{\partial x}Q_{s_0}(x) + (\lambda_w + \mu_s(x) + \delta)Q_{s_0}(x) = \delta Q_{s_1}(x) \tag{8}$$

$$\lambda_w Y = (1-p)\int_0^\infty Q_{l_0}{}^{(2)}(x)\mu_{l_2}(x)dx + (1-p)\int_0^\infty Q_0(x)\mu_q(x)dx + \int_0^\infty Q_{s_0}(x)\mu_s(x)dx \tag{9}$$

IV. INITIAL AND BOUNDARY CONDITIONS

$$Q_n(0) = (1-p)\int_0^\infty Q_{l_{n+1}}{}^{(2)}(x)\mu_{l_2}(x)dx + (1-p)\int_0^\infty Q_{n+1}(x)\mu_q(x)dx + \int_0^\infty Q_{s_{n+1}}(x)\mu_s(x)dx + \lambda_w U_{n+1} Y \tag{10}$$

$$Q_{l_n}{}^{(1)}(0) = p\int_0^\infty Q_n(x)\,\mu_q(x)dx \tag{11}$$

$$Q_{l_n}{}^{(2)}(0) = \int_0^\infty Q_{l_n}{}^{(1)}(x)\mu_{l_1}(x)dx \tag{12}$$

$$Q_{s_n}(0) = \varphi\int_0^\infty Q_{l_n}{}^{(2)}(x)\mu_{l_2}(x)dx \tag{13}$$

V. SUPPLEMENTARY VARIABLE TECHNIQUE

Apply the supplementary variable technique to the above governing equations.

Multiply (1) by z^n summing over n from 1 to ∞ and adding to (2), we obtain

$$\frac{\partial}{\partial x}Q_q(x,z) + \left(\lambda_w - \lambda_w U(z) + \mu_q(x)\right)Q_q(x,z) = 0 \tag{14}$$

$$\frac{\partial}{\partial x}Q_{l_q}{}^{(1)}(x,z) + \left(\lambda_w - \lambda_w U(z) + \mu_{l_1}(x)\right)Q_{l_q}{}^{(1)}(x,z) = 0 \tag{15}$$

$$\frac{\partial}{\partial x}Q_{l_q}{}^{(2)}(x,z) + \left(\lambda_w - \lambda_w U(z) + \mu_{l_1}(x)\right)Q_{l_q}{}^{(1)}(x,z) = 0 \tag{16}$$

$$\frac{\partial}{\partial x}Q_{s_q}(x,z) + \left(\lambda_w - \lambda_w U(z) + \mu_s(x) + \delta - \frac{\delta}{2}\right)Q_{s_q}(x,z) = 0 \tag{17}$$

Same procedure is applied for initial and boundary conditions

$$zQ_q(0,z) = (1-p)\int_0^\infty Q_{l_q}{}^{(2)}(x,z)\mu_{l_2}(x)dx + (1-p)\int_0^\infty Q_q(x,z)\mu_q(x)dx + \int_0^\infty Q_{s_q}(x)\mu_s(x)dx + \lambda_w Y(U(z)-1) \tag{18}$$

$$Q_{l_q}{}^{(1)}(0,z) = p\int_0^\infty Q_q(x,z)\,\mu_q(x)dx \tag{19}$$

$$Q_{l_q}{}^{(2)}(0,z) = \int_0^\infty Q_{l_q}{}^{(1)}(x,z)\mu_{l_1}(x)dx \tag{20}$$

$$Q_{s_q}(0,z) = \varphi\int_0^\infty Q_{l_q}{}^{(2)}(x,z)\mu_{l_2}(x)dx \tag{21}$$

Integrating (14) from 0 to x yields

$$Q_q(x,z) = Q_q(x,z)e^{-(\lambda_w - \lambda_w U(z))x - \int_0^\infty \mu_q(t)dt} \tag{22}$$

Integrating (22) dx by parts, we get

$$Q_q(x) = Q_q(0,z)\left[\frac{1-\bar{F}_1(a)}{a}\right],\quad a = \lambda_w - \lambda_w U(z) \tag{23}$$

Where $\bar{F}_1(a) = \int_0^\infty e^{-(\lambda_w - \lambda_w U(z))x}\,dF_1(x)$ is the Laplace Stieltjes transform of the service time $F_1(x)$

Multiply (22) in both sides by $\mu_q(x)$ and integrating, we get

$$\int_0^\infty Q_q(x,z)\mu_q(x)\,dx = Q_q(0,z)\bar{F}_1(a) \tag{24}$$

Similarly for long vacation and short vacation the same process is repeated. Hence we have the following:

$$Q_{l_q}{}^{(1)}(z) = pQ_q(0,z)\bar{F}_1(a)\left[\frac{1-\bar{F}_2(a)}{a}\right] \tag{25}$$

$$\int_0^\infty Q_{l_q}{}^{(1)}(x,z)\mu_l{}^{(1)}(x)dx = pQ_q(0,z)\bar{F}_1(a)\,\bar{F}_2(a) \tag{26}$$

$$Q_{l_q}{}^{(2)}(z) = pQ_q(0,z)\bar{F}_1(a)\bar{F}_2(a)\left[\frac{1-\bar{F}_3(a)}{a}\right] \tag{27}$$

$$\int_0^\infty Q_{l_q}{}^{(2)}(x,z)\mu_l{}^{(2)}(x)dx = pQ_q(0,z)\bar{F}_1(a)\,\bar{F}_2(a)\bar{F}_3(a) \tag{28}$$

$$Q_{s_q}(z) = \varphi pQ_q(0,z)\bar{F}_1(a)\,\bar{F}_2(a)\bar{F}_3(a)\left[\frac{1-\bar{F}_4(b)}{b}\right],\quad b = \lambda_w - \lambda_w U(z) + \delta - \frac{\delta}{2} \tag{29}$$

$$\int_0^\infty Q_{s_q}(x,z)\mu_s(x)dx = \varphi pQ_q(0,z)\bar{F}_1(a)\,\bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b) \tag{30}$$

Substituting (24), (28) and (30) in (18) we get

$$Q_q(0,z) = \frac{\lambda_w Y(U(x)-1)}{z-(1-\varphi)p\bar{F}_1(a)\,\bar{F}_2(a)\bar{F}_3(a)-(1-p)\bar{F}_1(a)-\varphi p\bar{F}_1(a)\,\bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b)} \tag{31}$$

Using (31) in (23), (25), (27) and (29), we have

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$$Q_q(z) = [z - (1-\varphi)p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a) - (1-p)\bar{F}_1(a) - \varphi p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b)][\tfrac{1-\bar{F}_1(a)}{a}] \quad (32)$$

$$Q_{l_q}{}^{(1)}(z) = p[z - (1-\varphi)p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a) - (1-p)\bar{F}_1(a) - \varphi p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b)]\bar{F}_1(a)\ [\tfrac{1-\bar{F}_2(a)}{a}] \quad (33)$$

$$Q_{l_q}{}^{(2)}(z) = p[z - (1-\varphi)p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a) - (1-p)\bar{F}_1(a) - \varphi p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b)]\bar{F}_1(a)\bar{F}_2(a)\ [\tfrac{1-\bar{F}_3(a)}{a}] \quad (34)$$

$$Q_{s_q}(z) = \varphi p[z - (1-\varphi)p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a) - (1-p)\bar{F}_1(a) - \varphi p\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a)\bar{F}_4(b)]\bar{F}_1(a)\ \bar{F}_2(a)\bar{F}_3(a)\,[\tfrac{1-\bar{F}_4(b)}{b}] \quad (35)$$

VI. PROBABILITY GENERATING FUNCTION

Let $M_q(z)$ be the probability generating function of the queue size

$$M_q(z) = Q_q(z) + Q_{l_q}{}^{(1)}(z) + Q_{l_q}{}^{(2)}(z) + Q_{s_q}(z)$$

$$= \frac{\lambda_w Y(U(x)-1)\{\left[\frac{1-\bar{F}_1(a)}{a}\right]+\bar{F}_1(a)\left[\frac{1-\bar{F}_2(a)}{a}\right]+p\bar{F}_1(a)\bar{F}_2(a)\left[\frac{1-\bar{F}_3(a)}{a}\right]+\varphi p\bar{F}_1(a)\bar{F}_2(a)\bar{F}_3(a)[\frac{1-\bar{F}_4(b)}{b}]\}}{z-\bar{F}_1(a)\bar{F}_2(a)\bar{F}_3(a)p[(1-\varphi)+\varphi\bar{F}_4(b)]-(1-p)\bar{F}_1(a)} \quad (36)$$

VII. IDLE TIME AND UTILIZATION FACTOR

Using Normalization Condition $L_q(1) + Y = 1$ gives the idle time Y (37)

At z=1 $M_q(z) = \frac{0}{0}$ (Indeterminate form) Applying L' Hopital's rule to (36), we get

$$\lim_{z\to 1} M_q(z) = \frac{N''(1)}{D''(1)} \quad (38)(a)$$

Where $Y(1) = 1,\ Y'(1) = E(I)$ is the average size of the batches of the arriving customers

$\bar{F}_1(0) = 1, \bar{F}_1{}'(0) = \lambda_w E(I)E(F_1), \bar{F}_2{}'(0) = \lambda_w E(I)\,E(F_2), \bar{F}_3{}'(0) = \lambda_w E(I)E(F_3), \bar{F}_4{}'(0) = \lambda_w E(I)\,E(F_4)$

Adding Y to (38), we get, $Y = \frac{D''(1)}{N''(1)+D''(1)}$ (38)(b)

The utilization factor ρ is determined form, $\rho = 1 - Y$.

VIII. MEAN LENGTH OF THE QUEUE AND THE MEAN WAITING TIME

Consider $M_q(z) = \frac{N(z)}{D(z)}$ where N(z) and D(z) are Numerator and Denominator of Eq. 36

Hence the average length of the queue $L_q = \frac{d}{dz} M_q(z)\,|_{z=1}$ (39)

$$L_q = \lim_{z\to 1} \frac{D''(z)N'''(1) - D'''(1)N''(1)}{2(D''(z))^2}$$

$$N''(1) = 2\lambda_w[E(F_1) + E(F_2) + pE(F_3) + \varphi pE(F_4)] \quad (40)$$

$$N'''(1) = 2\lambda_w\Big[\lambda_w E(F_1)^2 + \lambda_w E(F_2)E(F_1) + \lambda_w[E(F_1)E(F_2) + E(F_2)^2] + p\lambda_w E(F_3)\big(E(F_1) + E(F_2)\big) + p\lambda_w\{E(F_3)[E(F_1) + E(F_2)] + E(F_3)^2\} + \varphi p\lambda_w E(F_4)[E(F_1) + E(F_2) + E(F_3)] + \varphi p\lambda_w[E(F_4)[E(F_1) + E(F_2) + E(F_3)] + E(F_4)^2]\Big] \quad (41)$$

$$D''(1) = p\varphi\lambda_w E(F_4)(-\lambda_w + \delta)[E(F_1) + E(F_2) + E(F_3)] - p\lambda_w{}^2\big[E(F_1)^2 + 2\big(E(F_1)E(F_2) + E(F_1)E(F_3) + E(F_3)E(F_2)\big) + E(F_2)^2 + E(F_3)^2\big] + p\lambda_w\varphi E(F_4)(-\lambda_w + \delta)[E(F_1) + E(F_2) + E(F_3)] - p[2\delta\varphi E(F_4) + \varphi(-\lambda_w + \delta)^2E(F_4)^2] \quad (42)$$

$$D'''(1) = -p\lambda_w\varphi\{[-2\delta E(F_4) + (-\lambda_w + \delta)^2E(F_4)^2][E(F_1) + E(F_2) + E(F_3)] + (-\lambda_w + \delta)E(F_4)(-\lambda_w)[E(F_1)^2 + E(F_2) + E(F_3)^2 + 2E(F_1)E(F_2) + 2E(F_1)E(F_3) + 2E(F_3)E(F_2)]\} - p\big[\varphi(-\lambda_w + \delta)\big(-E(F_4)\big)\big](\lambda_w)^2[E(F_1)^2 + E(F_2)^2 + E(F_3)^2 + 2E(F_1)E(F_2) + 2E(F_1)E(F_3) + 2E(F_3)E(F_2)] - p(\lambda_w)^2\big[\lambda_w\big[E(F_1)^2[E(F_2) + E(F_3)] + E(F_2)^2[E(F_1) + E(F_3)] + E(F_3)^2[E(F_1) + E(F_2)] + 2\lambda_w[E(F_1)^2E(F_2) + E(F_2)^2E(F_1) + 3E(F_1)E(F_2)E(F_3) + E(F_1)^2E(F_3)]\big] + p\lambda_w{}^2[E(F_1)^2 + E(F_2)^2 + E(F_3)^2 + 2E(F_1)E(F_2) + 2E(F_1)E(F_3) + 2E(F_3)E(F_2)]\varphi E(F_4)(-\lambda_w + \delta)\big] - 2\varphi\delta E(F_4)p\lambda_w[E(F_1) + E(F_2) + E(F_3)] + \varphi E(F_4)^2(-\lambda_w + \delta)^2p\lambda_w[E(F_1) + E(F_2) + E(F_3)] + p\lambda_w[E(F_1) + E(F_2) + E(F_3)][2\delta\varphi E(F_4) + \varphi(-\lambda_w + \delta)^2E(F_4)^2] - p[-6\delta\varphi E(F_4) - 2\delta\varphi(-\lambda_w + \delta)E(F_4)^2 + \varphi E(F_4)^2(2\delta)^2] \quad (43)$$

Substituting eq.40-43 in equation 39, we get Lq in closed form.Other execution measures can be obtained from Little's formula

IX. NUMERICAL ILLUSTRATION

The model is well justified by means of numerical illustration. Consider the following:

$p = 0.6, \varphi = 0.8, \lambda_w = 3, \mu_q = 3, \mu_{l1} = 2.8, \mu_{l2} = 3.4, \mu_s = 4.2, \delta = 0.7, E(F_1) = \frac{1}{\mu_q},$

$E(F_2) = \frac{1}{\mu_{l1}}, E(F_3) = \frac{1}{\mu_{l2}}, E(F_4) = \frac{1}{\mu_s}, E(F_1)^2 = \frac{2}{\mu_q{}^2}, E(F_2)^2 = \frac{2}{\mu_{l1}{}^2}, E(F_3)^2 = \frac{2}{\mu_{l2}{}^2},$

$$E(F_4)^2 = \frac{2}{\mu_s{}^2}$$

Table1: The effect of change of Long vacation

p	Q	ρ	L_q	L	W_q	W
1	0.6119	0.3880	1.3033	1.6913	0.4344	0.5638
1.2	0.6287	0.3713	1.0833	1.4546	0.3611	0.4849
1.4	0.6412	0.3588	0.8971	1.2559	0.2990	0.4186
1.6	0.6509	0.3491	0.7320	1.0811	0.2440	0.3604
1.8	0.6586	0.3414	0.5810	0.9224	0.1937	0.3075

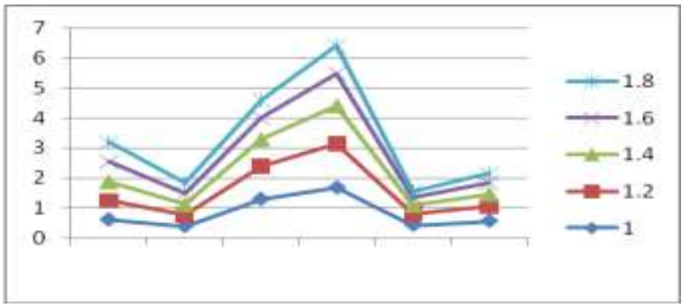

Table 2: The effect of change of short vacation

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φ	Q	ρ	L_q	L	W_q	W
2	0.5875	0.4125	1.2298	1.6423	0.4099	0.5474
2.5	0.5968	0.4032	1.0720	1.4752	0.3573	0.4917
3	0.6044	0.3956	0.9561	1.3517	0.3187	0.4506
3.5	0.6107	0.3893	0.8677	1.2570	0.2892	0.4190
4	0.6151	0.3849	0.7998	1.1847	0.2666	0.3949

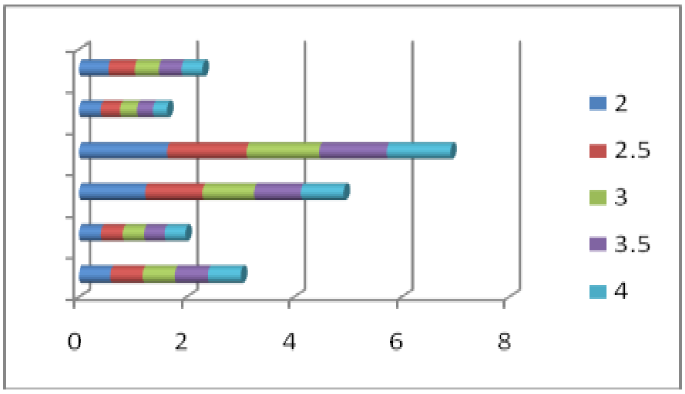
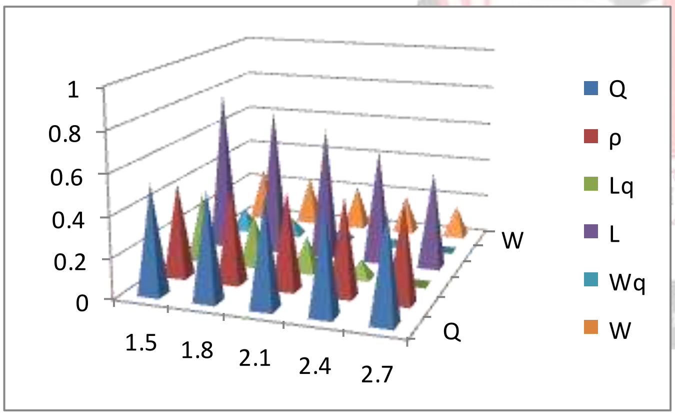


TABLE 3: THE EFFECT OF CHANGE OF STAND BY SERVER

δ	Q	ρ	L_q	L	W_q	W
1.5	0.5350	0.4649	0.3409	0.8058	0.1136	0.2686
1.8	0.5289	0.4711	0.2629	0.7341	0.0877	0.2447
2.1	0.5232	0.4768	0.1789	0.6557	0.0596	0.2185
2.4	0.5179	0.4821	0.0894	0.5715	0.0298	0.1905
2.7	0.5132	0.4868	0.0046	0.4822	0.0015	0.1607



X. ANALYSIS REPORT

Table 1 indicates the way that as the probability of taking long excursion increments, maximal upkeep work is completed during that time. Subsequently it decreases all the exhibition proportions of the lining model. In Table 2, increment in likelihood of taking short excursion builds the inert time and lessens the usage factor. Likewise, reserve server likelihood diminishes the length of the line and the various execution measures. Because of much time spend by the server for the administration, server inert time gets lessened. All the outcomes are true to form.

XI. CONCLUSION

In the above lining framework, we have considered a lining model comprising of single phase of administration, long get-away of two phases and short vacation. The idea of vacation acquainted here assists with run the framework easily with no administration interference as much upkeep work is completed during the hour of get-away. The model is all around legitimized by methods for numerical representation and graphical picture. Such sort of models are all around used in PC systems administration, creation and assembling enterprises.

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