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Special Classes of Coprime Irregular Graphs

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Abstract: **An *k-edge-weighting* of a graph *G = (V, E)* is a mapping φ: E(G) → {*1, 2, 3, ...k*}, where *k* is a positive integer. The sum of the edge-weighting appearing on the edges incident at the vertex *v* under the edge-weighting φ and is denoted by S_φ(v). An *k* edge-weighting of *G* is a *coprime irregular edge-weighting* if *gcd* (S_φ(u), S_φ(v)) = 1 for every pair of adjacent vertices *u* and *v* in G. A graph *G* admits a coprime irregular edge-weighting is called a *coprime irregular graph*. In this paper, we discuss the coprime irregular edge-weighting for some special classes of graphs.**

Keywords —Irregular edge weighting, friendship graph, Dutch windmill graph.

I. Introduction

By a graph *G = (V,E)*, we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak [3].

A graph labeling is an assignment of integers to the vertices or edges or both with respect to some conditions. A detailed survey of graph labeling is given by Gallian [4]. Basically the class of edge-weighting problems can be divided into two parts. The first one is the proper edge-weighting of a graph G, that is, a weighting of G in which no two incident edges receive the same labels and the other one is the non-proper edge-weighting in which the incident edges may receive same labels. For detailed study of this kind of edge-weightings, one may refer [1], [2], [5]. With respect to the non-proper edge-weighting, I. Sahul Hamid et.al[6] introduced the concept of equitable irregular edge-weighting of graphs and in which the authors discussed some properties of equitable irregular graphs. Motivated by the definition of equitable irregular graphs, S. Saravanakumar introduced the concept of coprime irregular graphs in [7] and found some coprime irregular graphs. Continuation of this work, in this paper, we prove that some special classes of graphs such as friendship graphs, Dutch windmill graphs, double quadrilateral snakes and pan graphs are coprime irregular.

II. Coprime Irregular Graphs

This section aims to prove that some special classes of graphs such as Friendship graphs, Dutch windmill graphs, Double quadrilateral graphs and Pan graphs are coprime irregular.

Definition 2.1. The *friendship graph* F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Theorem 2.2. The friendship graph F_n(n ≥ 3) is coprime irregular graph for all n.

Proof. Let V (F_n) = {$v_0, v_1, v_2, ..., v_n, u_1, u_2, ..., u_n$} and E($F_n$) = {$v_0 v_i$, $v_0 u_i$, $v_i u_i$: 1 ≤ i ≤ n}. We prove this theorem in the following cases.

Case (i): *n* is odd

For all i = 1, 2, 3, …, n, let $\varphi(v_0 v_i) = \frac{n-1}{2}$, $\varphi(v_0 u_i) = \frac{n+1}{2}$ and $\varphi(v_i u_i) = \frac{n+3}{2}$. Then $S_\varphi(v_0) = n^2$, $S_\varphi(v_i)$ = n + 1 and $S_\varphi(u_i)$ = n + 2 for all n. Clearly, the weights of any two adjacent vertices of F_n is relatively prime and so F_n is coprime irregular (For instance, the graph F_3 and its coprime irregular edge weighting φ is given in Figure 1.). This proves Case (i).

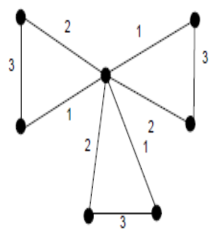

Figure 1: The graph F_3 and its coprime irregular edge-weighting under ϕ

Case (ii): *n* is even

For all i = 1, 2, 3, …, n, let $\varphi(v_0 v_i) = \frac{n-2}{2}$, $\varphi(v_0 u_i) = \frac{n+2}{2}$ and $\varphi(v_i u_i) = \frac{n}{2}$. Then $S_\varphi(v_0) = n^2$, $S_\varphi(v_i)$ = n - 1 and $S_\varphi(u_i)$ = n + 1 for all n. It is easy to verify that the weight of any two adjacent vertices of F_n are relatively prime (The graph F_4 and its coprime irregular edge weighting φ is illustrated in Figure 2.)

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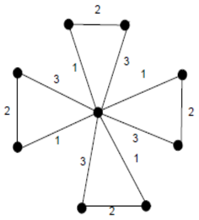

Figure 2: The graph F_4 and its coprime irregular edge-weighting under ϕ

Definition 2.3. A *double quadrilateral snake* DQ_n is a graph consists of two quadrilateral snakes that have a common path.

Theorem 2.4. For all $n \geq 3$, the graph double quadrilateral snake DQ_n is coprime irregular.

Proof. Let $V(DQ_n) = \{v_1, v_2, \ldots, v_{n+1}\} \cup \{u_i, w_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(DQ_n) = \{v_iu_i, u_iw_i, v_{i+1}w_i, v_ix_i, x_iy_i, v_{i+1}y_i : 1 \leq i \leq n\}$. Define an edge weighting φ of DQ_n as follows.

$$\varphi(v_iv_{i+1}) = \begin{cases} 2, & \text{if } i \equiv 1 \pmod 3 \\ 1, & \text{if } i \equiv 2 \pmod 3 \\ 4, & \text{other wise} \end{cases}$$

and $\varphi(v_iu_i) = \varphi(v_ix_i) = 1$ and $\varphi(u_iw_i) = \varphi(x_iy_i) = 2$ and $\varphi(v_{i+1}w_i) = 3$ for all $i = 1, 2, \ldots, n-1$. Further, assign $\varphi(v_{i+1}y_i) = 3$ for all $i = 1, 2, 3\ldots, n-2$ and for $i = n-1$, assign

$$\varphi(v_{i+1}y_i) = \begin{cases} 3, & \text{if } i \equiv 2 \pmod 3 \\ 5, & \text{if } i \equiv 0 \pmod 3 \\ 6, & \text{if } i \equiv 1 \pmod 3 \end{cases}$$

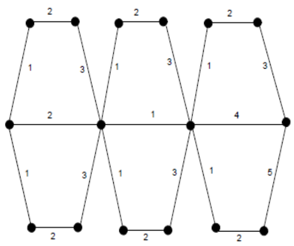

Figure 3: The graph DQ_4 and its coprime irregular edge-weighting under ϕ

Clearly, $S_\varphi(v_1) = 4$ and then for all $i = 2, 3\ldots, n-1$, we have

$$S_\varphi(v_i) = \begin{cases} 11, & \text{if } i \equiv 2 \pmod 3 \\ 13, & \text{if } i \equiv 0 \pmod 3 \\ 14, & \text{if } i \equiv 1 \pmod 3 \end{cases}$$

and

$$S_\varphi(v_n) = \begin{cases} 7, & \text{if } i \equiv 0 \pmod 3 \\ 12, & \text{if } i \equiv 1 \pmod 3 \\ 11, & \text{if } i \equiv 2 \pmod 3 \end{cases}$$

Also, for all $i = 1, 2, 3\ldots, n-1$, we have $S_\varphi(u_i) = S_\varphi(x_i) = 3$ and for all $i = 1, 2, 3\ldots, n-2$, we get $S_\varphi(y_i) = 5$. Further, for $i = n-1$,

$$S_\varphi(y_i) = \begin{cases} 5, & \text{if } i \equiv 2 \pmod 3 \\ 7, & \text{if } i \equiv 0 \pmod 3 \\ 8, & \text{if } i \equiv 1 \pmod 3 \end{cases}$$

Clearly, the weights of any two adjacent vertices of DQ_n are relatively prime. Therefore DQ_n is coprime irregular graph [The graph DQ_4 and the coprime irregular edge weighting φ is illustrated in Figure 3].

Definition 2.5. The *Dutch windmill graph* $D_n^{(m)}$ can be constructed by joining m copies of the cycle graph C_4 with a common vertex

Theorem 2.6. The Dutch windmill graph $D_n^{(m)}$ $(m \geq 3)$ is coprime irregular for all m.

Proof. Let $V(D_n^{(m)}) = \{v_0, v_i, u_i, w_i : 1 \leq i \leq m\}$ and $E(D_n^{(m)}) = \{v_0v_i, v_iu_i, u_iw_i, v_0w_i; 1 \leq i \leq m\}$. Now, let us define an edge weighting φ of $D_n^{(m)}$ as follows. We prove this theorem in the following two cases.

Case(i): *m* is odd

For all $i = 1, 2, \ldots, m$, define $\varphi(v_0v_i) = \frac{m-1}{2}$, $\varphi(v_iu_i) = \frac{m+3}{2}$ and $\varphi(u_iw_i) = \varphi(v_0w_i) = \frac{m+1}{2}$.

Then $S_\varphi(v_0) = m^2$, $S_\varphi(u_i) = m+2$, $S_\varphi(v_i) = S_\varphi(w_i) = m+1$. Thus any two adjacent vertices of $D_n^{(m)}$ whose weights are coprime and so φ is a coprime irregular edge-weighting of $D_n^{(m)}$. Hence the Case(i) is follows.

[The graph $D_4^{(3)}$ and its coprime irregular edge weighting φ is illustrated in Figure 4].

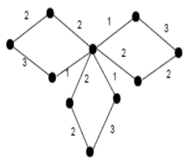

Figure 4: The graph $D_4^{(3)}$ and its coprime irregular edge-weighting under ϕ

Case(ii): *m* is even

For all $i = 1, 2, \ldots, m$, define $\varphi(v_0v_i) = \frac{m-2}{2}$, $\varphi(v_iu_i) = \varphi(u_iw_i) = \frac{m}{2}$ and $\varphi(v_0w_i) = \frac{m+1}{2}$. Then $S_\varphi(v_0) = m^2$, $S_\varphi(u_i) = m$, $S_\varphi(v_i) = m-1$, $S_\varphi(w_i) = m+1$. Clearly, any two adjacent vertices of $D_n^{(m)}$ whose weights are relatively prime and thus φ is coprime irregular edge weighting of $D_n^{(m)}$. This proves Case(ii). (The graph $D_4^{(4)}$ and its coprime irregular edge weighting φ is illustrated in Figure 5).

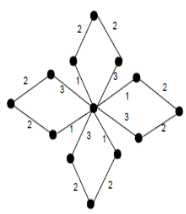

Figure 5: The graph $D_4^{(4)}$ and its coprime irregular edge-weighting under ϕ

Definition 2.7. The *pan graph* $(Pn)_n$ is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

Theorem 2.8. For all $n \geq 4$, the pan graph $(Pn)_n$ is coprime irregular.

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Proof. We prove the result in the following two cases. Let $V((Pn)_n) = \{v_0, v_1, v_2, ..., v_n\}$ and $E((Pn)_n) = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_1 v_n\} \cup \{v_1 v_0\}$. Consider the edge weighting φ of $((Pn)_n)$ define as follows.

Case(i): *n* is even

For all $1 \le i \le n-1$, let

$$\varphi(v_i v_{i+1}) = \begin{cases} 1, & \text{if } i \equiv 0 \pmod 4 \\ 2, & \text{if } i \equiv 1 \pmod 4 \\ 3, & \text{if } i \equiv 2 \pmod 4 \\ 4, & \text{if } i \equiv 3 \pmod 4 \end{cases}$$

and

$$\varphi(v_1 v_n) = \begin{cases} 1, & \text{if } n \equiv 0 \pmod 4 \\ 3, & \text{if } n \equiv 2 \pmod 4 \end{cases}$$

and assign $\varphi(v_1 v_0) = 1$. Then, we have

$$S_\varphi(v_1) = \begin{cases} 4, & \text{if } n \equiv 0 \pmod 4 \\ 6, & \text{if } n \equiv 2 \pmod 4 \end{cases}$$

and for all i = 2, 3, ..., n,

$$S_\varphi(v_i) = \begin{cases} 3, & \text{if } i \equiv 1 \pmod 4 \\ 5 & \text{if } i \equiv 0 \text{ (or)} 2 \pmod 4 \\ 7, & \text{if } i \equiv 3 \pmod 4 \end{cases}$$

and $S_\varphi(v_0) = 1$.

This proves that the weights of any two adjacent vertices of $(Pn)_n$ is relatively prime and so the graph $(Pn)_n$ is coprime irregular .[For instance, the graph $(Pn)_4$ and the coprime irregular egde weighting under φ is given Figure 6.]

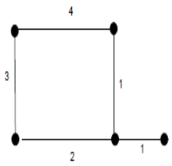

Figure 6: The graph $(Pn)_4$ and its coprime irregular edge-weighting under ϕ

Case(ii): *n* is odd

For all $1 \le i \le n-1$, let

$$\varphi(v_i v_{i+1}) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod 4 \\ 2, & \text{if } i \equiv 2 \pmod 4 \\ 3, & \text{if } i \equiv 3 \pmod 4 \\ 4, & \text{if } i \equiv 0 \pmod 4 \end{cases}$$

and

$$\varphi(v_1 v_n) = \begin{cases} 1, & \text{if } n \equiv 1 \pmod 4 \\ 3, & \text{if } n \equiv 3 \pmod 4 \end{cases}$$

and also assign

$$\varphi(v_1 v_0) = \begin{cases} 3, & \text{if } n \equiv 3 \pmod 4 \\ 5, & \text{if } n \equiv 1 \pmod 4 \end{cases}.$$

Then, we have

$$S_\varphi(v_0) = \begin{cases} 3, & \text{if } n \equiv 1 \pmod 4 \\ 5, & \text{if } n \equiv 3 \pmod 4 \end{cases}$$

and for all i = 2, 3, ..., n,

$$S_\varphi(v_i) = \begin{cases} 3, & \text{if } i \equiv 2 \pmod 4 \\ 5 & \text{if } i \equiv 1 \text{ (or)} 3 \pmod 4 \\ 7, & \text{if } i \equiv 0 \pmod 4 \end{cases}$$

Now it is easy to observe that the weights of any two adjacent vertices of $(Pn)_n$ is coprime and so φ is coprime irregular edge weighting of $(Pn)_n$. Hence the graph $(Pn)_n$ is

coprime irregular. [For example, the graph $(Pn)_3$ and its coprime irregular edge weighting φ is shown in Figure 7.]

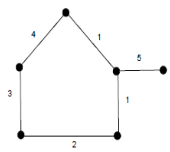

Figure 7: The graph $(Pn)_5$ and its coprime irregular edge-weighting under ϕ

Conclusion

In this paper, we proved some special classes of graphs such as friendship graphs, Dutch windmill graph, double quadrilateral snakes and Pan Graphs are coprime irregular graphs. However, there is a wide scope for further research in this topic. Here we list some of them.

1. Obtaining a characterization of coprime irregular graphs.

2. Characterizing trees which are coprime irregular is worthy trying.

3. Obtain more families of coprime irregular graphs using graph operations.

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