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Roughness in G-modules and its Properties

*Aklesh Kumar, #Amit Kumar, #Dr. Sunil Kumar Sah

*Research Scholar & Assistant Professor, Dept of Mathematics, S.S.V. College Kahalgoan (Bihar) India.

#Research Scholar & Programmer, Govt. Polytechnic Jamui, T.M B. U, Bhagalpur (Bihar), India.

$SPM, Jharkhand, India & Ex-Faculty Member, UDCA, Computer Centre, T.M B. U, Bhagalpur (Bihar), India.

*kumarakleshssv@gmail.com, #amitmcapatna74@gmail.com, $drsksahtmbu@gmail.com

Abstract:-Rough set theory is direct mathetical approach to deal with uncertainly and indeterminateness in information systems. By connecting this theory with abstract algebra, many rough algebraic structures were introduced. In this paper we shall first introdue the concept of rough G-modules, homomorphism in rough G-modules and prove some related important results. In this paper we consider G-module as the universal set and we introduce the notion of rough G-module with respect to a G-submodule of a G-module.

Keywords : Rough set, rough group, rough field, rough coset, rough G-modules, rough vector space, rough G-modules homomorphism, upper rough G-module, lower rough G-module.

I. INTRODUCTION

Rough set theory is generalizations of classical set theory. Z. Pawlak introduced rough set theory as a framework for the constructions of approximations of conceptswhen only incomplete information is available [1]. It has proved to be an effective mathematical tool to dead with vague, uncertain and imperfect knowledge. In this set theory the main concept is an equivalence relation and the equivalence classes and the building blocks for the construction of lower and upper approximations in terms of which is a rough set is defined.

In the area of mathematical research rough set theory with abstract algebra is an emerging trend. Some papers substitued an algebraic structure for the universal set and investigated the roughness in algebraic structures. Also some papers directly introduced the concepts of rough algebraic structure into an approximation space. The concepts of rough group, rough subgroup, rough quotient group, rough coset, rough normal subgroup and rough homomorphism an studied in [2,3,4]. Also roughness in rings and modules by B. Davvaz studied in [5,6]. Some properties of rough subrings and rough ideas are studied in [7,8] by P. Isaac. The concept of rough modules in an approximation space and investigated their properties in [9] by Q. Zhang.

In the 19th century the theory of group representation was developed by G. Frobenius. The works of Emmy Noether on representation theory led to the absorption of the theory of group representation into the study of modules over ring algebra. Modules theoretic approach especially group module structure has been extensively used for the study of group repretation. By S. Fernandez the concept of fuzzy G-module and its properties and studied in [10]. The aim of this paper is to introduce the concept of rough G-module in an approximation space and its some properties.

In section 3, we define the concepts of rough field and rough vector space and then introduce the motion of rough G-module. In section 2, we see some basic definition of rough algebraic structures and results that will be needed in the sequel. In section 4, we define the homomorphims of two rough G-modules and study some of its properties. In section 5, we conclude with possible future work in the area of G-modules.

II. PRELIMINARIES

In this section we define approximation space, rough group, rough subgroup, commutative rough group, rough coset, rough normal subgroup, rough quotient group.

Definition 2.1[1] A pair (μ,ψ) when$\mu\neq\phi$ and ψ is an equivalence relation on μ is known as approximation space .

Definition 2.2 [1] For an approximation space (μ,ψ) and a subset A of μ, then the sets

$$\overline{A} = \{x\in\mu \mid [x]\psi \cap A \neq\phi\}$$

$$\underline{A} = \{x\in\mu \mid [x]\psi \subseteq A\}$$

Boundary region i.e BN (X) = $\overline{A}$ _ $\underline{A}$ are known as respectively the upper approximation lower appoximation and boundary region of A in (μ,ψ).

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Definition 2.3[2] let (μ,ψ) is an approximation space and let ∗ be a binary operation on μ. A subset G of μ is said to be rough group if the following postulates are satisfies :-

(1) ∀a,b∈G, a∗b ∈G
(2) ∀a,b,c∈G, a∗(b∗c) = (a∗b)∗c
(3) ∀ a∈G, ∃ e∈G s.t a∗e = a = e∗a ; e is known as the rough identity.
(4) ∀ a∈G, ∃ b∈G s.t a∗b = e = b∗a ; b is known as the rough inverse of a.

Definition 2.4 [2] A non-empty subset H of a rough group G is said to be rough subgroup if it also a rough group itself with respect to the same binary operation on G.

Definition 2.5 [2] A n.a.s.c for a subset H of a rough group G to be a rough subgroup if

(i) ∀a,b∈ H, a*b ∈ H
(ii) ∀ a ∈ H, a-1∈ H

Definition 2.6[2] A rough group G is said to be commutative rough group if ∀ a, b ∈ G we have a*b = b*a .

Definition 2.7[2] Let G be a rough group and it be a rough subgroup of G. If we define a relationship O of elements of G asaobiff a-1*b ∈ H μ {e} then O is a compatible relation.

The rough left coset of H in G with respect to a∈G is the compatible category a*H = {a*h : h ∈H, a*h ∈ G} μ {a}. And rough right coset of H in G with respect to a ∈ G is the compatible category H*a = {h*a : h ∈H, h*a ∈ G} μ {a}.

Definition 2.8[2] A rough subgroup N of rough group G is said to be rough normal (invariant) subgroup if ∀ a ∈ G, a*N = N*a .

Definition 2.9[9] Let N be a rough normal subgroup of a rough group G and let G/N = {g*N : g ∈ G}. Then (g/N.*') be a rough group which is known as the rough quotient group of G with respect to N where the binary operation *' is defined as (g1*N) *' (g2*N) = (g1*g2) * N.

III. ROUGH G-MODULE

In this section we define the basic concepts of rough field , rough vector space , rough subspace and then introduce the notion of rough G-module.

Definition 3.1 An approximation space (μ,ψ) and +, * be two binary operation μ. A non-empty subset F of μ is said to be a rough field if it satisfied the following postulates;

(i) (F, +) is a rough commutative additive group.
(ii) (F, *) is a rough commutative multiplication group.
(iii) ∀ a, b, c ∈F , (a + b)*c = a*c + b*c and a*(b + c) = a*b + a*c .

Definition 3.2 Let (μ1,ψ) and (μ2,ψ2) be two approximation space with the binary operations + and * on μ1 and + on μ2.

Let F ⊆υ1 be a rough field and M ⊆μ2 be a rough commutative group then M is said to be a rough vector space over the rough field F if there is a mapping $\overline{F}_{\times}$ $\overline{M}_{\rightarrow}$ $\overline{M}$; (x,m) →xm such that

(i) x(m + n) = xm + xn
(ii) (x + y)m = xm + ym
(iii) (x*y)m = x(ym)
(iv) 1m = m Where x,y∈ F ; m,n∈ M and I is the rough multiplicative identity of F.

It can be easily verified that xo = o ;∀ x ∈K.

Definition 3.3 A non-empty subset N of M ie. N ⊆ M is said to be a rough subspace of M if an1 + n2∈ $\overline{N}$,∀ a ∈ F and n1,n2∈ N.

Definition 3.4 Let (μ1,ψ1) , (μ2,ψ2) and (μ3,ψ3) be the approximation spaces with the binary operation * on μ1, + on μ2 and + and * on μ3. Let G ⊆μ1 be a rough group. A rough vector space V ⊆μ2 over a rough field F ⊆μ3 is called a rough G-module if there is a mapping G × V →V ; (g,v) →g.v such that

(i) IG. ν = ν,∀ν∈V Where IG be the rough identity element of G.
(ii) (g*h). ν = g.(h.ν) ;∀ν∈ V ; g,h∈ G.
(iii) g.(f1ν1 + f2ν2) = f1(g.ν1) + f2(g.ν2) ; ∀ f1,f2∈ F; ν1, ν2∈ V; g ∈G.

It can be easily verified that g.o = o ;∀ g∈G.

Definition 3.5 A non-empty subset S of rough G-module M is said to be a rough G-module of M if

(i) S is a rough subspace of M.
(ii) g.s∈S ,∀ g ∈G and s ∈S.

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Theorem 3.6 IfA and B be two rough G-modules of M then $A \cap B$ is a rough G-submodule of M if $\overline{A} \cap \overline{B} = \overline{A \cap B}$

Proof Let a, b $\in A \cap B$

$\Rightarrow$a,b$\in$ A and a,b$\in$ B

$\Rightarrow$ ca + b $\in \overline{A}$ and ca + b $\in \overline{B}$

$\Rightarrow$ ca + b $\in \overline{A} \cap \overline{B} = \overline{A \cap B}$

Now consider g $\in$G and a $\in A \cap B$

$\Rightarrow$ a $\in$A and a $\in$ B

$\Rightarrow$g.a$\in \overline{A}$ and g.a $\in \overline{B}$

$\Rightarrow$g.a$\in \overline{A} \cap \overline{B} = \overline{A \cap B}$

$\therefore$ $A \cap B$ is a rough G-submodule of M.

IV. ROUGH G-MODULE HOMOMORPHISM

In this section we define G-module homomorphism, G-module isomorphism ,kernel of homomorphism and study some of its properties. Let (μ1,ψ1) , (μ2,ψ2) be two approximation spaces.

Definition 4.1 If M1 $\subset\mu$2 and M2$\subseteq\mu$2 be two rough G-modules then a mapping ϕ : $\overline{M}_1 \rightarrow \overline{M}_2$ is said to be a rough G-module homomorphism if (i) ϕ (f1m1 + f2m2) = f1ϕ (m1) + f2ϕ (m2)

and (ii) ϕ (g.m) = g.ϕ (m) ,$\forall$ f1,f2$\in$ F ; m,m1,m2$\in$ M and g $\in$G.

Definition 4.2 A rough G-module homomorphism ϕ : M1 $\rightarrow$ M2 is said to be a rough G-module isomorphism if ϕ : $\overline{M}_1 \rightarrow \overline{M}_2$ is both one-one and onto.

Definition 4.3 The two rough G-modules M1$\subseteq\mu$1 ,M2$\subseteq\mu$2and ϕ : M1$\rightarrow$M2 a rough G-module homomorphism.

Then {x $\in \overline{M}_1$:ϕ (x) = o} Where o is the rough identity element of M2 is said to be rough G-module homomorphism Kernel of ϕ denoted by Ker ϕ.

Theorem 4.4 The two rough G-modules M1$\subseteq\mu$1 , M2$\subseteq\mu$2 and ϕ : M1$\rightarrow$ M2 a rough G-module homomorphism.

7 Ker ϕ is a subset of M, then it is a rough G-submodule of M1.

Proof : Let a,b$\in$kerϕ and c $\in$K

$\Rightarrow$a,b$\in$ M1

$\Rightarrow$ ca + b $\in \overline{M}_1$

Also ϕ (ca + b) = c ϕ (a) + ϕ (b) = co + o = o

$\therefore$ ca + b $\in$Ker ϕ

Now, let a $\in$Ker ϕ and g $\in$G

$\Rightarrow$g.a$\in \overline{M}_1$

Also ϕ (g.a) = g.ϕ (a) = g.o = o

$\therefore$g.a$\in$Ker ϕ

Thus Ker ϕ is a rough G-submodule of M1.

Theorem 4.5 The two rough G-modules M1$\subseteq\mu$1 , M2$\subseteq\mu$2 and ϕ : M1$\rightarrow$ M2 a rough G-module homomorphism. Let S be a rough G-module of M1. Then ϕ (S) be a rough G-submodule of M2 if $\phi(\overline{S}) = \overline{\phi(S)}$.

Proof :Let y1,y2$\in\phi$ (S) , then $\exists$ x1, x2$\in$ S such that ϕ (x1) = y1 and ϕ (x2) = y2.

Consider cy1 + y2 = c ϕ (x1) + ϕ (x2)

= ϕ (cx1 + x2) $\in \phi(\overline{S}) = \overline{\phi(S)}$

$\therefore$ cy1 + y2$\in \overline{\phi(S)}$

Now , consider g $\in$G

g.y1 = g.ϕ (x1) = ϕ (g.x1) $\in \phi\overline{(S)} = \overline{\phi(S)}$

$\therefore$ g.y1$\in \overline{\phi(S)}$

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Thus ϕ (S) is a rough G-submodule of M2.

V. LOWER AND UPPER APPROXIMATIONS IN A G-MODULE

Definition 5.1 Let N be a G-submodule of M and X ⊆ M. The sets $\underline{N}(X) = \{ x \in M \mid x + N \subseteq X\}$ and $\overline{N}(X) = \{ x \in M \mid (x + N) \cap X \neq \phi\}$ and respectively called the lower and upper approximations of a set X with respect to the G-submodule N and (M,N) is called the approximation space.

Theorem 5.2 Let N be a G-submodule of M and X and Y be non-empty subsets of M. Then

(a) $\underline{N}(X) \subseteq X \subseteq \overline{N}(X)$

(b) $\underline{N}(X \cap Y) = \underline{N}(X) \cap \underline{N}(Y)$

(c) $\overline{N}(X \cup Y) = \overline{N}(X) \cup \overline{N}(Y)$

(d) $\underline{N}(X \cup Y) = \underline{N}(X) \cup \underline{N}(Y)$

(e) $\overline{N}(X \cap Y) \subseteq \overline{N}(X) \cap \overline{N}(Y)$

(f) If X ⊆ Y then $\underline{N}(X) \subseteq \underline{N}(Y)$,

$\overline{N}(X) \subseteq \overline{N}(Y)$

(g) $\overline{N}(X) = X + N$

(h) If N ⊆ X then $N \subseteq \underline{N}(X)$ and $\underline{N}(X) \neq \phi$

(i) $\overline{N}(X + Y) = \overline{N}(X) + \overline{N}(Y)$

(j) $\underline{N}(X + Y) = \underline{N}(X) + \underline{N}(Y)$

(k) $\underline{N}(k\,x) = k\,\underline{N}(x)$; $\forall\, k \in K$

(l) $\overline{N}(k\,x) = k\,\overline{N}(x)$; $\forall\, k \in K$

(m) $\underline{N}(g\,.\,x) = g\,.\,\underline{N}(x)$; $\forall\, g \in G$

(n) $\overline{N}(g\,.\,x) = g\,.\,\overline{N}(x)$; $\forall\, g \in G$

Proof : We prove only (m) and (n). The proof of other conclusions are similar to the conclusions in [16].

Let x ∈ g. $\underline{N}(X) \Rightarrow y \in \underline{N}(X)$ such that x = g.y

Since (y + N) ⊆ X we have

g.(y + N) ⊆ g.X ⇒ g.y + g.N ⊆ g.X

⇒ x + N ⊆ g.X ⇒ $x \in \underline{N}(g\,x)$

Conversely, if $x \in \underline{N}(g.x)$ then x + N ⊆ g.X

Now , x + N = g.(g-1.x) + N = g.(g-1.x + N) ⊆ g.X

⇒ g-1.x + N ⊆ X ⇒ g-1.x ∈ $\underline{N}(X)$ ⇒ x ∈ g. $\underline{N}(X)$

Thus $\underline{N}(g.x) = g.\underline{N}(X)$.

From (g) $\overline{N}(g.X) = g.X + N = g.X + g.N = g.(X + N) = g.\overline{N}(X)$.

Definition 5.3 Let (M,N) be an approximation space, a non-empty subset X of M is called a N – lower rough G-submodule, if $\underline{N}(X)$ is a G-submodule of M. X is known as a N – upper rough G-submodule, if $\overline{N}(X)$ is a G-submodule of M.X is known as a N – rough G-submodule if both $\underline{N}(X)$ and $\overline{N}(X)$ are G-submodule of M.

Theorem 5.4 Let A and B be two G-submodule of M. Then $\underline{A}(B)$ is non-empty then $\underline{A}(B) = B$

Proof : We know that $\underline{A}(B) \subseteq B$.

Now we will show that $B \subseteq \underline{A}(B)$

Since $\underline{A}(B) \neq \phi$ then ∃ an element $x \in \overline{A}(B)$

⇒ (x + A) ⊆ B.

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Since o ∈ A we have x + o = x ∈ B then –x ∈ B.

So A = -x + x +A ⊆ -x + B = B

Let b ∈ B since A ⊆ B and B is a G-submodule

We have b + A ⊆ B ⇒ b ∈ $\underline{A}$(B)

i.e. b ∈ B we have b ∈ $\underline{A}$(B) ⇒ B ⊆ $\underline{A}$(B)

Thus $\underline{A}$(B) = B

VI. CONCLUSION

In this paper, we have introduced the concept of rough G-module. We have also defined homomorphism in rough G-module, Rough G-submodule and investigated some properties. The theory and properties of rough sets can be extended to other field in the traditional module theory in a similar manner. The theory of rough sets and fuzzy sets can be extended in a similar way to other algebraic structures.

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