A New Approach of Generalized Pre-Closed Sets With Respect to an Ideal

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Abstract— **An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. The concept of generalized closed sets was introduced by Levine. In this paper, I introduce and investigate the concept of generalized pre-closed sets with respect to an ideal**

Keywords— **g-closed sets, gp-closed sets, Ig-closed sets, Igp-closed sets, ideal, Topological spaces.**

I. Introduction

Indeed ideals are very important tools in General Topology. It was the works of Newcomb[8], Rancin [9], Samuels [10] and Hamlet and Jankovic (see [1, 2, 3, 4, 5]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology. A nonempty collection I of subsets on a topological space (X,τ) is called a topological ideal [6] if it satisfies the following two conditions:

1. If $A \in I$ and $B \subset A$ implies $B \in I$ (heredity)
2. If $A \in I$ and $B \in I$, then $A \cup B \in I$ (finite additivity)

If A is a subset of a topological space (X,τ), cl(A) and int(A) denote the closure of A and the interior of A, respectively. Let $A \subset B \subset X$. Then $cl_B(A)$ (resp. $int_B(A)$) denotes closure of A (resp. interior of A) with respect to B. Levine [7] introduced the concept of generalized closed sets. This notion has been studied extensively in recent years by many topologists.

II. PRELIMINARIES

Definition 2.1. A subset A of a topological space (X,τ) is said to be generalized closed (briefly g-closed) if $cl(A) \subset U$ whenever $A \subset U$ and U is open in (X,τ).

Definition 2.2. A subset A of a topological space (X,τ) is said to be generalized pre-closed (briefly gp-closed) if $pcl(A) \subset U$ whenever $A \subset U$ and U is open in (X,τ).

Definition 2.3. Let (X,τ) be a topological space and I be an ideal on X. A subset A of X is said to be generalized closed with respect to an ideal (briefly Ig-closed) if and only if $cl(A) - U \in I$, whenever $A \subset U$ and U is open in (X,τ).

In this paper, I introduce and study the concept of gp-closed sets with respect to an ideal, which is the extension of the concept of g-closed sets.

III. GENERALIZED PRE-CLOSED SETS WITH RESPECT TO AN IDEAL

Definition 3.1 Let (X,τ) be a topological space and I be an ideal on X. A subset A of X is said to be generalized pre-closed with respect to an ideal (briefly Igp-closed) if and only if $pcl(A) - U \in I$, whenever $A \subset U$ and U is open in (X,τ).

Remark 3.1 Every g-closed set is Igp-closed, but the converse need not be true, as this may be seen from the following example.

Example 1. Let X={a, b, c}with topology ={Φ, {a},{c},{a,c}, X} and I={Φ, {a}, {b}, {a, b}}.Clearly, the set {a} is Igp-closed but not g-closed in (X,τ).

Remark 3.2 Every gp-closed set is Igp-closed, but the converse need not be true, as this may be seen from the following example.

Example 2. Let X={a, b, c}with topology ={Φ, {a},{c},{a,c}, X} and I={Φ, {a}, {b}, {a, b}}.Clearly, the set {a,c} is Igp-closed but not gp-closed in (X,τ).

Theorem 3.1. A set A is Igp-closed in (X,τ) if and only if $F \subset pcl(A) - A$ and F is closed in X implies $F \in I$.

Proof. Let A be Igp-closed. Suppose that $F \subset pcl(A) - A$. If F is closed then $A \subset X - F$. Since A is Igp-closed. Then $pcl(A) - (X - F) \in I$. But $F \subset pcl(A) - (X - F)$ and hence $F \in I$.Conversely, Assume that $F \subset pcl(A) - A$ and F is closed in X implies that $F \in I$. Suppose $A \subset U$ and U is open. Then $pcl(A) - U = pcl(A) \cap (X-U) \subset pcl(A) - A$. But $pcl(A) - U$ is closed. By assumption, $pcl(A) - U \in I$. Hence A is Igp-closed

Theorem 3.2. If A and B are Igp-closed sets of (X,τ), then their union $A \cup B$ is also Igp-closed.

Proof. Let A and B be Igp-closed sets in (X,τ).If $A \cup B \subset U$ and U is open, then $A \subset U$ and $B \subset U$. Since A and B is Igp-closed sets. Then $pcl(A) - U \in I$ and $pcl(B) - U \in I$

 DOI : 10.35291/2454-9150.2020.0453

ISSN : 2454-9150 Vol-06, Issue-03, June 2020

and hence $pcl(A \cup B) - U = (pcl(A)-U) \cup (pcl(B)-U) \in I$. That is $A \cup B$ is Igp-closed.

Theorem 3.3. If A and B are Igp-closed sets of (X,τ), then their intersection $A \cap B$ is also Igp-closed.

Proof. Let A and B be Igp-closed sets in (X,τ).Then $pcl(A) - U \in I$ whenever $A \subset U$ and U is open and $pcl(B) - U \in I$ whenever $B \subset U$ and U is open and hence $pcl(A \cap B) - U = (pcl(A) - U) \cap (pcl(B)-U) \in I$ whenever $A \cap B \subset U$ and U is open . That is $A \cap B$ is Igp-closed.

Theorem 3.4. If A is Igp-closed and $A \subset B \subset pcl(A)$ in (X,τ), then B is Igp-closed in (X,τ).

Proof. Let A is Igp-closed and $A \subset B \subset pcl(A)$ in (X,τ).Suppose $B \subset U$ and U is open. Then $A \subset U$. Since A is Igp-closed, then we have $pcl(A)-U \in I$. Now $B \subset pcl(A)$. This implies that $pcl(B)-U \subset pcl(A)-U \in I$. Hence B is Igp-closed in (X,τ).

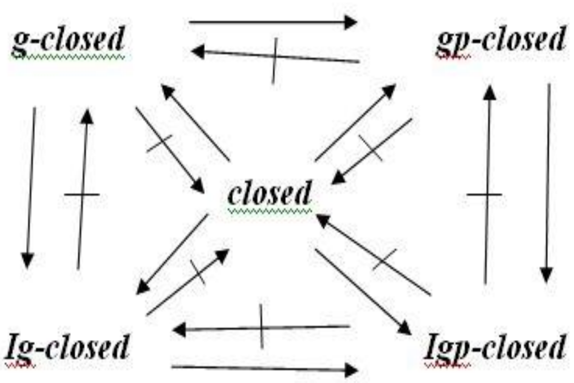
Theorem 3.5. Let $A \subset Y \subset X$ and A be Igp-closed in (X,τ). Then A is Igp-closed relative to the subspace Y of X, with respect to the ideal $I_Y = \{F \subset Y: F \in I\}$.

Proof. Let $A \subset U \cap Y$ and U be open in (X,τ), then $A \subset U$. Since A is Igp-closed in (X,τ), we have $pcl(A)-U \in I$. $(pcl(A)\cap Y) - (U\cap Y) = (pcl(A)-U)\cap Y \in I$, whenever $A \subset U \cap Y$ and U is open. Hence A is Igp-closed relative to the subspace Y.

Theorem 3.6. Let A be an Igp-closed set and F be a closed set in (X,τ), then $A \cap F$ is an Igp-closed set in (X,τ).

Proof. Let A be an Igp-closed set and in (X,τ).Then $pcl(A) - U \in I$, whenever $A \subset U$ and U is open. Let F be a closed set in (X,τ). Since Every closed set is Igp-closed. Hence F is Igp-closed. By theorem 3.3, intersection of any two Igp-closed sets is also Igp-closed. Thus $A \cap F$ is an Igp-closed set in (X,τ).

Remark 3.3. The following diagram holds for any subset of a topological space (X,τ) with respect to an ideal



Definition 3.2. Let (X,τ) be a topological space and I be an ideal on X. A subset $A \subset X$ is said to be generalized pre-open with respect to an ideal (briefly Igp-open) if and only if X–A is Igp-closed.

Theorem 3.7. A set A is Igp-open in (X,τ) if and only if $F-U \subset pint(A)$, for some $U \in I$, whenever $F \subset A$ and F is closed.

Proof. Let A be Igp-open. Suppose $F \subset A$ and F is closed. We have $X - A \subset X - F$. Since A is Igp-open Then $pcl(X-A) \subset (X-F) \cup U$, for some $U \in I$. This implies $X-((X-F) \cup U) \subset X-(pcl(X-A))$ and hence $F - U \subset pint(A)$.Conversely, assume that $F \subset A$ and F is closed imply $F-U\subset pint(A)$, for some $U\in I$. Consider an open set H such that $X-A \subset H$. Then $X-H \subset A$. By assumption, $(X-H)-U\subset pint(A) = X-pcl(X-A)$. This gives that $X - (H \cup U) \subset X-pcl(X-A)$. Then, $pcl(X-A) \subset H \cup U$, for some $U\in I$. This shows that $pcl(X-A)-G\in I$. Hence X–A is Igp-closed.

Theorem 3.8. If $A \subset Y \subset X$, A is Igp-open relative to Y and Y is Igp-open relative to X, then A is Igp-open relative to X.

Proof. Suppose $A \subset Y \subset X$, A is Igp-open relative to Y and Y is Igp-open relative to X. Suppose $H \subset A$ and H is closed. A is Igp-open relative to Y, by Theorem 3.7, $H-U \subset pint_Y(A)$, for some $U \in I$. This implies there exists an open set G such that $H-U \subset G \cap Y \subset A$, for some $U \in I$. Since Y is Igp-open relative to X, $H \subset Y$ and H is closed; we have $F-V \subset pint(Y)$, for some $V \in I$. This implies there exists an open set K such that $H-V \subset K \subset Y$, for some $V \in I$. Now $H - (U \cup V) \subset (H-U) \cap (H-V) \subset G \cap K \subset G \cap Y \subset A$. This implies that $H - (U \cup V) \subset pint(A)$, for some $U \cup V \in I$ and hence A is Igp-open relative to X.

Theorem 3.9. Let $pint(A) \subset B \subset A$ and A be Igp-open in (X,τ), then B is Igp-open in X.

Proof. Let $pint(A) \subset B \subset A$ and A be Igp-open in (X,τ). Then $X-A \subset X-B \subset pcl(X-A)$ and X–A is Igp-closed. By Theorem 3.4, X–B is Igp-closed and hence B is Igp-open.

Theorem 3.10. A set A is Igp-closed in (X,τ) if and only if pcl(A)–A is Igp-open.

Proof. Suppose $F \subset pcl(A)-A$ and F is closed. Then $F \in I$.This implies that $F-U = \Phi$ for some $U \in I$. Clearly, $F-U \subset pint(pcl(A)-A)$. By Theorem 3.7, pcl(A)–A is Igp-open.Conversely, Suppose $A \subset G$ and G is open in (X,τ). Then $pcl(A)\cap(X-G) \subset pcl(A) \cap (X-A) = pcl(A)-A$. By hypothesis, $(pcl(A)\cap(X-G)) - U \subset pint(\alpha cl(A)-A) = \Phi$ for some $U \in I$. This implies that $pcl(A) \cap (X-G) \subset U \in I$ and hence $pcl(A)-G \in I$. Thus, A is Igp-closed.

Theorem 3.11. Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be continuous and closed. If $A \subset X$ is Igp-closed in X, then f(A) is f(I)-gp-closed in (Y,σ), where $f(I) = \{f(U): U \in I\}$.

 DOI : 10.35291/2454-9150.2020.0453

Proof. Let A ⊂ X and A be Igp-closed. Suppose f(A) ⊂ G and G is open. Then A ⊂ f^{-1}(G). By definition, pcl(A) – f^{-1}(G) ∈I and hence f(pcl(A)) – G ∈ f(I). Since f is closed, pcl(f(A)) ⊂ pcl(f(pcl(A))) = f(pcl(A)). Then pcl(f(A)) – G ⊂ f(pcl(A)) – G ∈ f(I) and hence f(A) is f(I)-gp-closed.

IV. Conclusion

Our work is an step forward to strengthen the theoretical foundation of topological spaces of generalized pre-closed with respect to an ideal. For further work one can quickly go for application of these theoretical developments, I list some important topics for further theoretical work.

Acknowledgment

The author is grateful to the editor and the referee(s) for their fruitful comments, valuable suggestions and careful corrections.

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 DOI : 10.35291/2454-9150.2020.0453